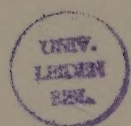


Receivers for CPM

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1985



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Front cover:

The figure on the front page shows phase variations with time for a binary CPM scheme with a 3RC pulse and modulation index $h=1/2$. For those unfamiliar with these notions, see text.

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RECEIVERS FOR CPM

SUMMARY

ABSTRACT

The problem of finding simple and good receivers for constant amplitude Continuous Phase Modulation (CPM) schemes is addressed in this thesis. CPM signals are suitable for satellite and mobile radio channels. The receivers considered are of coherent type, and the channel is a white Gaussian noise or a Rayleigh fading channel.

By using the properties of the CPM signals it is possible to find receivers with reduced complexity compared to the optimum receiver. Four different types are considered. Two of them are general receivers that works for all CPM schemes, while two of them only works for binary schemes with modulation index $h=1/2$. It is shown that the complexity can be significantly reduced while still maintaining a good error probability behaviour, i.e. the loss compared to the optimum receiver is low. Thus, digital constant amplitude modulation schemes with a more narrow bandwidth and much lower sidelobes than e.g. Minimum Shift Keying (MSK), with simple receivers can be constructed.

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I. INTRODUCTION

In recent years the demand for communication by digital signaling has increased considerable. Digital communication systems have many advantages over other techniques. One advantage is the very low error probability and high fidelity that can be obtained with error detection and correction. Another advantage is the privacy that can be obtained by using encryption. A major advantage is the flexibility of digital hardware implementation which permits the use of microprocessors, digital switching and very large scale integrated (VLSI) circuits.

Applications for digital communication systems are among others for satellite communication and also for mobile radio communications. In some of these applications, the available radio frequency spectrum is limited. Thus it is important that each user consumes only a small portion of the available spectrum. In this way many users can be accommodated in a given frequency band. This means that the signals should be narrow-banded.

The digital modulation systems used today are mainly Phase Shift Keying (PSK), Quaternary Phase Shift Keying (QPSK) and Minimum Shift Keying (MSK). The drawback of these systems is the wide bandwidth they occupy. Thus these signals need to be filtered before they are transmitted through the channel. This introduces both amplitude and phase distortion. A constant amplitude is however desired, since many amplifiers and repeaters are highly nonlinear. The distortion results in deteriorated performance on the receiver side. To compensate for this the transmitted power has to be increased, but in some applications the available power is limited. Alternatively error control coding can be used but then the bandwidth of the signal is increased, if the information rate and the modulation scheme are kept unchanged.

Recently much attention has been paid to power efficient digital modulation schemes that do not have the above disadvantages [1], [2], [10], [11], [17]. One class of such schemes is the Continuous Phase Modulation (CPM) schemes [1]. These schemes have constant envelope and a continuous phase variation. Controlled intersymbol interference is introduced to produce very smooth phase variations. Together with the continuous phase property this results in narrow power spectra and very low sidelobes. Thus these schemes are well suited for satellite and mobile radio channels.

The drawback of the CPM schemes is receiver complexity. Due to the correlation introduced in the phase, the receiver has to exploit the phase structure, to be able to detect the transmitted symbols in an optimum manner [1]. Optimum receivers are derived in [1], [2].

In this thesis various receivers are investigated and analysed. The channel is assumed to be a white Gaussian noise channel or a Rayleigh fading channel. All the receivers have in common, that they have reduced complexity compared to the optimum receiver. They are all of coherent type, which means that the receiver must know the carrier phase exactly. This can not be obtained in practice, but by using one or more Phase Locked Loops (PLL's) a very good estimate can be obtained in many applications.

II. REDUCED-COMPLEXITY VITERBI DETECTORS

Previously it has been shown that the CPM signals can be detected by means of the Viterbi algorithm in an optimum manner [1]. The receiver consists of a bank of matched filters and a Viterbi processor. The number of states and filters are however large for the CPM schemes based on smooth pulses. Thus the receiver is complex for CPM schemes with power spectrum having narrow mainlobe and very low sidelobes.

In this thesis it has been observed that the phase tree of a complex scheme, i.e. the ensemble of transmitted phases, can be approximated with a phase tree of a less complex CPM scheme. Hence the transmitted scheme can be approximated with another less complex signaling scheme. Thus the receiver for this less complex scheme can be used also to the more complex signals. The approximation should however not be too large. A Euclidean distance giving the asymptotic error probability (large signal-to-noise ratios) is derived and used for calculating the performance of these reduced-complexity receivers. It is also shown that this distance can be tightly upper bounded, by identifying so called merges in the receiver phase tree.

A specific class of receiver phase trees has been considered and the reduced-complexity receiver has been asymptotically optimized within this class. In fact the upper bound on the Euclidean distance has been maximized. These optimum receivers have been found for various transmitted signals with various values of the modulation index, for several complexity reduction factors.

The reduced-complexity receiver is a general receiver and can be used for all CPM schemes, both binary and M-ary, all modulation indices and all type of transmitted frequency pulses. In fact the Euclidean distance analysis hold for all types of signals, not only CPM signals. Hence, this performance measure can always be used when a signal set is approximated by another signal set. The results show that for many CPM schemes a complexity reduction of 4 can be obtained with a power degradation of about 0.5dB. The results also show that the receiver pulse shape is not especially sensitive to small variations. This new idea of approximating a signal set with another signal set is not the only way to find simplified receivers.

Another way of reducing the complexity of the receiver to a CPM scheme has been proposed by Simmons and Wittke [3]. The idea is to let the Viterbi decoder use a predetermined processing order and to work with a reduced number of survivor signals [3]. This receiver is analysed for piecewise linear phase pulses. It is claimed that the decoder can perform equally well as the optimum receiver for large signal-to-noise ratios for the schemes considered, while being computationally faster. It is not known how this receiver compares with ours since it is not analysed for the smooth CPM schemes.

The work on reduced-complexity receivers has been presented at two major communication conferences, in part at NTC'81 [4] and in part at GLOBECOM'83 [5]. The general idea of reducing the complexity and the distance analysis is published in part in Electronics Letters [6]. It is also accepted in part for publication in the IEEE Trans. on Communications [7].

III. PARALLEL MSK-TYPE RECEIVERS

When the modulation is binary and the modulation index is $h=1/2$, the CPM schemes have some nice properties that can be used to simplify the receiver. MSK is a simple scheme of this type. MSK is actually linear modulation in the quadrature arms. Thus a linear receiver can be employed in the quadrature arms. This receiver consists of only two filters and very simple decision logic. When the more complex CPM schemes are used the transmission is no longer linear in the quadrature arms [1]. The quadrature signals can however still be detected in a linear receiver of the above type. The problem is to find the receiver filter optimizing the error probability.

Many authors [8]-[11], [13]-[15] have worked parallel on this problem and various receiver filters have been analysed. We have derived a distance measure giving the performance for large SNR's for this parallel MSK-type receiver on the Gaussian channel. An exact expression for the error probability in detecting the phase node has been derived both for the white Gaussian noise channel and also for the Rayleigh fading channel. The symbol error probability can be calculated from this error probability in detecting the phase node. Diversity schemes have also been considered. Some alternative methods for analysing these MSK-type receivers are found in the literature [9]-[11], [13].

Various attempts have been made to find the receiver filter giving the best performance at various SNR's. [10], [14]. A major contribution in optimizing the receiver filters for large SNR's has been given by Galko et.al. [8]. In this thesis some of this theory is used and an alternative algorithm giving the asymptotic optimum filter is given. By using the properties of the signals numerical methods are developed that give the optimum filter also for intermediate signal-to-noise ratios as well as for the Rayleigh fading channel.

Various numerical results are given for many different pulse shapes. These results show that very bandwidth efficient schemes can be used with the MSK-type receiver with an asymptotic loss less than 1dB compared to MSK. The loss at intermediate error probability levels is even smaller. The drawback for this receiver is that it can only be used for binary $h=1/2$ CPM schemes. Many of the interesting schemes are however in this class, and thus it is a minor disadvantage.

The work on the parallel MSK-type receiver has been presented in part at ICC'82 [14] and is accepted for presentation in part at ICC'84 [15]. It is also submitted for publication in the Proceedings of IEE, Part F [16].

IV. SERIAL MSK-TYPE DETECTION

The MSK-type detector can also be realized as a serial type of detector. This has been observed by Amoroso et.al. [17] and Ziemer et.al. [18] for MSK. They have shown that this serial receiver is less sensitive to phase errors and they conclude that this receiver has advantages in implementation.

This idea has been generalized to partial response CPM schemes in this thesis. A Euclidean distance giving the asymptotic performance both for perfect synchronization and when timing and phase errors are present has been derived as well as an exact error probability in detecting the phase node. The similarities with the parallel MSK-type receiver are discussed. The optimum filters for various signal-to-noise ratios are calculated. These can easily be calculated from the optimum filters for the parallel MSK-type receiver.

Numerical results are given for a variety of transmitted schemes, receiver filters, phase and timing errors. It is shown that the advantages for the serial receiver over the parallel receiver to partial response CPM schemes are the same as for MSK, i.e. in sensitivity to phase errors. It is also shown that the filters for the serial receiver can be calculated from the filter for the parallel receiver. Furthermore under the assumption of perfect synchronization both the receivers have the same performance with these filters. It is also shown that the optimum filters for the serial receiver can be calculated from the optimum filter for the parallel receiver. Thus the optimum performance is also equal. Thus all the results under the assumption of perfect synchronization given in part II of this thesis also apply to the serial receiver.

This work has been presented at two conferences, i.e. MELECON'83 [19] and GLOBECOM'83 [20]. It is also submitted for publication in the IEEE Trans. on Communications [21].

V. AMF RECEIVERS

The optimum correlation receiver for M-ary schemes has previously been derived by Osborne and Luntz [2]. If the signaling is of full response type this receiver can be approximated by an Average Matched Filter (AMF) receiver for low signal-to-noise ratios as noticed by Hirt and Pasupathy [12]. This AMF receiver has been analysed for various M-ary full response schemes.

In this thesis the idea of an AMF receiver is generalized to partial response CPM schemes. These schemes have memory more than through the continuous phase. The signal in a given symbol interval also depends on previous symbols and thus special consideration has to be given to these symbols. A multiple-symbol AMF receiver, having a larger filterbank, has also been introduced. This is a new idea of increasing the performance of the AMF receiver. A Euclidean distance giving the performance for large SNR's has been introduced. Upper bounds on the symbol error probability are given for the general case of M-ary CPM schemes. This error probability result is exact for binary modulation. Comparisons of the complexity are also given for the receivers considered in this thesis. Numerical results are given for a variety of schemes.

The results show that the generalized AMF receiver performs equal to the optimum receiver for schemes with low complexity and low modulation indices. For the more complex schemes the receiver has a tendency to choose wrong filters and this degrades the performance.

This work has been presented in part at the International Symposium on Information Theory 1982 [22] and has been submitted in part to GLOBECOM'84 [23].

VI. DISCUSSION AND CONCLUSIONS

This thesis presents different types of receivers with reduced complexity compared to the optimum receiver for CPM schemes. All the receivers given are of coherent type. The receivers have been analysed both at large signal-to-noise ratios and some also for intermediate signal-to-noise ratios. All except the AMF receiver have been optimized within the subclass they belong and very good performance has been obtained. For some schemes a performance close to optimum has been obtained and in many cases the complexity has been significantly reduced but still yielding a performance loss less than 1dB.

The applications for the receivers considered in this thesis are on mobile radio and satellite channels. With the development of VLSI circuits the Viterbi detector might be implemented on a chip and the filtering done by using SAW (Surface Acoustic Wave) devices. Thus the complexity of this receiver is expected to be a manageable problem.

The MSK-type receivers are very simple and they are used for MSK today in various applications. Especially the serial MSK-type receiver is used in very high speed transmission (760Mb/s) at the 14.7GHz carrier [24]. Thus it is expected that this can as well be done for partial response schemes. The problem might be the serial generation of a partial response scheme. MSK-type receivers are also considered for land mobile radio applications [10], [11].

The AMF receiver has a large filterbank in some cases. Work on filters based on SAW devices for full response schemes has been done [25]. It is expected that these techniques could be used also for the AMF receivers presented in this thesis.

In this thesis perfect synchronization is assumed in many cases. It is not clear how well this can be obtained because it depends highly on the channel. This thesis does not consider any synchronization schemes.

In some applications phase coherence can not be established. Parallel with this work, some work considering noncoherent receivers have been done [26].

This work shows that the same asymptotic performance as with coherent receivers can be obtained with noncoherent receivers. At present, work is done on differential and discriminator detection [27].

Many of the detection properties for coherent detectors are investigated under the assumption of perfect synchronization in this thesis. The synchronization problems are however not solved in great detail in the literature and need further work. Co-channel interference, i.e. crosstalk for different channel spacings should also be considered in more detail. This is a problem in e.g. cellular mobile radio systems.

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RECEIVERS FOR CPM

PART I

A CLASS OF REDUCED-COMPLEXITY VITERBI DETECTORS

ABSTRACT

Partial response continuous phase modulated signals (sometimes also referred to as partial response digital FM or correlative phase modulation) give constant envelope digital modulation schemes with excellent power spectra. Both narrow main-lobe and low spectral tails can be achieved. When these signals are detected in an optimum coherent maximum likelihood sequence detector (Viterbi detector), power efficient schemes can also be designed, sometimes at the expense of receiver complexity.

This part of the thesis defines a general class of simple Viterbi detectors with reduced-complexity compared to the optimum case. The asymptotically optimum reduced-complexity receiver in a specific class of receivers is found for a variety of transmitted schemes with various modulation indices. This is done for various complexity reduction factors. A new distance measure is introduced for the performance analysis. Smooth schemes based on raised cosine pulses are analysed and simulated for the case of simplified reception. A graceful performance degradation occurs with the reduction of complexity.

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I. INTRODUCTION

Due to limited available radio frequency spectrum, spectrally efficient digital modulation schemes have recently gained increased attention. For some applications, notably where nonlinearities are present in repeaters and power amplifiers, it is favourable if the signaling scheme can be of constant envelope type, [8]. It has recently been demonstrated that constant envelope digital modulation methods with excellent power spectra can be constructed, see for example [2], [17], [19]. A wide class of such schemes are contained among the Continuous Phase Modulation (CPM) schemes. By using partial response techniques and keeping the information carrying phase continuous, the power spectra become very attractive, see examples in [2], [19].

It has also been shown that schemes with combined good spectral properties and detection properties can be obtained in the CPM-family [2]. The schemes with attractive combined spectra and error probability often require a maximum likelihood sequence detector [12] implemented by means of a Viterbi detector. The optimum receiver exploits the structure of the transmitted phase trajectories, and uses the Viterbi algorithm to perform maximum likelihood sequence estimation (MLSE). These receivers typically consist of a filterbank followed by a Viterbi processor, where sometimes the number of states is quite large. This means that the receiver is often complex.

In this part a reduced-complexity Viterbi detector is considered. It is shown that a good performance can be obtained with a Viterbi receiver, where the number of filters and states are reduced. The idea is to approximate the phase tree with a phase tree based on a shorter frequency response, and use the Viterbi receiver for this shorter scheme instead as a receiver. This scheme, used by the receiver, is optimized to give the best possible performance for large signal-to-noise ratios (SNR's).

The aim of part I is to find the optimum reduced-complexity receiver for large SNR's in a specific class to a transmitted scheme, given a specific complexity reduction. The error probability is difficult to calculate and also tight bounds on the error probability is difficult to derive [4]. Therefore a Euclidean distance concept is introduced. This distance gives the error probability performance for large SNR's. The minimum distance is optimized by means of numerical optimization routines on a computer. Any ana-

lytical results for the optimum reduced-complexity receiver have not been found and this seems difficult. Also some simulations on the error probability are presented.

Throughout part I coherent detection and perfect carrier recovery and synchronization is assumed. It is also assumed that the channel is an additive white Gaussian noise channel.

This part is organized in the following manner. In section II the transmitted scheme and the optimum receiver is described. The reduced-complexity receiver is presented in section III. The performance measure is given in section IV and section V presents some properties of the minimum Euclidean distance for the mismatched signal sets. The optimization procedure is explained in section VI. A variety of results, both minimum distance and simulated error probabilities, are given in section VII. In section VIII the tradeoff between complexity reduction and performance degradation is discussed and finally this part is terminated with a discussion and conclusions section.

II. THE MODULATION SCHEME

The approach to receiver simplification is introduced in section III. First we will give the definition of partial response continuous phase modulated signals [1], [2], [6]. These signals are also sometimes referred to as partial response digital FM signals [7], [9] or correlative phase modulation [13], [15], [16]. In this section the general optimum coherent receiver structure will also be given [7], [9].

First an introduction to the CPM signaling scheme is given [1], [2]. Let $\underline{\alpha} = \dots, \alpha_{n-2}, \alpha_{n-1}, \alpha_n, \alpha_{n+1}, \alpha_{n+2}, \dots$ be an infinitely long sequence of statistically independent data symbols, taking the values $\pm 1, \pm 3, \dots, \pm(M-1)$ with equal probability $1/M$. The transmitted constant envelope signal is given by

$$s(t, \underline{\alpha}) = \sqrt{\frac{2E}{T}} \cos(2\pi f_0 t + \varphi(t, \underline{\alpha}) + \varphi_0) \quad (1)$$

where the information carrying phase is

$$\varphi(t, \underline{\alpha}) = 2\pi h \sum_{i=-\infty}^{\infty} \alpha_i \cdot q(t-iT) \quad (2)$$

The symbol energy is denoted E , the symbol time T and the carrier frequency f_0 . h is the modulation index and φ_0 an arbitrary constant phase shift, and can be set to zero without loss of generality. Sometimes the bit energy E_b is used instead of the symbol energy E and the relation between them is

$$E = \log_2(M) \cdot E_b \quad (3)$$

The phase response $q(t)$ above, is defined by

$$q(t) = \int_{-\infty}^t g(\tau) d\tau \quad (4)$$

where $g(t)$ is the frequency response. A conceptual modulator is shown in figure 1 [7], [9]. This frequency response is normalized in such a way that the maximum absolute phase change $\varphi(t, \underline{\alpha})$ over any symbol interval T is $(M-1) \cdot h\pi$ radians. This is obtained if

$$\int_{-\infty}^{\infty} g(\tau) d\tau = \frac{1}{2} \quad (5)$$

The length and shape of the pulse $g(t)$ determines the performance of the modulation system. Figure 2 shows some examples of time limited and positive pulses $g(t)$ of types which will be considered below. The corresponding phase responses are also shown in figure 2. The 1REC scheme gives Minimum Shift Keying (MSK) for $h=1/2$. The 2REC is a partial response scheme [15], [16], [2] with linear phase trajectories. This scheme is sometimes also referred to as duobinary linear phase. The raised cosine pulse of length $L=3$ is defined by equation (7) below. For further details, see [1], [2].

The frequency responses of the transmitter, used in this part are the Rectangular scheme of length L symbol intervals, denoted LREC, and given by

$$g(t) = \begin{cases} \frac{1}{2LT} & ; \quad 0 \leq t \leq LT \\ 0 & ; \quad \text{otherwise} \end{cases} \quad (6)$$

and the Raised Cosine scheme of length L symbol intervals, denoted LRC, given by

$$g(t) = \begin{cases} \frac{1}{2LT} \left[1 - \cos\left(\frac{2\pi t}{LT}\right) \right] & ; \quad 0 \leq t \leq LT \\ 0 & ; \quad \text{otherwise} \end{cases} \quad (7)$$

Figure 3 shows the so-called phase tree for a scheme with a raised cosine pulse with $L=3$. The phase tree is the ensemble of $\varphi(t, \underline{\alpha})$, equation (2), with a common prehistory (i.e. $\alpha_i = M-1 = 1$, $i < 0$) up to $t=0$.

It is assumed that the pulse $g(t)$ has finite length LT , i.e. $g(t) \equiv 0$ for $t < 0$ and $t > LT$.

Since $g(t)$ is time limited, $q(t)$ is 0 for $t \leq 0$ and constant $q(LT)$ for $t \geq LT$. For positive pulses $g(t)$, e.g. the raised cosine pulses, $q(LT) = 1/2$, [2]. Thus considering the time interval $nT \leq t < (n+1)T$ the information carrying phase can be written as

$$\begin{aligned} \varphi(t, \underline{\alpha}_n) &= 2\pi h \sum_{i=-\infty}^n \alpha_i q(t-iT) = \\ &= 2\pi h \sum_{i=n-L+1}^n \alpha_i q(t-iT) + h\pi \sum_{i=-\infty}^{n-L} \alpha_i ; \quad nT \leq t < (n+1)T \end{aligned} \quad (8)$$

where $\underline{\alpha}_n = \dots, \alpha_{n-2}, \alpha_{n-1}, \alpha_n$.

Hence for given h and $g(t)$ and for any symbol interval n , the phase $\varphi(t, \underline{\alpha}_n)$ is uniquely defined by $\underline{\alpha}_n$, the correlative state vector $(\alpha_{n-1}, \alpha_{n-2}, \dots, \alpha_{n-L+1})$ and the phase state θ_n where

$$\theta_n = h\pi \sum_{i=-\infty}^{n-L} \alpha_i \text{ mod. } 2\pi \quad (9)$$

where mod. 2π means modulo 2π .

The number of correlative states is finite and equal to $M^{(L-1)}$. For rational modulation indices the phase tree is reduced to a phase trellis [7].

For $h = 2k/p$, where k and p are integers having no common divisor, there are p different phase states with values $0, 2\pi/p, 2 \cdot 2\pi/p, \dots, (p-1) \cdot 2\pi/p$.

The total number of states is

$$S = p \cdot M^{(L-1)} \quad (10)$$

Figure 3 shows the phase tree for a binary system with a raised cosine pulse of length $L=3$ (3RC) for $h = 4/5$. Phase states and correlative states are assigned to the nodes in the phase tree. The root node is arbitrarily given phase state 0. Figure 4 shows the state trellis.

Equation (8) can also be written

$$\begin{aligned} \varphi(t, \underline{\alpha}_n) = & 2\pi h \left[\sum_{i=n-L+1}^n \alpha_i q(t-iT) - \frac{1}{2} \sum_{i=n-L+1}^{n-L+j} \alpha_i \right] + \\ & + h\pi \sum_{i=-\infty}^{n-L+j} \alpha_i ; \quad nT \leq t < (n+1)T \\ & j = 0, 1, \dots, L-1 \end{aligned} \quad (11)$$

For different values of j the second term in (11) gives alternative definitions of phase states [10]. Note that the number of different phase states is the same, compare figure 3.

The transmitted signal can always be written

$$s(t, \underline{\alpha}) = \sqrt{\frac{2E}{T}} [I(t, \underline{\alpha}) \cos(2\pi f_0 t) - Q(t, \underline{\alpha}) \sin(2\pi f_0 t)] \quad (12)$$

where

$$\begin{cases} I(t, \underline{\alpha}) = \cos[\varphi(t, \underline{\alpha})] \\ Q(t, \underline{\alpha}) = \sin[\varphi(t, \underline{\alpha})] \end{cases} \quad (13)$$

From equation (8) we have

$$\varphi(t, \underline{\alpha}_n) = \theta(t, \underline{\alpha}_n) + \theta_n \quad ; \quad nT \leq t < (n+1)T \quad (14)$$

where

$$\begin{cases} \theta(t, \underline{\alpha}_n) = 2\pi h \sum_{i=n-L+1}^n \alpha_i q(t-iT) \\ \theta_n = h\pi \sum_{i=-\infty}^{n-L} \alpha_i \end{cases} \quad ; \quad nT \leq t < (n+1)T \quad (15)$$

Hence for $nT \leq t < (n+1)T$

$$\begin{cases} I(t, \underline{\alpha}_n) = \cos[\theta(t, \underline{\alpha}_n)] \cos \theta_n - \sin[\theta(t, \underline{\alpha}_n)] \sin \theta_n \\ Q(t, \underline{\alpha}_n) = \cos[\theta(t, \underline{\alpha}_n)] \sin \theta_n + \sin[\theta(t, \underline{\alpha}_n)] \cos \theta_n \end{cases} \quad (16)$$

The receiver observes the signal $r(t) = s(t, \underline{\alpha}) + n(t)$ where the noise $n(t)$ is Gaussian and white, with one sided spectral density N_0 . The MLSE receiver maximizes the log likelihood function

$$\ln[p_{r(t)} | \underline{\tilde{\alpha}}] \sim - \int_{-\infty}^{\infty} [r(t) - s(t, \underline{\tilde{\alpha}})]^2 dt \quad (17)$$

with respect to the infinitely long estimated sequence $\underline{\tilde{\alpha}}$. The maximizing sequence $\underline{\tilde{\alpha}}$ is the maximum likelihood sequence estimate and $p_{r(t)} | \underline{\tilde{\alpha}}$ is the probability density function for the observed signal $r(t)$ conditioned on the infinitely long sequence $\underline{\tilde{\alpha}}$ ($\sim$ means proportional to).

Maximizing (17) is equivalent to maximize the correlation

$$J(\underline{\tilde{\alpha}}) = \int_{-\infty}^{\infty} r(t) \cdot \cos[2\pi f_0 t + \varphi(t, \underline{\tilde{\alpha}})] dt \quad (18)$$

Now define

$$J_n(\underline{\tilde{\alpha}}_n) = \int_{-\infty}^{(n+1)T} r(t) \cdot \cos[2\pi f_0 t + \varphi(t, \underline{\tilde{\alpha}}_n)] dt \quad (19)$$

Thus it is possible to write

$$J_n(\underline{\tilde{\alpha}}_n) = J_{n-1}(\underline{\tilde{\alpha}}_{n-1}) + Z_n(\underline{\tilde{\alpha}}_n) \quad (20)$$

where

$$Z_n(\underline{\tilde{\alpha}}_n) = \int_{nT}^{(n+1)T} r(t) \cos(2\pi f_0 t + \varphi(t, \underline{\tilde{\alpha}}_n)) dt \quad (21)$$

Using the above formulas it is possible to calculate the function $J(\underline{\tilde{\alpha}})$ recursively through equation (20) and the metric $Z_n(\underline{\tilde{\alpha}}_n)$.

The Viterbi algorithm works recursively using a metric to choose those sequences that maximizes the log likelihood function up to the n -th symbol interval. A trellis is used to choose among possible extensions of those sequences. We now define the set of states in the trellis by the L -tuple $\underline{\alpha}_n = (\alpha_n, \alpha_{n-1}, \alpha_{n-2}, \dots, \alpha_{n-L+1})$. The total number of such states is $p \cdot M^{L-1}$. It is seen that the state space consists both of the phase states and the $L-1$ previous data symbols (the correlative states). An example of a state trellis is shown in figure 4.

The receiver computes $Z_n(\underline{\tilde{\alpha}}_n, \tilde{\theta}_n) = Z_n(\underline{\tilde{\alpha}}_n, \underline{\tilde{\alpha}}_n)$ for all M^L possible sequences $\underline{\tilde{\alpha}}_n = \{\tilde{\alpha}_n, \tilde{\alpha}_{n-1}, \dots, \tilde{\alpha}_{n-L+1}\}$ and all p possible $\tilde{\theta}_n$. This makes $p \cdot M^L$ different Z_n values. Rewriting (21) using (14) yields

$$Z_n(\underline{\tilde{\alpha}}_n, \tilde{\theta}_n) = \int_{nT}^{(n+1)T} r(t) \cos(\omega_0 t + \theta(t, \underline{\tilde{\alpha}}_n) + \tilde{\theta}_n) dt \quad (22)$$

where $\omega_0 = 2\pi f_0$.

It is seen that $Z_n(\underline{\tilde{\alpha}}_n, \tilde{\theta}_n)$ is obtained by feeding the signal $r(t)$ into a filter and sampling the output of the filter at $t=(n+1)T$. In this case a bank of bandpass-filters must be used, see [10].

After passing the noise $n(t)$ through a bandpass filter with bandwidth larger than the signal bandwidth the filtered noise $n'(t)$ can be decomposed in baseband noise components according to

$$n'(t) = \sqrt{2} n_c(t) \cdot \cos \omega_0 t - \sqrt{2} n_s(t) \cdot \sin \omega_0 t \quad (23)$$

Using the basic quadrature receiver from figure 5 the received quadrature components are

$$\begin{cases} \hat{I}(t, \underline{\alpha}_n) = \frac{1}{2} \left[\sqrt{\frac{2E}{T}} \cdot I(t, \underline{\alpha}_n) + \sqrt{2} n_c(t) \right] \\ \hat{Q}(t, \underline{\alpha}_n) = \frac{1}{2} \left[\sqrt{\frac{2E}{T}} \cdot Q(t, \underline{\alpha}_n) + \sqrt{2} n_s(t) \right] \end{cases} \quad (24)$$

By inserting these components in (22) and omitting double frequency terms we have

$$\begin{aligned} Z_n(\tilde{\underline{\alpha}}_n, \tilde{\theta}_n) = & \cos(\tilde{\theta}_n) \cdot \int_{nT}^{(n+1)T} \hat{I}(t, \underline{\alpha}_n) \cdot \cos(\theta(t, \tilde{\underline{\alpha}}_n)) dt + \\ & + \cos(\tilde{\theta}_n) \cdot \int_{nT}^{(n+1)T} \hat{Q}(t, \underline{\alpha}_n) \cdot \sin(\theta(t, \tilde{\underline{\alpha}}_n)) dt + \\ & + \sin(\tilde{\theta}_n) \cdot \int_{nT}^{(n+1)T} \hat{Q}(t, \underline{\alpha}_n) \cdot \cos(\theta(t, \tilde{\underline{\alpha}}_n)) dt - \\ & - \sin(\tilde{\theta}_n) \cdot \int_{nT}^{(n+1)T} \hat{I}(t, \underline{\alpha}_n) \cdot \sin(\theta(t, \tilde{\underline{\alpha}}_n)) dt \end{aligned} \quad (25)$$

This can be interpreted as $4 \cdot M^L$ baseband filters with the impulse response

$$h_c(t, \tilde{\underline{\alpha}}_n) = \begin{cases} \cos \left[2\pi h \cdot \sum_{j=-L+1}^0 \tilde{\alpha}_j \cdot q((1-j)T-t) \right] \\ 0 \quad \text{for } t \text{ outside } [0, T] \end{cases} \quad (26)$$

and

$$h_s(t, \tilde{\underline{\alpha}}_n) = \begin{cases} \sin \left[2\pi h \cdot \sum_{j=-L+1}^0 \tilde{\alpha}_j \cdot q((1-j)T-t) \right] \\ 0 \quad \text{for } t \text{ outside } [0, T] \end{cases} \quad (27)$$

The number of filters required can be reduced by a factor of two by observing that every $\tilde{\underline{\alpha}}_n$ -sequence has a corresponding sequence with reversed sign. Figure 6 shows an optimum receiver with

$$F = 2 \cdot M^L \quad (28)$$

matched filters. The outputs of these filters are sampled once every symbol interval. The metrics $Z_n(\tilde{\underline{\alpha}}_n, \tilde{\theta}_n)$ are obtained by the use of equation (25) and are used by the Viterbi algorithm. A delay of N_T symbol intervals is introduced. Alternative receiver structures are given in [9].

III. THE COMPLEXITY REDUCTION CONCEPT

The class of coherent suboptimum receivers considered in this part is of maximum likelihood sequence estimating type with reduced complexity compared to the optimum receiver. The approach is quite general. As we will see, for any rational h , number of levels M and pulse shape $g(t)$ approximate receivers will be found. The principle will be illustrated by the following examples.

From the optimum receiver presentation in section II and from [7], [9] it is clear that the receiver complexity in terms of the number of states S and the number of filters F grows exponentially with the pulse length L , i.e. $\sim M^L$ where M is the number of levels. The aim with this part is to obtain suboptimum receiver structures with a reduced complexity compared to the optimum receiver. The principle idea in this part is the following. The receiver uses a shorter pulse than the transmitter, thus approximating the phase tree. The receiver pulse $g_R(t)$ must be chosen in such a way that the tree generated by $g_R(t)$ is a reasonably good approximation of the phase tree generated by the transmitter pulse $g_T(t)$. The receiver based on $g_R(t)$ is built according to the optimum receiver principles in section II. It is of course the optimum receiver for the case of $g_T(t) = g_R(t)$. This part deals with the analysis and simulation of the performance of receivers with short pulses $g_R(t)$. The aim is also to find the optimum pulse $g_R(t)$ for a given complexity reduction. If the transmitter pulse length is L_T and the receiver pulse length is L_R , then the complexity reduction factor is $M^{(L_T - L_R)}$ both in terms of the number of receiver states and receiver filters.

The basic idea and some of the problems that arises will be illustrated by means of the following examples. Figures 7-10 show the phase trees for the binary 1REC, 2REC, 3RC and 4RC schemes. The corresponding pulses and phase responses except for 4RC are given in figure 2. Figure 11 shows a transmitter tree based on 3RC and a simplified receiver with a phase tree based on the shorter 1REC pulse. The two trees are shown with a small phase offset relative to the best alignment. Thus the complete trees can be seen. From figure 11 it is clear that the 1REC tree, with properly chosen phase and time offset (T in this case), approximates the 3RC scheme fairly well. The quality of this approximation is analysed in the following sections of this paper. The best values of time and phase offset for best match of the 2 phase trees will also be derived for the general case.

Figure 12 shows another similar example, i.e. the 3RC scheme received in a 2REC receiver. This means a complexity reduction of a factor of 2, compared with a factor of 4 in figure 11.

Several interesting observations can be made by studying figures 11 and 12. For example, in the general case a certain transmitted sequence of binary symbols e.g. +1, -1, -1, +1,... etc does not in general correspond to the same phase path in the transmitter and receiver phase trees. This is of course a consequence of the approximation made. Thus, in some cases, even if there is no noise at all in the transmission link, there is no perfect match between the transmitted phase path and the correct phase path as generated by the receiver. We will say that the transmitter and receiver signal sets are mismatched. This mismatch should of course not be too large, because then detection errors are made also when the transmission is noiseless.

It is also clear from the examples above that the phase synchronization and timing between the transmitter and receiver trees is of great importance. This problem will be addressed below.

IV. THE PERFORMANCE MEASURE

In [1], [2] the minimum normalized squared Euclidean distance is used as a performance measure. For the optimum detector the minimum normalized squared Euclidean distance in the signal space is [1], [2]

$$\frac{d_{\min}^2}{2E_b} = d_{\min}^2 = \min_{\substack{\underline{\alpha}, \underline{\beta} \\ \underline{\alpha}_0 \neq \underline{\beta}_0}} \left\{ \frac{1}{2E_b} \int_0^{NT} [s(t, \underline{\alpha}) - s(t, \underline{\beta})]^2 dt \right\} \quad (29)$$

where NT is the length of the observation interval and $\underline{\alpha}$ and $\underline{\beta}$ are data sequences which are different in the first symbol. The distance depends on the difference sequence $\underline{\gamma} = \underline{\alpha} - \underline{\beta}$ rather than the individual data sequences themselves. Assuming $2\pi f_0 T \gg 1$, equation (29) can be rewritten

$$d_{\min}^2 = \min_{\underline{\gamma}, \gamma_0 \neq 0} \left\{ \log_2(M) \frac{1}{T} \int_0^{NT} [1 - \cos(\varphi(t, \underline{\gamma}))] dt \right\} \quad (30)$$

where

$$\varphi(t, \underline{\gamma}) = 2\pi h \sum_{i=-\infty}^{N-1} \gamma_i q(t-iT) \quad (31)$$

and $\gamma_i = \alpha_i - \beta_i$. Note that the normalization is made in bit energy E_b . It is well-known that the error probability behaviour for large signal-to-noise ratios (SNR's) is given by the minimum Euclidean distance for coherent detection and a Gaussian channel.

A brute-force method for calculating the minimum distance is computing d^2 for all allowed $2(M-1) (2M-1)^{N-1}$ difference sequences of length N symbol intervals. Since the cosine is an even function this number can be reduced by a factor of 2. However, the computational effort is unrealistic even for fairly small values of M and N .

One of the key properties of the minimum distance is that an upper bound can easily be calculated [1], [2]. The method of calculating this bound is based on the observation that merges occur in the phase tree for all h -values. These merges occur for the infinitely long difference sequences

$$\gamma_i = \begin{cases} 0 & ; i < 0, \quad i \geq m+1 \\ 2, 4, 6, \dots, 2(M-1) & ; i = 0 \\ 0, \pm 2, \pm 4, \dots, \pm 2(M-1) & ; 0 < i < m+1 \end{cases} \quad (32)$$

and

$$\sum_{i=0}^m \gamma_i = 0 \quad (33)$$

where m is a positive integer. The merges occur at $t = (L+m)T$. The upper bound is obtained by taking the minimum of all the distances associated by the merge sequences for all $m \leq m_0$. For all the schemes considered, a tight upper bound is always obtained when $m_0 \approx L$ [2]. This bound is essential for the fast algorithm for calculating the minimum distance [2].

The first step in the algorithm is to calculate all distances for $N=1$, given a specific h . If any of these distances are larger than the upper bound for this h -value, the difference sequences having the corresponding γ_0 -values will not be used anymore. This can be done since the distance is a non-decreasing function of N for fixed h . The minimum of the calculated distances gives the minimum distance for $N=1$. For $N=2$ only those γ_0 -values whose corresponding distances did not exceed the upper bound are used. The second component is chosen to be $\gamma_1 = 0, \pm 2, \dots, \pm 2(M-1)$ and the corresponding distances are calculated. Again only those sequences whose corresponding distance do not exceed the upper bound will be used as possible start difference sequences for $N=3$, etc. Through this procedure subsets of all possible difference sequences are deleted in each step. It has been observed that the computation time grows approximately linearly with N instead of exponentially as for the brute-force method. The fast algorithm works for all h -values and it is recursive in N . It is not required that the upper bound is tight. The computation time increases using a looser bound, however.

Let us now consider the reduced-complexity receiver. Here a distance measure between mismatched signal sets is introduced. By using this new minimum Euclidean distance, the error probability behaviour for large signal-to-noise ratios can be estimated for a given scheme and a suboptimum reduced-complexity receiver.

The receiver is assumed to correlate the received signal with all alternatives in the receiver library of signals and choose the alternative with the largest correlation. The receiver principle and the critical parameters are illustrated in figure 13, see explaining text below.

Let the transmitted signal for a given $\underline{\alpha}$ be

$$s_T(t, \underline{\alpha}) = \sqrt{\frac{2E}{T}} \cos[\omega_0 t + \varphi(t, \underline{\alpha})] \quad (34)$$

Below we are considering all signals over a time interval of length NT .

The receiver generates its library of possible signal alternatives (where $\underline{\alpha}$ is varying)

$$s_R(t, \underline{\alpha}) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \psi(t, \underline{\alpha})) \quad (35)$$

These signals are of course also viewed over the same time interval as the transmitted signal. $\varphi(t, \underline{\alpha})$ and $\psi(t, \underline{\alpha})$ are given through eq. (2).

A vector representation of the signals can be obtained in two ways both giving the same distance result. Since the receiver works in the space spanned by the signals in the receiver library, one way is to use this signal space. Then the transmitted signals have to be projected into this signal space and these projections of the signals have to be considered. Alternatively the joint space spanned by both the transmitted signals and the signals in the receiver library can be used. Then the receiver signal space is a subspace of the joint space.

We have chosen to work with the joint space described above. Let $\underline{s}_T$ denote the transmitted signal in this space and let $\underline{s}_0$ represent the receiver alternative which corresponds to the correct sequence $\underline{\alpha}$, i.e. the transmitted data sequence. Let $\underline{s}_1, \underline{s}_2, \underline{s}_3, \dots$ represent other receiver alternatives.

Let the signal alternative $\underline{s}_1$ correspond to the signal

$$s_R(t, \underline{\beta}) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \psi(t, \underline{\beta})) \quad (36)$$

The receiver selects the signal alternative which corresponds to the largest correlation. This corresponds to the signal point which has the smallest distance to the received signal point.

Figure 13 shows the situation in the noiseless case. With additive white Gaussian noise, the signal point $\underline{s}_T$ is added to a noise vector. When the magnitude of the noise vector is such that the received signal point is located on the other side of the decision boundary, an error occurs. Thus the distance parameter D is of interest for all pairs of receiver signals $(\underline{s}_0, \underline{s}_j)$ $j = 1, \dots$ for a given transmitted signal $\underline{s}_T$. This distance should of course be positive otherwise the mismatch is too large. It would in principle be possible to analyse schemes with $D < 0$ but these are not interesting and are not considered. Note that $\underline{s}_0$, $\underline{s}_j$ and $\underline{s}_T$ are located on a 2-dimensional plane in the signal space. D is the distance in the $\underline{s}_j - \underline{s}_0$ direction of this plane. For large signal-to-noise ratios the smallest D is of particular interest. This is the minimum Euclidean distance for the mismatched signal set. This parameter determines the symbol error probability for large signal-to-noise ratios.

Simple geometry (see figure 13) yields that

$$\frac{1}{2} D = \frac{1}{2} D_R - D_P \quad (37)$$

where

$$D_R^2 = |\underline{s}_0 - \underline{s}_1|^2 \quad (38)$$

is the Euclidean distance between the receiver alternatives and $|\cdot|$ is a norm in the Euclidean space. This is referred to as the receiver distance below. D_P is the projection of the signal $\underline{s}_T - \underline{s}_0$ on the signal $\underline{s}_1 - \underline{s}_0$, i.e.

$$D_P = \langle \underline{s}_T - \underline{s}_0, \frac{\underline{s}_1 - \underline{s}_0}{|\underline{s}_1 - \underline{s}_0|} \rangle \quad (39)$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product.

From equation (29) and (30) we know that the Euclidean distance for the receiver signal set is

$$D_R^2 = \frac{2E}{T} \int_0^{NT} \left(1 - \cos(\psi(t, \underline{\beta}) - \psi(t, \underline{\alpha})) \right) dt \quad (40)$$

for $\omega_0 T \gg 1$.

The projection D_P is

$$D_P = \int_0^{NT} \sqrt{\frac{2E}{T}} \left[\cos(\omega_0 t + \varphi(t, \underline{\alpha})) - \cos(\omega_0 t + \psi(t, \underline{\alpha})) \right] \cdot \frac{\sqrt{\frac{2E}{T}} \left[\cos(\omega_0 t + \psi(t, \underline{\beta})) - \cos(\omega_0 t + \psi(t, \underline{\alpha})) \right]}{D_R} dt \quad (41)$$

The factor D_R in the denominator can be viewed as a normalizing factor.

Assuming $\omega_0 T \gg 1$ we have

$$D_P = \frac{2E}{D_R} \cdot \frac{1}{2} \cdot \left[\frac{1}{T} \int_0^{NT} \left(1 - \cos(\psi(t, \underline{\beta}) - \psi(t, \underline{\alpha})) \right) dt - \left\{ \frac{1}{T} \int_0^{NT} \cos[\varphi(t, \underline{\alpha}) - \psi(t, \underline{\alpha})] dt - \frac{1}{T} \int_0^{NT} \cos[\varphi(t, \underline{\alpha}) - \psi(t, \underline{\beta})] dt \right\} \right] \quad (42)$$

Equation (42) can also be written

$$D_P = \frac{1}{D_R} \cdot \frac{1}{2} [D_R^2 - D_A^2] \quad (43)$$

where

$$D_A^2 = \frac{2E}{T} \int_0^{NT} \cos(\varphi(t, \underline{\alpha}) - \psi(t, \underline{\alpha})) dt - \frac{2E}{T} \int_0^{NT} \cos(\varphi(t, \underline{\alpha}) - \psi(t, \underline{\beta})) dt \quad (44)$$

also can be written

$$D_A^2 = |\underline{s}_1 - \underline{s}_T|^2 - |\underline{s}_0 - \underline{s}_T|^2 \quad (45)$$

This is by definition a positive quantity. Note that D_A^2 is the difference between 2 squared Euclidean distances, compare figure 13. Thus

$$D_A^2 = D_1^2 - D_0^2 \quad (46)$$

where

$$D_1^2 = |\underline{s}_1 - \underline{s}_T|^2 \quad (47)$$

and

$$D_0^2 = |s_0 - s_T|^2 \quad (48)$$

Thus, from (37) and (43) we arrive at the formula for D

$$D = \frac{D_A^2}{D_R} \quad (49)$$

In the following the normalized squared distances will be used, i.e.

$$d^2 = \frac{D^2}{2E_b} = \left(\frac{D_A^2}{D_R} \right)^2 = \frac{d_A^4}{d_R^2} \quad (50)$$

where

$$\begin{cases} d_R^2 = \frac{D_R^2}{2E_b} \\ d_A^2 = \frac{D_A^2}{2E_b} \end{cases} \quad (51)$$

For comparisons between multi-level systems the normalization in (50), (51) will be made with the bit energy E_b given in eq. (3).

Thus, from the analysis above, the receiver performance for large SNR's is determined by the minimum distance $d_{\min}^2$, where the minimization is carried out both over transmitter sequences and receiver sequences.

Note that for the special case of no mismatch, i.e. $\varphi(t, \underline{\alpha}) = \psi(t, \underline{\alpha})$, we have

$$\begin{cases} d_0^2 = 0 \\ d_A^2 = d_R^2 = d_1^2 \end{cases} \quad (52)$$

and thus

$$d^2 = d_R^2 \quad (53)$$

In this case, the error probability is determined by the minimum distance in the signal set, i.e. $\min d_R^2$.

The formula

$$d^2 = \frac{d_A^4}{d_R^2} = \frac{(d_A^2)^2}{d_R^2} \quad (54)$$

with

$$d_A^2 = \log_2(M) \cdot \frac{1}{T} \int_0^{NT} \left\{ \cos[\varphi(t, \underline{\alpha}) - \psi(t, \underline{\alpha})] - \cos[\varphi(t, \underline{\alpha}) - \psi(t, \underline{\beta})] \right\} dt \quad (55)$$

and

$$d_R^2 = \log_2(M) \cdot \frac{1}{T} \int_0^{NT} \left(1 - \cos(\psi(t, \underline{\beta}) - \psi(t, \underline{\alpha})) \right) dt \quad (56)$$

is the key to the analysis of the behaviour of mismatched signal sets and thus reduced-complexity Viterbi receivers for large SNR's. The minimum of d^2 gives the asymptotic performance of the considered signal scheme with the given receiver.

Thus, for large signal-to-noise ratios, the symbol error probability behaviour is given by

$$P_e \sim Q \left(\sqrt{\frac{E_b \cdot d_{\min}^2}{N_0}} \right) \quad (57)$$

where $Q(\cdot)$ is the error function associated with the normal distribution given by [8]

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy \quad (58)$$

By studying (55), (56) we note that d_A^2 and d_R^2 can be calculated recursively when N is increased, [1], [2], [6]. This can be used for computing d^2 (54) efficiently.

It is clear that the above reduced-complexity receiver is suboptimum, i.e. the minimum Euclidean distance for the reduced-complexity receiver $\min(d^2)$ is less or equal to the minimum Euclidean distance of the transmitted signal scheme $\min(d_T^2)$, where

$$d_T^2 = \log_2(M) \left\{ N - \frac{1}{T} \int_0^{NT} \cos[\varphi(t, \underline{\alpha}) - \varphi(t, \underline{\beta})] dt \right\} \quad (59)$$

Otherwise the reduced-complexity receiver would perform better than the optimum receiver for large signal-to-noise ratios and thus the optimum receiver would not be optimum, which is a contradiction. Thus we have the inequality

$$\min(d^2) = \min\left(\frac{d_A^4}{d_R^2}\right) \leq \min(d_T^2) \quad (60)$$

It is interesting to note that it is possible to have

$$\min(d^2) = \min\left(\frac{d_A^4}{d_R^2}\right) > \min(d_R^2) \iff d_A > d_R \text{ for that specific } \underline{\alpha} \text{ and } \underline{\beta} \quad (61)$$

This implies that by starting from an optimum scheme with matched receiver and transmitter, it is possible to obtain a larger minimum Euclidean distance by exchanging the transmitter into another one with a larger $\min(d_T^2)$. The resulting minimum distance is of course smaller than $\min(d_T^2)$ but the reduction depends upon the receiver.

The inequality in eq (60) has been motivated by the discussion above. The inequality seems difficult to prove strictly mathematically since it is an inequality between two minima. For simplicity it will be proven only for binary full response schemes with an one bit observation interval. Then there is only one distance value d^2 and d_T^2 respectively. The notation

$$\begin{cases} \varphi^+ = \varphi(t, \underline{\alpha}) \text{ with } \alpha_0 = +1 \\ \varphi^- = \varphi(t, \underline{\alpha}) \text{ with } \alpha_0 = -1 \\ \psi^+ = \psi(t, \underline{\beta}) \text{ with } \beta_0 = +1 \\ \psi^- = \psi(t, \underline{\beta}) \text{ with } \beta_0 = -1 \end{cases} \quad (62)$$

will be used.

Thus

$$d_A^2 = \frac{1}{T} \int_0^T \cos(\varphi^+ - \psi^+) dt - \frac{1}{T} \int_0^T \cos(\varphi^+ - \psi^-) dt \quad (63)$$

Using trigonometrical identities this can be written

$$d_A^2 = \frac{2}{T} \int_0^T \sin \frac{\psi^+ - \psi^-}{2} \cdot \sin \frac{2\varphi^+ - \psi^+ - \psi^-}{2} dt \quad (64)$$

and thus

$$d^2 = \frac{d_A^4}{d_R^2} = \frac{4}{T} \frac{\left[\int_0^T \sin \frac{\psi^+ - \psi^-}{2} \cdot \sin \frac{2\varphi^+ - \psi^+ - \psi^-}{2} dt \right]^2}{\int_0^T [1 - \cos(\psi^+ - \psi^-)] dt} \leq$$

$$\leq \frac{1}{T} \int_0^T [1 - \cos(2\varphi^+ - \psi^+ - \psi^-)] dt \quad (65)$$

where the Cauchy-Schwartz inequality for integrals was used. This expression can also be written

$$d^2 \leq \frac{1}{T} \int_0^T [1 - \cos(2\varphi^+ - \psi^+ - \psi^-)] dt =$$

$$= d_T^2 + \frac{1}{T} \int_0^T [\cos(\varphi^+ - \varphi^-) - \cos(2\varphi^+ - \psi^+ - \psi^-)] dt \quad (66)$$

In the full response case we have

$$\begin{cases} \varphi^+ = -\varphi^- \\ \psi^+ = -\psi^- \end{cases} \quad (67)$$

Thus

$$\begin{cases} 2\varphi^+ - \psi^+ - \psi^- = 2\varphi^+ + \psi^- - \psi^- = 2\varphi^+ \\ \varphi^+ - \varphi^- = \varphi^+ + \varphi^+ = 2\varphi^+ \end{cases} \quad (68)$$

The integral above equals zero and $d^2 \leq d_T^2$.

V. PROPERTIES OF THE MINIMUM EUCLIDEAN DISTANCE FOR MISMATCHED SIGNAL SETS

The properties of the distance measure introduced in section IV will now be investigated more precisely. First we will look into the usefulness of difference sequences, [2]. Then upper bounds d_B^2 on the minimum distance are calculated. The new distance is applied to some simple but instructive examples at the end of this section.

For calculation of Euclidean distances for optimum receivers only the phase difference $\varphi(t, \underline{\alpha}) - \varphi(t, \underline{\beta}) = \varphi(t, \underline{\alpha} - \underline{\beta})$ has to be considered. This simplifies the calculations since only the difference sequence $\underline{\gamma} = \underline{\alpha} - \underline{\beta}$ has to be considered.

It will now be investigated if and when difference sequences also can be used for calculation of Euclidean distances for mismatched receivers. The crucial quantities which have to be investigated are $\varphi(t, \underline{\alpha}) - \psi(t, \underline{\alpha})$ and $\varphi(t, \underline{\alpha}) - \psi(t, \underline{\beta})$, see eq (55) above. The first difference is

$$\begin{aligned} \varphi(t, \underline{\alpha}) - \psi(t, \underline{\alpha}) &\stackrel{\Delta}{=} \varepsilon(t, \underline{\alpha}) = 2\pi h \sum_i \alpha_i [q_T(t-iT) - q_R(t-iT)] = \\ &= 2\pi h \sum_i \alpha_i q_{\Delta}(t-iT) \end{aligned} \quad (69)$$

where

$$q_{\Delta}(t) = q_T(t) - q_R(t) \quad (70)$$

The symbol timing between the transmitter and the receiver is defined by letting

- 1) $g_R(t)$ occupy the interval $[0, L_R \cdot T]$
- 2) $g_T(t)$ occupy the interval $[-L_{\Delta} \cdot T, (L_T - L_{\Delta}) \cdot T]$

where $L_T \geq L_R$ and

$$L_{\Delta} = \frac{L_T - L_R}{2} \quad (71)$$

By proper phase synchronization ψ_0 this will assure the best fit between the two phase trees given by the ensemble of $\varphi(t, \underline{\alpha})$ and $\psi(t, \underline{\alpha})$ respectively. This yields

$$q_R(t) = \begin{cases} 0 & ; \quad t \leq 0 \\ \frac{1}{2} & ; \quad t \geq L_R \cdot T \end{cases} \quad (72)$$

and

$$q_T(t) = \begin{cases} 0 & ; \quad t \leq -L_\Delta T \\ \frac{1}{2} & ; \quad t \geq (L_T - L_\Delta)T \end{cases} \quad (73)$$

and thus $q_\Delta(t) \equiv 0$ when $t \leq -L_\Delta T$ and $t \geq (L_T - L_\Delta)T$, since $L_T - L_\Delta \geq L_R$. Using the properties of $q_\Delta(t)$ yields over the interval $nT \leq t < (n+1)T$

$$\begin{aligned} \epsilon(t, \underline{\alpha}) &= 2\pi h \left[\alpha_{n+\Lambda_\Delta} q_\Delta(t - (n+\Lambda_\Delta)T) + \dots + \alpha_n q_\Delta(t - nT) + \dots \right. \\ &\quad \left. \dots + \alpha_{n-L_R+1-\Lambda_\Delta} q_\Delta(t - (n-L_R+1-\Lambda_\Delta)T) \right] = \\ &= 2\pi h \sum_{i=-L_R+1-\Lambda_\Delta}^{\Lambda_\Delta} \alpha_{n+i} q_\Delta(t - (n+i)T) \end{aligned} \quad (74)$$

where

$$\Lambda_\Delta = \left\lceil \frac{L_T - L_R}{2} \right\rceil = \text{smallest integer larger than or equal to } (L_T - L_R)/2 \quad (75)$$

Thus it can be seen that the dependence of $\underline{\alpha}$ in $\epsilon(t, \underline{\alpha})$ lies in the $(2\Lambda_\Delta + L_R)$ -ary vector

$$(\alpha_{n-L_R+1-\Lambda_\Delta}, \dots, \alpha_n, \dots, \alpha_{n+\Lambda_\Delta}) = \underline{\alpha}_{n+\Lambda_\Delta} \quad (76)$$

Now the expression $\varphi(t, \underline{\alpha}) - \psi(t, \underline{\beta})$ is considered.

$$\varphi(t, \underline{\alpha}) - \psi(t, \underline{\beta}) = 2\pi h \sum_i \alpha_i q_T(t - iT) - 2\pi h \sum_i \beta_i q_R(t - iT) \quad (77)$$

By constraining t to lie in the interval $nT \leq t < (n+1)T$ we have

$$\varphi(t, \underline{\alpha}) = h\pi \sum_{i \leq n-L_R-\Lambda_\Delta} \alpha_i + 2\pi h \sum_{i=n-L_R-\Lambda_\Delta+1}^{n+\Lambda_\Delta} \alpha_i q_T(t-iT) \quad (78)$$

and

$$\psi(t, \underline{\beta}) = h\pi \sum_{i \leq n-L_R} \beta_i + 2\pi h \sum_{i=n-L_R+1}^n \beta_i q_R(t-iT) \quad (79)$$

where the properties of $q_T(t)$ and $q_R(t)$ have been used.

This directly gives

$$\begin{aligned} \varphi(t, \underline{\alpha}) - \psi(t, \underline{\beta}) &= \\ &= h\pi \sum_{i \leq n-L_R-\Lambda_\Delta} (\alpha_i - \beta_i) - h\pi \sum_{i=n-L_R-\Lambda_\Delta+1}^{n-L_R} \beta_i + \\ &+ 2\pi h \sum_{i=n-L_R-\Lambda_\Delta+1}^{n-L_R} \alpha_i q_T(t-iT) + 2\pi h \sum_{i=n-L_R+1}^n [\alpha_i q_T(t-iT) - \beta_i q_R(t-iT)] + \\ &+ 2\pi h \sum_{i=n+1}^{n+\Lambda_\Delta} \alpha_i q_T(t-iT) \end{aligned} \quad (80)$$

and only the first term depends upon $\underline{\alpha} - \underline{\beta}$. For the other terms the two $(2\Lambda_\Delta + L_R)$ -ary vectors

$$\begin{cases} \underline{\alpha}_{n+\Lambda_\Delta} = (\alpha_{n-L_R-\Lambda_\Delta+1}, \dots, \alpha_n, \dots, \alpha_{n+\Lambda_\Delta}) \\ \underline{\beta}_{n+\Lambda_\Delta} = (\beta_{n-L_R-\Lambda_\Delta+1}, \dots, \beta_n, \dots, \beta_{n+\Lambda_\Delta}) \end{cases} \quad (81)$$

must be used.

Thus for an observation interval $N \leq L_R + \Lambda_\Delta$ symbol intervals, both the $\underline{\alpha}$ and $\underline{\beta}$ sequences must be used to obtain all the Euclidean distances. For $N > L_R + \Lambda_\Delta$ symbol intervals however, the phase separation created by the initial $N - L_R - \Lambda_\Delta$ data symbols can be expressed by means of the sum of the difference sequence $\underline{\alpha} - \underline{\beta}$.

Upper bound $d_B^2(h)$

The minimum Euclidean distance for optimum receivers has been shown to be upper bounded, [1], [2]. This holds for all observation lengths N , including infinity. The upper bound is calculated by identifying so-called merges in the phase tree. From the initial phase separation up to the merge the two phase trajectories are different. Outside this interval however, the two phase trajectories coincide and give no contribution to the Euclidean distance.

For mismatched receivers merges do not exist in general, at least not merges of the kind defined in [2], [6]. It is possible however to choose sequences $\underline{\alpha}$ and $\underline{\beta}$ such that the calculation of d_A^2 only has to be performed over a finite interval, although $\underline{\alpha}$ and $\underline{\beta}$ are infinitely long. The sequences $\underline{\alpha}$ and $\underline{\beta}$ are chosen exactly as the merge sequences given in [2], [7] e.g.

$$\begin{cases} \underline{\alpha} = \dots \alpha_{-2}, \alpha_{-1}, +1, -1, \alpha_2, \dots \\ \underline{\beta} = \dots \beta_{-2}, \beta_{-1}, -1, +1, \beta_2, \dots \end{cases} \quad (82)$$

where $\alpha_i = \beta_i$ when $i \neq 0, 1$ for the first merge in the binary case. Now this pair of sequences yields a merge at $t = (L_R + 1)T$ in the phase tree given by the ensemble of $\psi(t, \underline{\alpha})$.

Now the expression for d_A^2 will be studied for this pair of sequences. Since $\alpha_i = \beta_i$, $i < 0$ it is clear that for an observation interval $-\infty < t < \infty$

$$\begin{aligned} d_A^2 &= \log_2(M) \frac{1}{T} \int_{-\infty}^{\infty} \left\{ \cos[\varphi(t, \underline{\alpha}) - \psi(t, \underline{\alpha})] - \cos[\varphi(t, \underline{\alpha}) - \psi(t, \underline{\beta})] \right\} dt = \\ &= \log_2(M) \frac{1}{T} \int_0^{\infty} \left\{ \cos[\varphi(t, \underline{\alpha}) - \psi(t, \underline{\alpha})] - \cos[\varphi(t, \underline{\alpha}) - \psi(t, \underline{\beta})] \right\} dt \quad (83) \end{aligned}$$

since

$$\psi(t, \underline{\alpha}) = \psi(t, \underline{\beta}) \iff \varphi(t, \underline{\alpha}) - \psi(t, \underline{\alpha}) = \varphi(t, \underline{\alpha}) - \psi(t, \underline{\beta}) ; \quad t \leq 0 \quad (84)$$

Now for all $t > (L_R + 1)T$ we know from the previous discussion about difference sequences that $\psi(t, \underline{\alpha}) = \psi(t, \underline{\beta})$ since the influence of α_0, α_1 and β_0, β_1 does not affect the shape of these two phase trajectories. The separation between these two phase trajectories is affected by these symbols. This separation equals zero since

$$\sum_i \alpha_i = \sum_i \beta_i \quad (85)$$

and thus

$$\psi(t, \underline{\alpha}) = \psi(t, \underline{\beta}) \quad ; \quad t \leq 0, \quad t \geq (L_R + 1)T \quad (86)$$

Thus it is sufficient to calculate d_A^2 over the time interval where the trajectories in the receiver phase tree are unmerged.

In the M-ary case there are more merge sequences, and of course there are also merges later than the first. In some cases the upper bound on d^2 can be improved by also taking such merges into account. Unlike the case for matched receivers, the upper bound can also depend upon a finite number of the prehistory data symbols $\alpha_{-1}, \alpha_{-2}, \dots$ and $\beta_{-1}, \beta_{-2}, \dots$. Only one pre-history sequence must be considered however since $\alpha_i = \beta_i$; $i < 0$, and not a pair of sequences.

Simple examples of the minimum Euclidean distance between mismatched signal sets

The concept of minimum Euclidean distance between mismatched signal sets is illustrated by some simple examples of binary schemes.

First consider the transmitter and receiver phases given in figure 14a. The transmitted phase response is

$$q_T(t) = \frac{t}{2T} \quad 0 \leq t \leq T \quad (87)$$

and the receiver phase response is

$$q_R(t) = \frac{1}{2} \quad 0 < t \leq T \quad (88)$$

For this case we have after straight forward application of the distance formula (54) for $N=1$

$$d^2 = \frac{4 \cdot \left(\frac{\sin(h\pi)}{h\pi} - \frac{\sin(2h\pi)}{2h\pi} \right)^2}{1 - \cos(2\pi h)} \quad (89)$$

The result is plotted as a function of modulation index h in figure 15. For comparison, the result for the correct phase response is also given, i.e. $q_R = q_T$ for which

$$d^2 = d_T^2 = 1 - \frac{\sin(2\pi h)}{2\pi h} \quad (90)$$

This is the minimum distance of the transmitter signal set. The loss due to the mismatched receiver is shown in figure 15. For $h=1/2$ we note a loss of 0.9dB when FSK is received in a PSK receiver rather than in an FSK receiver. Note that the receiver signal set degenerates into 1 signal point for $h=1$, see figure 15.

By interchanging q_T and q_R (see figure 14a) given in equations (87) and (88) we have

$$d^2 = \frac{4 \cdot \left(\frac{\sin(\pi h)}{\pi h} - \frac{\sin(2\pi h)}{2\pi h} \right)^2}{1 - \frac{\sin(2\pi h)}{2\pi h}} \quad (91)$$

and

$$d_T^2 = 1 - \cos(2\pi h) \quad (92)$$

This result is also shown in figure 15. For $h=1/2$ it is seen that there is a 0.9dB degradation when PSK is received in an FSK receiver. Note that for some h -values, PSK received in an FSK receiver is much better than the matched FSK signal received in the same FSK receiver. PSK is of course a better signal set than FSK.

Figure 16 shows the result when a transmitted FSK, i.e. linear phase, is received in a plateau function [1], see figure 14b. For this case the minimum (mismatched) distance is

$$d^2 = \frac{\left(3 \cdot \frac{\sin(\frac{h\pi}{2})}{h\pi} - \frac{\sin(\frac{3h\pi}{2})}{h\pi} \right)^2}{1 - \cos(h\pi)} \quad (93)$$

with d_T^2 given in equation (90). Reversing q_R and q_T we have

$$d^2 = \frac{\left(3 \cdot \frac{\sin(\frac{h\pi}{2})}{h\pi} - \frac{\sin(\frac{3h\pi}{2})}{h\pi}\right)^2}{1 - \frac{\sin(2\pi h)}{2\pi h}} \quad (94)$$

and

$$d_T^2 = 1 - \cos(h\pi) \quad (95)$$

These results are also shown in figure 16. An example of a case with a lower degree of mismatch is shown in figure 14c. First, let the transmitted phase response be linear phase, i.e. FSK, and let the receiver have the phase response

$$q_R(t) = q_0(t) = \begin{cases} 0 & 0 \leq t < \frac{T}{4} \\ \frac{t}{T} - \frac{1}{4} & \frac{T}{4} \leq t < \frac{3T}{4} \\ \frac{1}{2} & \frac{3T}{4} \leq t \leq T \end{cases} \quad (96)$$

For this case the minimum distance is

$$d^2 = \frac{\left[\frac{3}{4} \cdot \frac{\sin(\frac{h\pi}{4})}{(\frac{h\pi}{4})} - \frac{\sin(\frac{7h\pi}{4}) - \sin(\frac{h\pi}{4})}{3h\pi} - \frac{\sin(2h\pi) - \sin(\frac{7h\pi}{4})}{h\pi}\right]^2}{\frac{3}{4} - \frac{1}{4} \cos(2h\pi) - \frac{1}{2} \frac{\sin(2h\pi)}{2h\pi}} \quad (97)$$

Reversing transmitter and receiver phase response gives

$$d^2 = \frac{\left[\frac{3}{4} \cdot \frac{\sin(\frac{h\pi}{4})}{\frac{h\pi}{4}} - \frac{\sin(\frac{7h\pi}{4}) - \sin(\frac{h\pi}{4})}{3h\pi} - \frac{\sin(2h\pi) - \sin(\frac{7h\pi}{4})}{h\pi}\right]^2}{1 - \frac{\sin(2\pi h)}{2\pi h}} \quad (98)$$

and

$$d_T^2 = \frac{3}{4} - \frac{1}{4} \cos(2h\pi) - \frac{1}{2} \cdot \frac{\sin(2h\pi)}{2h\pi} \quad (99)$$

These results are shown in figure 17.

Note that the simple examples in figure 14-17 do not provide complexity reduction. However, the examples give a first rough idea how the distance between signal sets is associated with the modulation index h and the degree of mismatch between q_T and q_R .

One way to construct simple receivers is to make $\psi(t, \underline{\beta})$ constant over a sub-symbol interval T/q . Since the receiver correlates the received quadrature components $\hat{I}(t, \underline{\alpha})$ and $\hat{Q}(t, \underline{\alpha})$ with [9]

$$\begin{cases} I(t, \underline{\beta}) = \cos\psi(t, \underline{\beta}) \\ Q(t, \underline{\beta}) = \sin\psi(t, \underline{\beta}) \end{cases} \quad (100)$$

it follows that only one receiver filter is needed in each of the two quadrature branches. This receiver filter has a rectangular-shaped impulse response and its output is sampled once every T/q seconds. These samples are then weighed together, q of them every symbol interval T , to obtain the desired metric for the sub-optimum receiver.

Now assume that the transmitter uses a full response rectangular shaped pulse $g_T(t)$. This gives $q_T(t)$ according to equation (87) and corresponds to full response 1REC (CPFSK) [6]. The receivers phase response $q_R(t)$ is now constructed by sampling $q_T(t)$ at the sampling points

$$t_i = \left[\frac{1}{2q} + (i-1) \frac{1}{q} \right] \cdot T \quad ; \quad i = 1, 2, \dots, q \quad (101)$$

and let $q_R(t)$ equal $q_T(t_i)$ over T/q long sub-symbol interval, centered around t_i , $i = 1, 2, \dots, q$. In this case $q_R(t)$ will be staircase shaped, i.e.

$$q_R(t) = \begin{cases} 0 & ; \quad t \leq 0 \\ \frac{1}{4q} + (i-1) \frac{1}{2q} & ; \quad \frac{(i-1)}{q} < \frac{t}{T} < \frac{i}{q} \quad i = 1, 2, \dots, q \\ \frac{1}{2} & ; \quad t \geq T \end{cases} \quad (102)$$

To calculate d^2 , d_A^2 and d_R^2 must be calculated. Assume first that $N=1$ observed bits in the receiver. Thus

$$\begin{aligned}
 d_R^2 &= 1 - \frac{1}{T} \int_0^T \cos[2 \cdot 2\pi h q_R(t)] dt = \\
 &= 1 - \frac{1}{T} \sum_{i=1}^q \frac{\frac{iT}{q}}{\frac{(i-1)T}{q}} \cos\left[\frac{h}{q} (2\pi i - \pi)\right] = 1 - \frac{1}{q} \sum_{i=1}^q \cos\left[\frac{h}{q} (2\pi i - \pi)\right] \quad (103)
 \end{aligned}$$

In a similar way it is easy to show that

$$d_A^2 = \frac{\sin\left(\frac{h}{q} \cdot \frac{\pi}{2}\right)}{\left(\frac{h}{q} \cdot \frac{\pi}{2}\right)} - \frac{2}{h\pi} \sum_{i=1}^q \cos[h\pi(2i-1) \left(1 + \frac{1}{q}\right)] \sin[h\pi(2i-1)] \quad (104)$$

and d^2 is found from d_A^4/d_R^2 .

Figure 18 shows d^2 as a function of h for different values of q . It is seen that when $h=2$ and $q=1$, $d^2=0$. For these parameters the receiver signal set degenerates into one signal point and detection is impossible.

As can also be seen from figure 18, d^2 is fairly large for small h -values already for $q=1$. Note that $q=1$ corresponds to the distance in equation (93), see also figure 14b. For low h -values d^2 grows monotonically with q , but it can also be seen that for $h \approx 2$, $q=2$ is better than $q=3$.

In [1] it is shown that for optimum full response receivers the upper bound d_B^2 is twice the squared Euclidean distance d_1^2 for $N=1$ observed symbol intervals, if the pulse $g(t)$ is symmetric. As will be shown, this is also true for full response suboptimum receivers, if the pulses $g_T(t)$ and $g_R(t)$ are symmetric.

In the M-ary full response case, the pair of sequences

$$\begin{cases} \underline{\alpha} = \dots \alpha_{-1}, \alpha_0, -\alpha_0, \alpha_2, \alpha_3, \dots \\ \underline{\beta} = \dots \beta_{-1}, -\alpha_0, \alpha_0, \beta_2, \beta_3, \dots \end{cases} ; \quad \begin{aligned} \alpha_i &= \beta_i \text{ when } i \neq 0, 1 \\ \alpha_0 &= \pm 1, \pm 3, \dots, \pm(M-1) \end{aligned} \quad (105)$$

gives merges at $t=2T$ in both the transmitter phase tree and the receiver phase tree. Thus for $N \rightarrow \infty$ it is sufficient to calculate both d_A^2 and d_R^2 over the interval $[0, 2T]$. It is sufficient to show that d_A^2 over this interval equals twice the d_A^2 over the interval $[0, T]$, since this property has already been shown for d_R^2 in [1].

Using the results from the discussion about difference sequences, see equation (74) and (80) ($\Lambda_A=0$) we have

$$d_A^2 = 2 \cdot \frac{1}{T} \int_0^T \cos 2\pi h k q_A(t) dt - \frac{1}{T} \int_0^T \cos \left[2\pi h k \left(q_T(t) - (-q_R(t)) \right) \right] dt - \\ - \frac{1}{T} \int_T^{2T} \cos \left[2\pi h k \left(1 - q_T(t-T) - q_R(t-T) \right) \right] dt \quad (106)$$

where $k = 1, 3, 5, \dots, (M-1)$.

By using the variable substitution $t-T \rightarrow t$ in the last integral, and using the symmetry of $q_T(t)+q_R(t)$, this can also be written

$$d_A^2 = 2 \left[\frac{1}{T} \int_0^T \cos 2\pi h k q_A(t) dt - \frac{1}{T} \int_0^T \cos [2\pi h k (q_T(t) + q_R(t))] dt \right] \quad (107)$$

and the upper bound on the minimum Euclidean distance can thus be written

$$d_B^2 = \min_{\substack{1 \leq k \leq M-1 \\ k \text{ odd}}} \left\{ \frac{\left[2 \left(\frac{1}{T} \int_0^T \cos 2\pi h k q_A(t) dt - \frac{1}{T} \int_0^T \cos [2\pi h k (q_T(t) + q_R(t))] dt \right) \right]^2}{2 \left(1 - \frac{1}{T} \int_0^T \cos 2\pi h k 2q_R(t) dt \right)} \right\} \quad (108)$$

and it is easily seen that this expression equals twice the minimum squared Euclidean distance for $N=1$ observed symbol intervals.

From this discussion it follows that the curves shown in figure 18, if multiplied by two, also can be thought of as upper bounds d_B^2 for a receiver observing many symbol intervals.

In figure 19 this upper bound is shown dashed for $q=1$, and actual minimum Euclidean distances have also been calculated for $N = 1, 2, \dots, 5$ observed bit intervals. The latter curves are shown solid. It is seen that the upper bound has been reached in some regions, for $h < 1/2$ already with $N=2$.

The minimum Euclidean distances when $q=6$ is shown in figure 20. They have been calculated for $N = 1, 2$ observed bit intervals and the upper bound is shown dashed. With this relatively small value of q , the minimum Euclidean distances are very close to those for the optimum receiver, see [1].

This kind of suboptimum receiver will be analysed also for the quaternary case ($M=4$). Figure 21 shows the minimum squared normalized Euclidean dis-

tances for an observation interval of one symbol interval. The normalization is in E_b , compare [1], [2]. It is seen that larger values of q are required to get close to the distance for $q \rightarrow \infty$ compared to the binary case. For $h < 0.4$ however the result is the same as for the binary case. Large h -values require large q -values.

This figure can also be interpreted as upper bounds d_B^2 for a receiver observing many symbol intervals. The distances must then be multiplied by two, see discussion above.

VI. THE OPTIMIZED REDUCED-COMPLEXITY RECEIVER

The aim of this section is to find the optimum reduced-complexity receiver. In this case a natural function to optimize is the error probability at a given signal-to-noise ratio. The error probability is however difficult to calculate and thus unrealistic to use. Therefore the minimum Euclidean distance has been chosen instead. If the minimum Euclidean distance is optimized, the asymptotic error probability is also optimized.

Thus, the receiver frequency response $g_R(t)$ that maximizes the minimum Euclidean distance has to be found. The function, which is to be maximized is therefore given by the equations (54), (55) and (56). An analytic solution to this is difficult to find. Therefore a numerical maximization is done instead. We have chosen to optimize the receiver within a subclass of reasonable phase responses, defined below.

With a numerical maximization, it would in principle be possible to find the optimum reduced-complexity receiver, to a given transmitted scheme and a given complexity reduction, for every observation length of N symbol intervals. The computational efforts to do this are however very large. Therefore, instead the upper bound on the minimum Euclidean distance is maximized. A tight upper bound can always be found and therefore it is possible to find an observation length sufficiently large for the minimum distance to reach this upper bound.

To make a numerical maximization, the function that has to be maximized has to depend on a finite number of parameters. Then a maximization routine e.g. a steepest descent algorithm [20] can be used. In this paper we have chosen the phase response $q_R(t)$ to be a piecewise linear function for $0 < t < L_R T$, which is allowed to be discontinuous at $t=0$ and at $t=L_R T$. Since the transmitter pulse $g_T(t)$ is symmetric around $t = L_T \cdot T/2$ it is assumed that also the receiver pulse $g_R(t)$ is symmetric around $t = L_R \cdot T/2$. Maximizations, without this assumption have also shown that this is most probably the case. Therefore we have chosen $q_R(t)$ to be of the form

$$q_R(t) = \begin{cases} 0 & ; \quad t \leq 0 \\ q_i + \frac{2N_1(q_{i+1} - q_i)}{L_R T} \cdot \left(t - i \frac{L_R T}{2N_1} \right) & ; \quad i \cdot \frac{L_R T}{2N_1} < t \leq (i+1) \frac{L_R T}{2N_1}, \\ & i = 0, 1, \dots, N_1 - 1 \\ \frac{1}{2} - q_R(L_R T - t) & ; \quad \frac{L_R T}{2} < t < L_R T \\ \frac{1}{2} & ; \quad t \geq L_R T \end{cases} \quad (109)$$

where $q_{N_1} = \frac{1}{4}$ by definition. This means that the interval $0 \leq t \leq \frac{L_R T}{2}$ is divided into N_1 subintervals, over which $q_R(t)$ is linear. The parameters q_i are the values at the endpoints of these intervals and $q_{N_1} = q_R(L_R T/2) = \frac{1}{4}$ due to symmetry. Then $q_R(t)$ for $L_R T/2 < t \leq L_R T$ is given by symmetry. Now the set of q_i , $i = 0, \dots, N_1 - 1$ denoted $\{q_i\}$ maximizing the upper bound, has to be found. By choosing N_1 large, it is possible to approximate every continuous phase response $q_R(t)$ (except at $t=0$ and $t=L_R T$), that corresponds to a symmetric frequency response $g_R(t)$. The results show that $q_0 \neq 0$ in general give the optimum. Note that all the results given are for the optimum receiver in the subclass of receivers defined in eq (109).

VII. NUMERICAL RESULTS

In this section both distance results and simulated error probability results will be given.

Distance results

First distance results for some smooth raised cosine schemes with reduced-complexity receivers are presented. The distance measure introduced in section IV is used. The observation interval length is always measured in the receiver tree. The results are presented as the minimum normalized squared Euclidean distance as a function of modulation index h and observation interval length N . This is done for a number of pairs of transmitter and receiver pulse shapes $g_T(t)$ and $g_R(t)$. Results are given for both optimum pulses $g_R(t)$ and also for ad-hoc chosen pulses. Comparisons will always be made to the optimum receiver based on $g_T(t)$.

In [3], [4], [5] a variety of distance results are given for different receivers. One of these are based on the "truncated" Raised Cosine scheme of length L symbol intervals, and truncated to L_1 symbol intervals, denoted LRCL $_1$, defined by

$$q(t) = \begin{cases} 0 & ; t \leq 0 \\ \frac{1}{2LT} \left[\left(t + \frac{L-L_1}{2} T \right) - \frac{LT}{2\pi} \sin \left(\frac{2\pi \left(t + \frac{L-L_1}{2} T \right)}{LT} \right) \right] & ; 0 < t < L_1 \cdot T \\ \frac{1}{2} & ; t \geq L_1 \cdot T \end{cases} \quad (110)$$

Note that this is not equivalent to a truncation of $g(t)$ for a LRC scheme to length L_1 symbol intervals and then define $q(t) = \int_{-\infty}^t g(\tau) d\tau$. This is instead a kind of "truncation" of the phase response directly, with respect to the properties of $q(t)$ that $q(t) = 0$, $t < 0$ and $q(t) = \frac{1}{2}$, $t > L_1 T$.

Considering the results below for the optimum reduced-complexity receivers, note that the scheme is optimum only for a specific modulation index. In the graphs also the upper bound for the optimum receiver, the upper bound for the optimum reduced-complexity receiver and the upper bound for an ad-hoc reduced-complexity receiver are given. One figure shows the phase response of the

optimum mismatched receiver. In this figure the corresponding $L_{R\text{REC}}$ and $L_{T\text{RCL}_R}$ phase responses are given as comparison. The optimum reduced-complexity receiver schemes is denoted $L_{T\text{RC-L}_R\text{Th}}$, where L_T and L_R is the length in symbol intervals of the transmitter respectively receiver pulse and h is the modulation index, at which this receiver is optimum. $L_{T\text{RC}}$ is the transmitted scheme to which this receiver pulse is optimum. The ad-hoc reduced-complexity receivers are in the same manner referred to as transmitted scheme - receiver scheme, e.g. 3RC - 1REC. For all the piecewise linear pulses considered the maximization is done with $N_1=10$, i.e. the interval $0 \leq t \leq L_R T$ is divided into 20 subintervals.

The first transmitted scheme considered is 2RC with a receiver based on a pulse of length T . Table 1 gives the exact upper bounds for some selected schemes and also the optimum reduced-complexity scheme. It is seen that the gain is very small. The results for the 1REC receiver are very close to the corresponding results for the optimum reduced-complexity receiver, and also quite close to the overall optimum receiver.

Scheme h	2RC-1Th	2RC-1REC	2RC-2RC1	2RC-1RC	Opt 2RC
1/4	0.6537	0.6532	0.6515	0.6337	0.6639
1/2	1.9316	1.9316	1.9276	1.8690	1.9666
3/4	2.5753	2.5729	2.5677	2.4653	2.6559

Table 1. Upper bounds for various receivers to 2RC.

Figure 22 shows the result for 2RC-1T0.50 for various h -values. Here it is also seen that 1REC is close to optimum. It is also known [5] that the optimum pulse shape $g_R(t)$ is close to the 1REC shape. The observation length needed to reach the upper bound is also short. For all these schemes the reduced-complexity receiver performs asymptotically close to the optimum Viterbi receiver. It is also surprisingly, how little the distances varies when the phase response is changed.

Next we consider a transmitted 3RC scheme and a receiver of length one symbol interval, i.e. a complexity reduction factor of 4. The upper bounds are given in table 2 for various receivers. The optimum phase responses for these various h -values differ quite much [5], but nevertheless the distance is close to the values for the 1REC-receiver. Not only the upper bounds but also the minimum distances are close [4]. The optimum pulse shape is something inbetween 1REC and 3RC1 [5]. Figure 23 shows the result for 3RC-1T0.50. In this case the gain for the optimum reduced-complexity compared to a 1REC receiver is very small.

Scheme h	3RC-1Th	3RC-1REC	3RC-3RC1	3RC-1RC	Opt 3RC
1/4	0.4407	0.4384	0.4368	0.4181	0.5214
1/2	1.4866	1.4846	1.4630	1.4197	1.7647
3/4	2.4566	2.4511	2.3994	2.3403	2.9918

Table 2. Upper bounds for various receivers to 3RC.

The next results correspond to a transmitted 4RC scheme and a receiver based on a pulse of length one symbol interval. Thus the complexity reduction factor is 8. The upper bound for various receivers to this scheme is given in table 3.

Figure 24 shows the results for 4RC-1T0.50. The same conclusions as before still holds, only a very small gain compared to a 1REC receiver is obtained. Note that 4RC-1T0.50 gives a smaller upper bound than 4RC-1REC for large h -values.

Scheme h	4RC-1Th	4RC-1REC	4RC-4RC1	4RC-1RC	Opt 4RC
1/4	0.2396	0.2391	0.2310	0.2296	0.4189
1/2	0.8777	0.8773	0.8339	0.8487	1.5135
3/4	1.6523	1.6437	1.5509	1.5967	2.8689

Table 3. Upper bounds for various receivers to 4RC.

The conclusion for the above schemes, with a receiver pulse of length one symbol interval is that the 1REC receiver performs very close to the optimum reduced-complexity receiver. The optimized receivers give a very small gain compared to the 1REC receiver.

The following figures deal with receivers based on a pulse of length 2 symbol intervals. Figure 25 shows the results for 3RC-2T0.25. In this case the phase response is closer to a 3RC2 pulse than to 2REC [5]. It is also known from [4] that 3RC-3RC2 has better distances than 3RC-2REC. The exact upper bounds for these schemes are given in table 4. From [5] it is known that the phase tree given by the optimum $g_R(t)$ pulse, approximates the 3RC phase tree very good. In the distance graph it is also seen that the distance is very close to the upper bound for the optimum 3RC receiver. It is also seen that the optimum $g_R(t)$ at $h=1/4$ is not optimum for larger h -values. For these h -values 3RC-3RC2 performs better than 3RC-2T0.25.

Figure 26 shows the results for 3RC-2T0.50. Now the phase response is even closer to 3RC2 [5]. Then figure 27 shows the graph for 3RC-2T0.75. Now the branches in the phase tree are varying a little bit more [5]. From the distance graph it is seen that this scheme has a better upper bound than the previous two for larger h -values, as of course should be the case. This is also true for the minimum distances for observation length between 1 and 5 symbol intervals [5].

Scheme h	3RC-2Th	3RC-2REC	3RC-3RC2	3RC-2RC	Opt 3RC
1/4	0.5197	0.4855	0.5108	0.4505	0.5214
1/2	1.7588	1.6327	1.7339	1.5200	1.7647
3/4	2.9780	2.7296	2.9550	2.5445	2.9918

Table 4. Upper bounds for various receivers to 3RC.

Yet simpler receivers can be obtained by using phase responses which are piecewise constant over a sub-symbol interval T/q according to eq. (101). Distance calculations have been performed for 2 such simplified binary schemes, where the receiver phase tree is based on a pulse of length $2T$. Figure 28 shows the distances for a binary 3RC transmitter and a sampled 2REC receiver with $q = 1, 2$ and 4 . The case $q = \infty$ (2REC receiver) is also shown. It is seen that already $q = 2$ or 4 yields a system close to $q = \infty$. Sometimes the resulting $d_{\min}^2$ can in fact be larger (for certain h -values).

Previously the 3RC2 receiver was found to be a good suboptimum receiver for transmitted 3RC. In figure 29 the transmitted 3RC signal is considered to be detected in a sampled 3RC2 receiver. The cases $q = 1, 2$ and 4 are considered together with $q = \infty$. When $q=4$ the loss is only 0.12 dB compared to an optimum 3RC receiver. This could be compared to 0.07 dB for the 3RC2 receiver and $q = \infty$, considered previously. When $q=4$ only one filter in each of the quadrature components [4] is needed. Some extra processing is required however.

Now transmitted 4RC is considered instead. Figure 30 gives the results for 4RC-2T0.25. Now the phase response has larger discontinuities at $t=0$ and $t=2T$ than 4RC2 [5]. With the complexity reduction factor of 4 in mind, this phase tree also approximates the 4RC phase tree fairly well [5]. The loss compared to the optimum 4RC receiver is about 0.35 dB. As seen from the distance graph this upper bound is reached with an observation length of 4 symbol intervals.

Figure 31 to 33 show the results for 4RC-2T0.50. Figure 31 shows the phase response for the optimum reduced-complexity receiver generating the receiver phase tree in figure 32. In figure 31 also the 2REC and 4RC2 phase responses are given. The phase tree in figure 32 can be compared with the 4RC phase tree. It is seen that the approximation of the 4RC phase tree is fairly good. The same conclusions, as for the previous scheme, can be made also here. The loss compared to the optimum 4RC receiver is about 0.35 dB.

The distance results for 4RC-2T0.75 are given in figure 34. Still the above conclusions hold. The loss compared to the optimum 4RC receiver is about 0.35 dB. This modulation index is in a weak region, and the bound is not reached for $N=5$. The corresponding bound for 4RC-4RC2 is tight [5], but a large observation length is needed to reach the bound. 4RC-2T0.75 also seems

to have a slower distance increase than 4RC-4RC2 [5], and therefore an even larger observation length is needed for this scheme.

The upper bounds for the above three schemes are given in table 5. The loss compared to the optimum 4RC receiver, with a 4 times larger complexity is as small as 0.35 dB.

Scheme h	4RC-2Th	4RC-2REC	4RC-4RC2	4RC-2RC	Opt 4RC
1/4	0.3853	0.3170	0.3504	0.2462	0.4189
1/2	1.3928	1.1681	1.2830	0.9000	1.5135
3/4	2.6436	2.2894	2.4858	1.7164	2.8689

Table 5. Upper bounds for various receivers to 4RC.

Optimizations and distance calculations for some quaternary ($M=4$) schemes have also been done. Figure 35 shows the results for quaternary 2RC-1T0.25. The discontinuities at $t=0$ and $t=T$ in the receiver phase response are now larger than for the corresponding binary scheme [5]. The complexity reduction factor is now 4. Note that the calculated values are marked with $\bullet$, $\times$, $+$ or $\square$ and the lines only connect the corresponding values. The distances shown are normalized with the bit energy E_b and are therefore comparable with the binary results. The loss compared to the optimum 2RC receiver is about 0.35 dB. The upper bound at $h=1/4$ is reached with an observation length $N=3$.

The results for 2RC-1T0.50 are given in figure 36. From the upper bound it is seen that this receiver performs better than a 2RC1 receiver only around $h=1/2$ and around $h=0.9$ to $h=1.0$. Note also that there is a catastrophic point at $h=1/2$ and the bound will never be reached. However there is a small gain also in the minimum distances at $h=1/2$ [5].

A small gain is also achieved at $h=3/4$. The results for 2RC-1T0.75 are given in figure 37. For a quaternary scheme the interesting modulation indices are $h \lesssim 0.4$ and here it is seen that a gain is achieved. The upper bounds for these receivers are given in table 6.

Scheme h	2RC-1Th	2RC-1REC	2RC-2RC1	2RC-1RC	Opt 2RC
1/4	1.2266	1.0125	1.1097	0.5577	1.3279
1/2	3.2149	3.1799	3.1983	2.1210	3.7094
3/4	2.7616	2.6325	2.7448	0.6374	3.7094

Table 6. Upper bounds for various receivers to quaternary 2RC.

Simulated error probabilities

The minimum distance calculations give the error probability behaviour for large signal-to-noise ratios. For low signal-to-noise ratios it is necessary to perform computer simulations. This is so for the optimum Viterbi receiver also. However it seems even more appropriate for the optimum reduced-complexity receiver. From the distance considerations in the previous sections it follows that relatively speaking, the error event corresponding to the minimum Euclidean distance is a more unusual event for the reduced-complexity receiver than for the optimum receiver. A specific prehistory must have occurred in the phase tree for the minimum distance error event to take place. This is not so for the optimum receiver. A minimum distance can occur for any prehistory since the phase difference is the same independent of the prehistory.

Another reason for performing computer simulations is that both the optimum reduced-complexity receiver and the optimum receiver are optimum for large signal-to-noise ratios. Thus they are actually both suboptimum receivers for low signal-to-noise ratios, and are compared by means of computer simulations.

Simulations have been made for transmitted 2RC with $h=1/2$, transmitted 4RC with both $h=1/2$ and $h=3/4$ and also transmitted quaternary 2RC with $h=1/4$. The receivers used are the optimum Viterbi receiver, the optimum reduced-complexity receiver and the ad-hoc reduced-complexity receiver with the

largest $d_{\min}^2$. All the simulation results are based on 2500 errors and the path memory in the Viterbi detector is 20 symbol intervals, which for all the schemes is enough to reach the asymptotic performance given by $d_{\min}^2$. The lower bound on the error probability for the optimum Viterbi receiver [6] is also given as comparison. Note that the simulated error probabilities are marked with x, + or $\square$ and then connected with a straight line. This is also the case for the lower bound.

In figure 38 the simulation results are given for 2RC-1REC (x), 2RC-1T0.50 (+) and optimum 2RC ($\square$), for $h=1/2$. The lower bound for optimum 2RC is also given. For these schemes the error probabilities are very close, but also the minimum distances are close. The complexity reduction for these reduced-complexity receivers are 2.

Figure 39 shows the results for 4RC-4RC2 (+), 4RC-2T0.50 (x) and optimum 4RC ($\square$), together with the optimum lower bound for $h=1/2$. It is seen that the reduced-complexity receivers perform approximately equal for low SNR's, while the optimum receiver performs about 0.1 dB better at $P=10^{-2}$. The loss at low SNR's for the reduced-complexity receivers are however, not at all as large as the minimum distance tells us. This is due to the fact that the probability for the error event giving minimum distance to occur is very low.

The above conclusions also hold for the schemes considered in figure 40. The schemes are 4RC-4RC2 (+), 4RC-2T0.75 (x) and optimum 4RC ($\square$) for $h=3/4$. Thus, with a complexity reduction of 4 it is possible to obtain a performance close to the optimum receiver. Note that the optimum reduced-complexity receiver has a loss compared to the 4RC-4RC2 receiver for low SNR's. This depends most probably on the receiver filter. From [5] it is known that the phase response is varying up and down and therefore the spectrum of the receiver filter is wider than for 4RC-4RC2. This means that the contribution of noise will increase, which is seen at low SNR's.

Figure 41 gives the results for some quaternary schemes. These are 2RC-2RC1 (+), 2RC-1T0.25 (x) and optimum 2RC ($\square$) for $h=1/4$. The lower bound for the optimum scheme is also shown. In this case 2RC-1T0.25 performs inbetween the other two schemes. Still the loss compared to the optimum scheme is low.

VIII. TRADEOFF BETWEEN COMPLEXITY REDUCTION AND PERFORMANCE DEGRADATION

To illustrate the tradeoff between complexity reduction and performance degradation we have selected data for some of the schemes in the distance calculation above and in [3] - [5]. By complexity reduction we have defined the reduction factor $M^{(L_T-L_R)}$. The number of states and filters in the suboptimum receiver is reduced by this factor compared to the optimum receiver.

By performance degradation we define the factor $10 \cdot \log_{10}(d_T^2/d_{\min}^2)$ i.e. the difference in power in dB between the optimum scheme and the suboptimum scheme for large SNR's. This is the same loss or degradation that has been referred to above.

The performance degradation varies with h while the complexity reduction factor only is a function of M and L_T-L_R .

Figure 42 to 43 show the performance degradation in dB as a function of h for various levels of complexity reduction for some binary and quaternary schemes. Compare distance graphs in section VII. Note that the performance degradation varies (slightly) with h . Also note that the optimum schemes are optimum only at a specific modulation index.

Figure 44 to 46 show the asymptotic performance degradation for the optimum reduced-complexity receiver compared to the optimum receiver versus the corresponding complexity reduction factor. The figures give the results for $h=1/4$, $h=1/2$ and $h=3/4$. It is seen that the degradation is very small for a low complexity reduction factor while it is increasing for larger factors.

IX. DISCUSSION AND CONCLUSIONS

A general method of reducing the complexity of the general optimum receiver structure for partial response CPM (digital FM) has been presented. A new distance measure has been introduced for analysing the asymptotic performance of schemes with suboptimum receivers. Coherent transmission on the additive white Gaussian noise channel is considered and the distance results give error probability behaviour for large signal-to-noise ratios. For low signal-to-noise ratios, computer simulations have been performed for some selected schemes.

The optimum reduced-complexity Viterbi receiver for partial response continuous phase modulation (digital FM) has also been found. The optimization is performed by means of a numerical maximization of the upper bound on the squared minimum Euclidean distance. The phase tree of the CPM-scheme is approximated in the receiver by a phase tree based on a shorter frequency pulse. This shorter pulse is assumed to have a piecewise linear phase response. The piecewise linear phase response, giving a maximum upper bound on the Euclidean distance, is found. The optimum receiver phase response varies with the modulation index h for a given receiver phase response length and a given transmitter phase response. This does not give an optimum reduced-complexity receiver in a global sense, since a subclass of receiver phase responses is allowed in the optimization.

The results show that the number of filters and states can be reduced significantly at the expense of a power degradation of the order of 0.5 - 1.0 dB. A graceful degradation occurs. The larger the complexity reduction, the larger the performance degradation. Partial response schemes with very smooth power spectra can be received in Viterbi receivers with a rather low number of states. The metric can either be calculated in a reduced number of linear filters sampled each symbol interval or in a much smaller number of filters (perhaps only one) sampled a number of times each symbol interval. This case requires more processing, however.

The reduced-complexity receiver is a robust receiver and the idea of approximating the phase tree can also be applied to other type of smooth pulses, e.g. TFM, GMSK, SRC [2], [17]. Similar results are expected for these schemes.

It should be noted that the class of schemes analysed above in terms of the distance between mismatched signal sets has the following feature. The power spectrum of the received signal is narrower than the power spectrum of the "optimum" received signal for which the reduced-complexity receiver is designed. A noise reducing bandpass filter might improve the performance beyond what is indicated by the distance value, which is calculated by means of matched filter in white noise arguments. Simulations should be performed to validate this hypothesis.

Even without a noise reducing bandpass filter, the distance values for the suboptimum receivers seem to be a little pessimistic for low and intermediate error probability values (signal-to-noise ratios). This could be explained by the rare occurrence of error events (relatively speaking) corresponding to the minimum distance.

An analytic optimization has still not been done and seems very difficult to do. However these results seem to be very close to an absolute optimum reduced-complexity receiver. A comparison with the results for the ad-hoc pulses give at hand that the minimum distance is not especially sensitive to small variations in the pulse shape.

In [21] a "low-complexity" method is given where the Viterbi decoder has a predetermined processing order and a reduced number of survivor signals. This method is claimed to be asymptotically optimum for certain modulation indices. So far it has only been applied to very simple piecewise linear phase responses for binary schemes. It is not clear how this method compares to ours for the more interesting cases i.e. long smoothing pulses for the binary schemes and to the quaternary $M=4$ schemes. Further work is required.

With the reduced-complexity receiver the signal spectrum is unaffected. Thus the very low sidelobes of e.g. 4RC can be maintained and still received in a receiver with a rather low complexity. It should be observed that the technique above is quite general and applies to nonbinary CPM schemes and modulation indices not necessary $1/2$. It also applies to a wide range of pulse shapes.

APPENDIX. ANALYTICAL CALCULATION OF THE MINIMUM DISTANCE FOR TRANSMITTED DUOBINARY RECEIVED IN A FULL RESPONSE CPFSK RECEIVER

The distance measure introduced in section IV have some properties which are different from the distance defined in equation (29), i.e. the "ordinary" Euclidean distance in a continuous phase signal set. One such property is that the new minimum distance is not necessarily increasing with increasing observation interval length. On the contrary, we will see from the example in this appendix that the minimum distance might actually sometimes decrease with increasing observation interval. This has also been shown in some of the figures in section VII above.

Introduction

One example of a suboptimal system is a duobinary linear phase (DBLP = 2REC) transmitter and a continuous phase frequency shift keying (CPFSK) receiver based on the 1REC pulse. The phase response functions for this case are linear functions of length $2T$ and $1T$ respectively, see figure 2. The phase trees are given in figures in section III. This appendix gives a simple example on a suboptimal scheme, which includes a partial response transmitter and a simplified receiver where the difference in the length of the pulses is one symbol interval. The behaviour of the squared Euclidean distance for this reduced-complexity system will be studied. We are going to calculate this distance for all possible phase trajectories for one symbol interval, and then look a bit forward into the phase tree.

The formula for calculating the minimum Euclidean distance between mismatched signal sets is given in (50), section IV. For the 2REC transmitter, the phase trajectory depends on one previous symbol. When calculating the distance for all possible transmitted phase trajectories for $N=1$, it is assumed that the prehistory symbol is the same in the transmitted sequence as in the receiver sequence, i.e. the error event starts at time $t=0$. We must also take into consideration that there is a time offset of $T/2$ and a phase offset of $h\pi/2$ between the transmitter and the receiver phase trees, see section V. Below, the transmitter tree is chosen as reference for the distance calculations. This is different from above. In a real modulation system the observation length is related to the receiver phase tree and therefore the receiver tree is chosen as reference. For the general behaviour however, this does not matter, only the distances are changed according to the observation length.

Distances for $N=1$

Starting with the transmitted sequence $\underline{\alpha} = -1, -1$, there is only one possible error sequence, with the same prehistory, namely $\underline{\beta} = -1, +1$. The transmitter and receiver phase in this case is given in figure A1 for the first bit ($0 \leq t \leq T$).

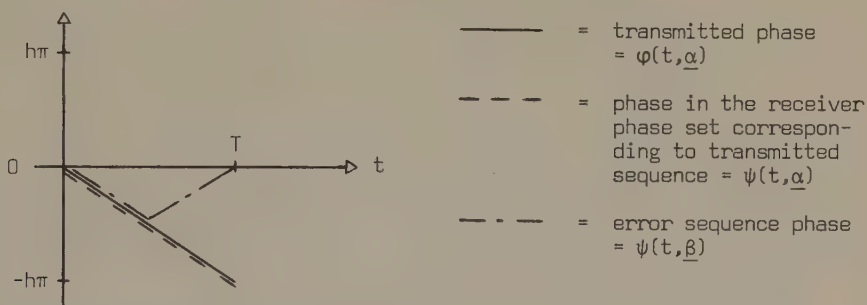


Figure A1. Transmitted $\underline{\alpha} = -1, -1$. Case 1.

For this case

$$e(t, \underline{\alpha}) = \varphi(t, \underline{\alpha}) - \psi(t, \underline{\alpha}) = 0 \quad ; \quad 0 \leq t \leq T \quad (A1)$$

and therefore

$$d_A^2 = \frac{1}{T} \int_0^T (1 - \cos(\psi(t, \underline{\alpha}) - \psi(t, \underline{\beta}))) dt = d_R^2 \quad (A2)$$

This reduces $d^2 = \frac{d_A^2}{d_R^2}$ to $d^2 = d_R^2$ and gives

$$d^2 = d_R^2 = \frac{1}{2} - \frac{\sin \pi h}{2\pi h} \quad (A3)$$

Next, for the transmitted sequence $\underline{\alpha} = -1, 1$ we have

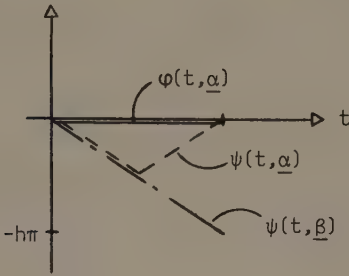


Figure A2. Transmitted sequence $\underline{\alpha} = -1, 1$. Case 2.

For this case

$$\begin{aligned} d_A^2 &= \frac{1}{T} \int_0^{T/2} \cos(\pi h \frac{t}{T}) dt + \frac{1}{T} \int_{T/2}^T \cos(\pi h \frac{T-t}{T}) dt - \frac{1}{T} \int_0^T \cos(\pi h \frac{t}{T}) dt = \\ &= 2 \frac{\sin \frac{\pi h}{2}}{\pi h} - \frac{\sin \pi h}{\pi h} \end{aligned} \quad (A4)$$

and

$$d_R^2 = \frac{1}{T} \int_0^{T/2} (1 - \cos(0)) dt + \frac{1}{T} \int_{T/2}^T \left(1 - \cos(-2\pi h \frac{t-T/2}{T}) \right) dt = \frac{1}{2} - \frac{\sin \pi h}{2\pi h} \quad (A5)$$

Thus we have

$$d^2 = \frac{\left[2 \frac{\sin \frac{\pi h}{2}}{\pi h} - \frac{\sin \pi h}{\pi h} \right]}{\frac{1}{2} - \frac{\sin \pi h}{2\pi h}} \quad (A6)$$

In the next case, the transmitted sequence is $\underline{\alpha} = 1, -1$ and the error receiver sequence is $\underline{\beta} = 1, 1$. The transmitter- and receiver-phases are given in figure A3 below.

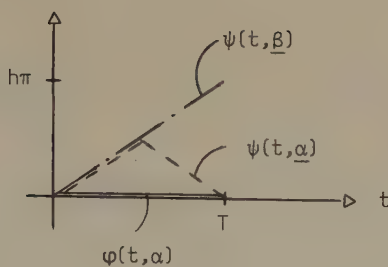


Figure A3. $\underline{\alpha} = 1, -1$. Case 3.

Here

$$d_A^2 = 2 \frac{\sin \frac{\pi h}{2}}{\pi h} - \frac{\sin \pi h}{\pi h} \quad (A7)$$

Due to symmetry (compare figures A2 and A3) this is the same result as in the previous case, see (A4). Thus the minimum Euclidean distance in this case is given by (A6). We see, that this case $\underline{\alpha} = (1, -1)$, which is an inversion of the previous case with $\underline{\alpha} = (-1, 1)$, gives the same Euclidean distance. It can be shown that it is always like this, for this mismatched system, and the conclusion is that the prehistory does not affect the distance and we need only calculate half of the distances, for instance all with prehistory -1.

This result is valid for all systems with symmetric frequency pulses $g(t)$ and therefore for all these systems we only have to calculate the Euclidean distance for the sequences with the first prehistory symbol set to +1, or in the M-ary case set to $-(M-1), -(M-3), \dots, -1$.

Using this result we don't have to calculate the Euclidean distance for case 4, in which $\underline{\alpha} = 1, 1$ and $\underline{\beta} = 1, -1$. In this case the phase is shown in figure A4.

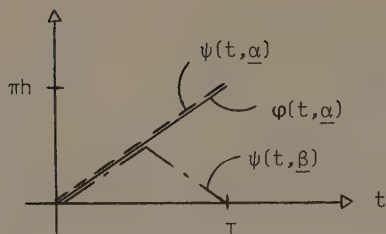


Figure A4. $\underline{\alpha} = 1, 1$. Case 4.

The minimum distance is given by equation (A3). Figure A5^{*)} summarizes the distance results above for the 4 cases. The minimum Euclidean distance for $N=1$ is the minimum of the two functions in figure A5. From the figure we see that case 1 and case 4 always give the minimum Euclidean distance in the interval $0 \leq h \leq 1$, see equation (A3).

Distances for $N=2$

Now we continue and calculate all Euclidean distances for the length $N=2$. There is a total of sixteen cases but only those with e.g. prehistory -1 have to be considered. Thus we can reduce the number to eight different cases. We can also use the result of d_A^2 and d_R^2 for $N=1$ because both d_A^2 and d_R^2 are additive each by itself in time but d^2 is not. Thus we only have to calculate the contribution from the second symbol interval, to both d_A^2 and d_R^2 , and then calculate d^2 .

We are not going through all the details for these eight cases. However the transmitter- and receiver-phases and the Euclidean distance are given for each of the 8 cases. We start with the transmitted sequence $\underline{\alpha} = -1, -1, -1$. To this sequence we can have two error receiver sequences, namely $\underline{\beta} = -1, 1, -1$ and $\underline{\beta} = -1, 1, 1$. These are the only ones, because the second symbol must be different in $\underline{\alpha}$ and $\underline{\beta}$, otherwise we are back in the length $N=1$ case. Starting with $\underline{\alpha} = -1, -1, -1$ and $\underline{\beta} = -1, 1, -1$ we have the phases

^{*)}Figure A5 is given on page I.107.

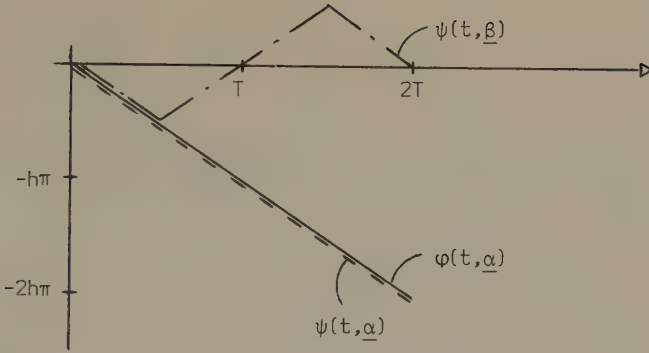


Figure A6. Case 1, $N=2$.

In this case $\epsilon(t, \underline{\alpha}) = 0$, $0 \leq t \leq 2T$ due to the fact that the phase corresponding to $\underline{\alpha}$ is the same in the receiver as in the transmitter. This also makes $d_A^2 = d_R^2$. Thus

$$d^2 = d_R^2 = \frac{3}{2} - \frac{\sin 2\pi h}{2\pi h} - \frac{1}{2} \cos 2\pi h \quad (A8)$$

In the next case we have $\underline{\alpha} = -1, -1, -1$, but $\underline{\beta} = -1, 1, 1$ and this gives the phases

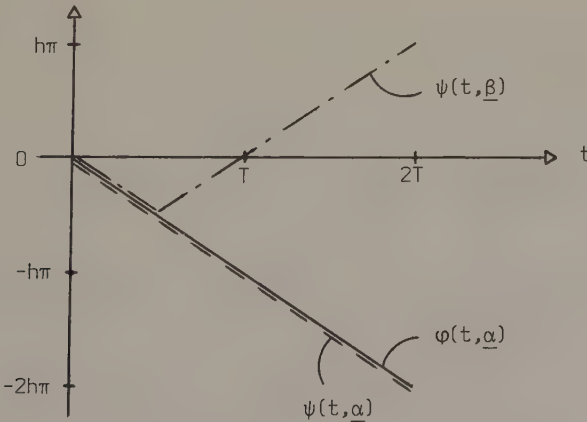


Figure A7. Case 2, $N=2$.

and the distance

$$d^2 = d_R^2 = \frac{3}{2} - \frac{\sin 2\pi h}{2\pi h} \quad (A9)$$

Next transmitted sequence is $\underline{\alpha} = -1, -1, 1$ and here we have the two error receiver sequences $\underline{\beta} = -1, 1, -1$ and $\underline{\beta} = -1, 1, 1$.

Starting with $\underline{\alpha} = -1, -1, 1$ and $\underline{\beta} = -1, 1, -1$ gives the phases

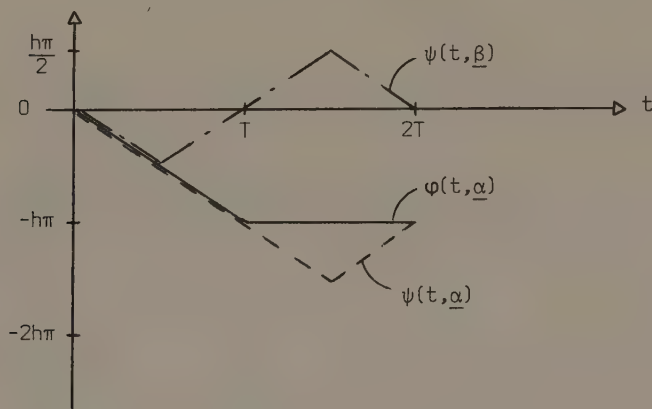
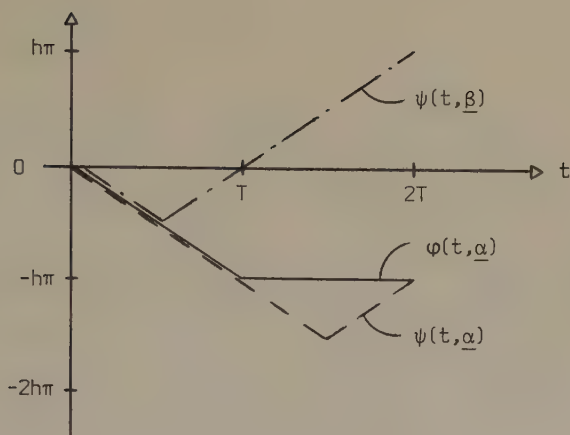


Figure A8. Case 3, $N=2$.

In this case $\epsilon(t, \underline{\alpha}) \neq 0$ and

$$d^2 = \frac{\left[\frac{1}{2} + 2 \frac{\sin \frac{\pi h}{2}}{\pi h} + 3 \frac{\sin \pi h}{2\pi h} - 2 \frac{\sin \frac{3\pi h}{2}}{\pi h} \right]^2}{\frac{3}{2} + \frac{\sin \pi h}{2\pi h} - \frac{\sin 2\pi h}{\pi h}} \quad (A10)$$

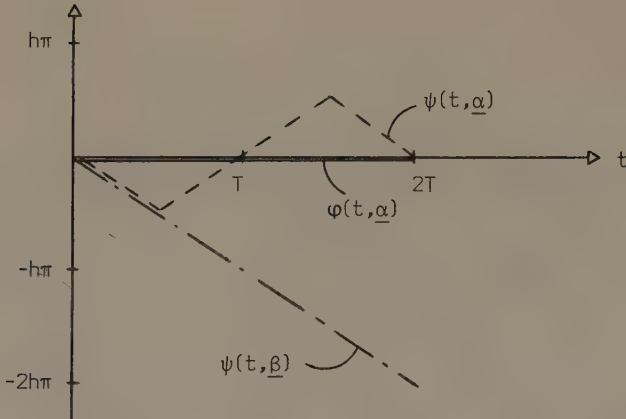
Changing $\underline{\beta}$ gives the sequences $\underline{\alpha} = -1, -1, 1$ and $\underline{\beta} = -1, 1, 1$ with the phases

Figure A9. Case 4, $N=2$.

This gives the Euclidean distance

$$d^2 = \frac{\left[\frac{1}{2} + 2 \frac{\sin \frac{\pi h}{2}}{\pi h} + \frac{\sin \pi h}{2\pi h} - \frac{\sin 2\pi h}{\pi h} \right]^2}{\frac{3}{2} - \frac{\sin 2\pi h}{2\pi h} - \frac{\cos 2\pi h}{2}} \quad (A11)$$

Next transmitted sequence is $\underline{\alpha} = -1, 1, -1$ and here we have the two error receiver sequences $\underline{\beta} = -1, -1, -1$ and $\underline{\beta} = -1, -1, 1$. Starting with $\underline{\alpha} = -1, 1, -1$ and $\underline{\beta} = -1, -1, -1$ yield the phases

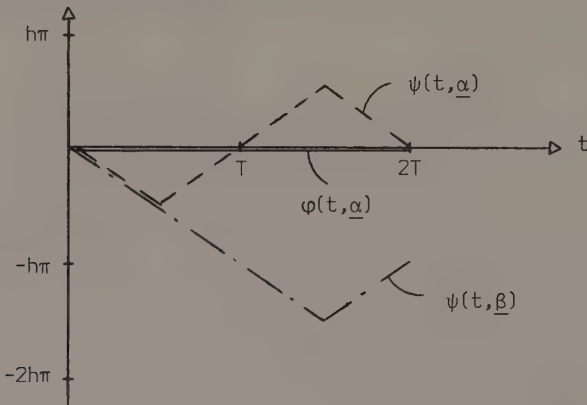
Figure A10. Case 5, $N=2$.

and the distance

$$d^2 = \left[4 \frac{\sin \frac{\pi h}{2}}{\pi h} - \frac{\sin 2\pi h}{\pi h} \right]^2 \quad (A12)$$

$$\frac{3}{2} - \frac{\sin 2\pi h}{2\pi h} - \frac{\cos 2\pi h}{2}$$

Now we change to $\underline{\alpha} = -1, 1, -1$ and $\underline{\beta} = -1, -1, 1$, which gives the phases

Figure A11. Case 6, $N=2$.

Here we have

$$d^2 = \frac{\left[4 \frac{\sin \frac{\pi h}{2}}{\pi h} + \frac{\sin \pi h}{\pi h} - 2 \frac{\sin \frac{3\pi h}{2}}{\pi h} \right]^2}{\frac{3}{2} + 2 \frac{\sin \pi h}{2\pi h} - \frac{\sin 2\pi h}{\pi h}} \quad (A13)$$

The last transmitted sequence we will take into consideration is $\underline{\alpha} = -1, 1, 1$ and here the two error receiver sequences are $\underline{\beta} = -1, -1, -1$ and $\underline{\beta} = -1, -1, 1$. The sequences $\underline{\alpha} = -1, 1, 1$ and $\underline{\beta} = -1, -1, -1$ give the phases

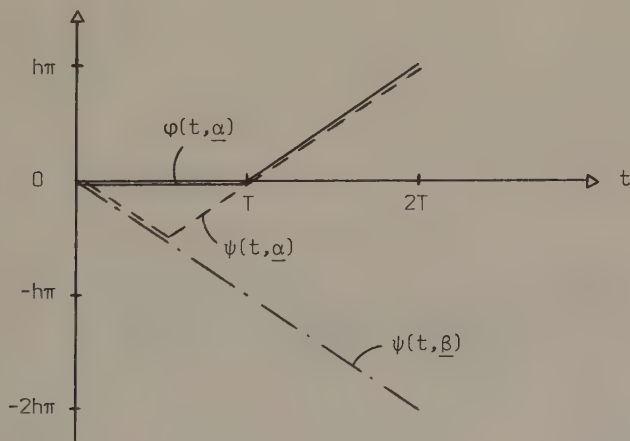
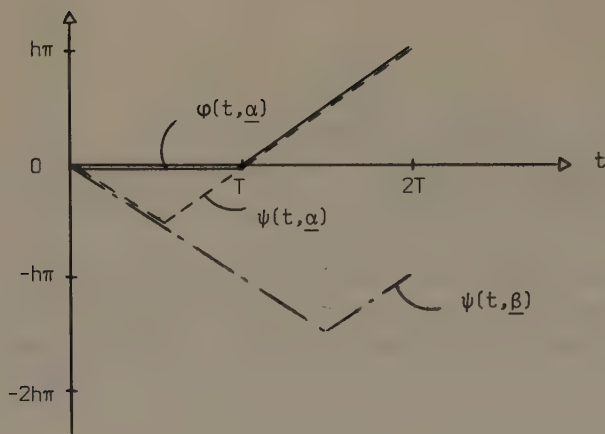


Figure A12. Case 7, $N=2$.

This gives

$$d^2 = \frac{\left[1 + 2 \frac{\sin \frac{\pi h}{2}}{\pi h} - \frac{\sin \pi h}{2\pi h} - \frac{\sin 3\pi h}{2\pi h} \right]^2}{\frac{3}{2} - \frac{\sin 3\pi h}{2\pi h}} \quad (A14)$$

Finally the sequences $\underline{\alpha} = -1, 1, 1$ and $\underline{\beta} = -1, -1, 1$ give the phases

Figure A13. Case 8, $N=2$.

This case gives

$$d^2 = \frac{\left[1 + 2 \frac{\sin \frac{\pi h}{2}}{\pi h} - \frac{\sin \pi h}{2\pi h} - \frac{\sin 2\pi h}{2\pi h} - \frac{\cos 2\pi h}{2} \right]^2}{\frac{3}{2} - \frac{\sin 2\pi h}{2\pi h} - \frac{\cos 2\pi h}{2}} \quad (\text{A15})$$

We can now summarize the $N=2$ distances. Figure A14 shows d^2 as a function of modulation index h in the interval $0 \leq h \leq 1$.

To get the minimum Euclidean distance for $N=2$ we have to take the minimum of the eight functions in figure A14^{*)}. As we can see from the diagram, it is not the same function which gives the minimum in the interval $0 \leq h \leq 1$. In the beginning it is case three that gives minimum and that was the transmitted sequence $\underline{\alpha} = -1, -1, 1$ and the error receiver sequence $\underline{\beta} = -1, 1, -1$. As we know the inverted sequence gives the same distance and therefore the sequences $\underline{\alpha} = 1, 1, -1$ and $\underline{\beta} = 1, -1, 1$ also give minimum Euclidean distance in this interval.

In the next interval it is case two which gives minimum Euclidean distance and the sequences belonging to this case are $\underline{\alpha} = -1, -1, -1$ and $\underline{\beta} = -1, 1, 1$ and the inverted sequences $\underline{\alpha} = 1, 1, 1$ and $\underline{\beta} = 1, -1, -1$. In the third interval

^{*)} See page I.108.

it is case one which gives minimum and therefore the sequences $\underline{\alpha} = -1, -1, -1$ and $\underline{\beta} = -1, 1, -1$ or the inverted sequences $\underline{\alpha} = 1, 1, 1$ and $\underline{\beta} = 1, -1, 1$. If we look at these phases we see that in the interval $3T/2 \leq t \leq T$ the transmitter and receiver phases are parallel and with a modulation index $h=1$ the distance between them are 2π . Looking at the phase modulo 2π we see that these phase trajectories coincide and therefore the distance between them are zero. If the modulation index is near $h=1$ these trajectories are very close and therefore give a small distance.

Distances for N=3

All the Euclidean distances for $N=3$ are not given because there are quite a lot of possibilities. We are going to study three possibilities of which two are especially interesting. A new phenomenon is happening, which is impossible for the Euclidean distance for matched signal sets.

Just as for $N=2$ we only have to calculate the increase in the d_A^2 and d_R^2 , depending on the third symbol interval and then add this to d_A^2 and d_R^2 for the case with $N=2$. We start with sequences $\underline{\alpha} = -1, -1, 1, 1$ and $\underline{\beta} = -1, 1, 1, 1$, thus giving the phase trajectories

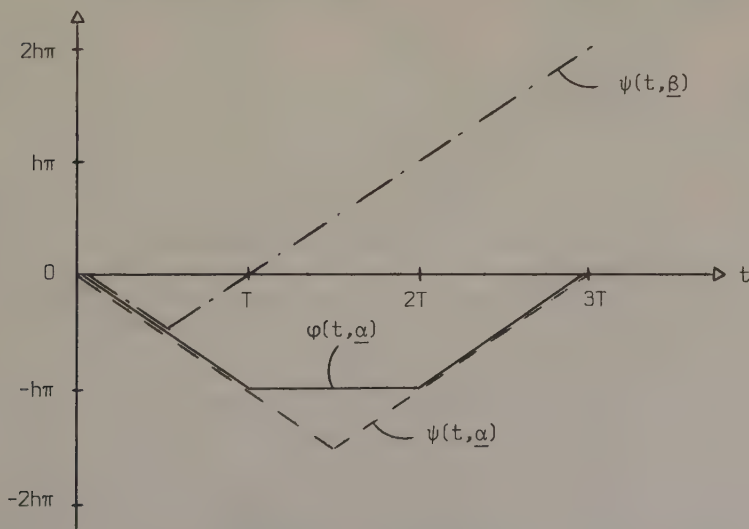


Figure A15: $N=3$, $\underline{\alpha} = -1, -1, 1, 1$; $\underline{\beta} = -1, 1, 1, 1$.

This case gives the distance

$$d^2 = \frac{\left[\frac{3}{2} + 2 \frac{\sin \frac{\pi h}{2}}{\pi h} + \frac{\sin \pi h}{2\pi h} - \frac{\sin 2\pi h}{\pi h} - \cos 2\pi h \right]^2}{\frac{5}{2} - \frac{\sin 2\pi h}{2\pi h} - \frac{3}{2} \cos 2\pi h} \quad (A16)$$

We can here make a diagram, that shows the Euclidean distance for these sequences for $N=2$ and $N=3$, see figure A16*). The behaviour of these distances is what we are used to from the distances for matched signal sets, [1], [2]. Choosing another pair of sequences, namely $\underline{\alpha} = -1, -1, -1, 1$ and $\underline{\beta} = -1, 1, 1, -1$, we have the phases

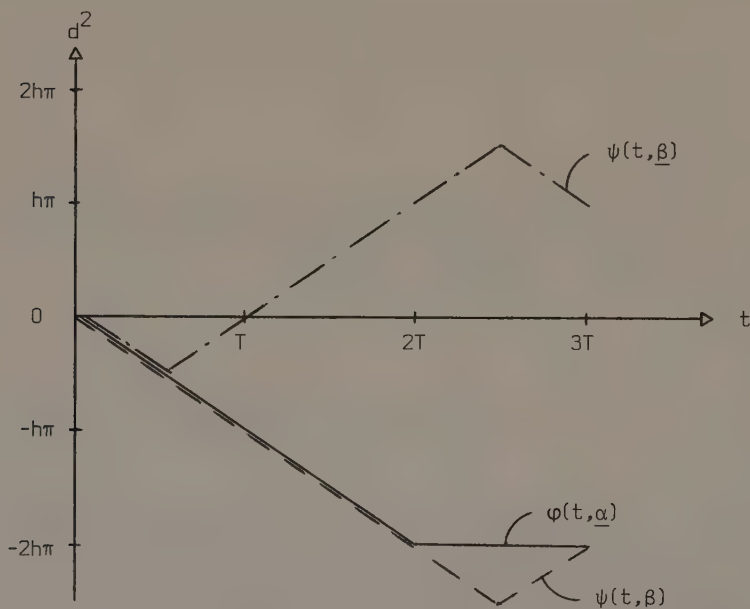


Figure A17. $N=3$, $\underline{\alpha} = -1, -1, -1, 1$; $\underline{\beta} = -1, 1, 1, -1$.

This gives

$$d^2 = \frac{\left[\frac{3}{2} + 2 \frac{\sin \frac{\pi h}{2}}{\pi h} + 3 \frac{\sin 3\pi h}{2\pi h} - 2 \frac{\sin 7\pi h}{\pi h} \right]^2}{\frac{5}{2} + \frac{\sin 3\pi h}{2\pi h} - \frac{\sin 5\pi h}{\pi h}} \quad (A17)$$

*) See page I.109.

The squared distance for $N=2$ and $N=3$ for the sequences in figure A17 are shown in figure A18*). Here we see that something happens in the interval $0.55 \lesssim h \lesssim 0.75$. The Euclidean distance for matched signal sets is an increasing function of N . Here we see that the Euclidean distance for mismatched signal sets is sometimes decreasing with N . The formula for d_A^2 shows that Δd_A^2 , the increase from the N :th symbol interval, is

$$\Delta d_A^2 = \frac{1}{T} \int_{(N-1)T}^{NT} \cos(\varphi(t, \underline{\alpha}) - \psi(t, \underline{\alpha})) dt - \frac{1}{T} \int_{(N-1)T}^{NT} \cos(\varphi(t, \underline{\alpha}) - \varphi(t, \underline{\beta})) dt \quad (A18)$$

and it is easily seen that Δd_A^2 can be negative, and therefore d^2 decreasing. The reason that this special sequence gives decreasing distance can be seen if we look at the phase modulo 2π . For modulation index $h=0.65$ the phases modulo 2π are shown in figure A19 (all phases drawn in the interval $[-2\pi, 0]$).

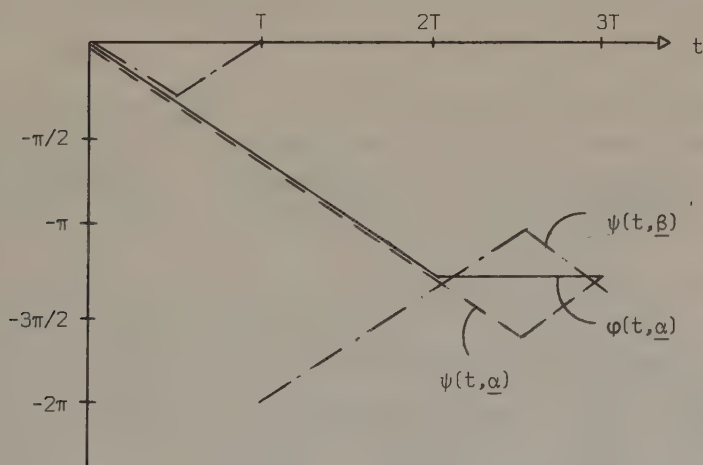


Figure A19. The case from figure A17 viewed modulo 2π with $h=0.65$.

In this case we see that the receiver phase trajectory corresponding to $\beta(\psi(t, \underline{\beta}))$ is closer to the transmitted phase trajectory ($\varphi(t, \underline{\alpha})$), than the receiver phase trajectory corresponding to the transmitted sequence $\alpha(\psi(t, \underline{\alpha}))$. In this case the receiver is going to choose the wrong symbol, which we can see from the decreasing distance. (We might say that the receiver gets more "confused" by observing 3 symbols rather than only 2.)

*) See page I.110.

One more example giving a decreasing distance is the sequences $\underline{\alpha} = -1, -1, 1, -1$ and $\underline{\beta} = -1, 1, 1, -1$ with the phases

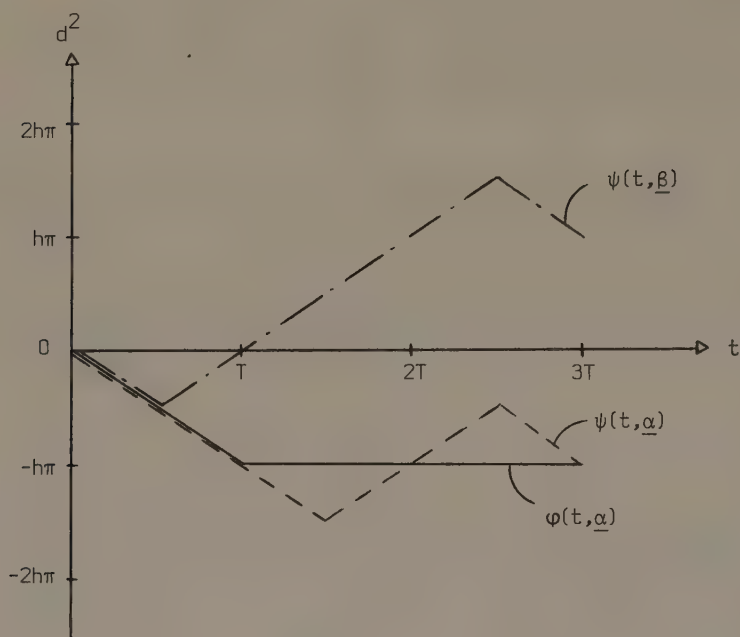


Figure A20. $N=3$, $\underline{\alpha} = -1, -1, 1, -1$; $\underline{\beta} = -1, 1, 1, -1$.

This gives

$$d^2 = \frac{\left[\frac{1}{2} + 4 \frac{\sin \frac{\pi h}{2}}{\pi h} + \frac{\sin \pi h}{2\pi h} + \frac{\sin 2\pi h}{\pi h} - 2 \frac{\sin \frac{5\pi h}{2}}{\pi h} \right]^2}{\frac{5}{2} - \frac{\sin 2\pi h}{2\pi h} - \frac{3}{2} \cos 2\pi h} \quad (\text{A19})$$

and the distances in figure A21^{*)}.

Here we have a decreasing distance in the interval $0.73 \lesssim h \lesssim 1$.

Choosing $h=0.9$ and drawing the phases modulo 2π we will have the figure A22.

^{*)} See page I.111.

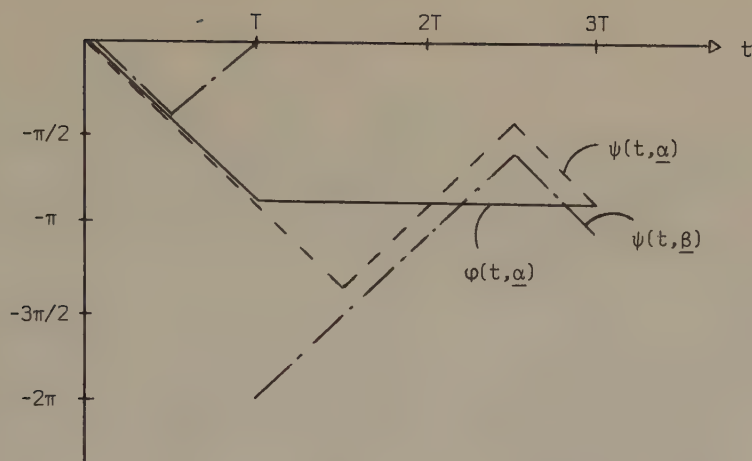


Figure A22. The case in figure A20 viewed modulo 2π for $h=0.90$.

It can be seen that the trajectory corresponding to $\underline{\beta}$ in the receiver is closer to the transmitted trajectory than the trajectory in the receiver corresponding to the transmitted sequence $\underline{\alpha}$. Thus, we will have a negative increase in d_A^2 and therefore a decrease in the Euclidean distance.

It is interesting to look at the sequences that give the minimum Euclidean distance for small modulation indices. The sequences are $\underline{\alpha} = -1, -1, 1, 1$ and $\underline{\beta} = -1, 1, -1, 1$ and the phases corresponding to these are given in figure A23.

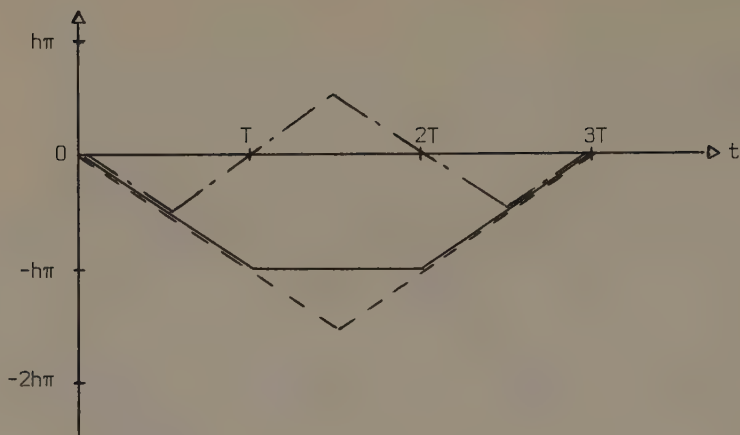


Figure A23. $N=3$, $\underline{\alpha} = -1, -1, 1, 1$; $\underline{\beta} = -1, 1, -1, 1$.

The Euclidean distance for this case is

$$d^2 = \frac{\left[1 + 2 \frac{\sin \frac{\pi h}{2}}{\pi h} + \frac{\sin \pi h}{\pi h} - \frac{\sin \frac{3\pi h}{2}}{\pi h} \right]^2}{2 - \frac{\sin 2\pi h}{\pi h}} \quad (\text{A20})$$

The distance is plotted in figure A24^{*)} together with the corresponding distance for $N=2$. Figure A24 also shows the performance degradation of the 2REC scheme with a 1REC receiver compared to the optimum receiver for 2REC.

^{*)} See page I.112.

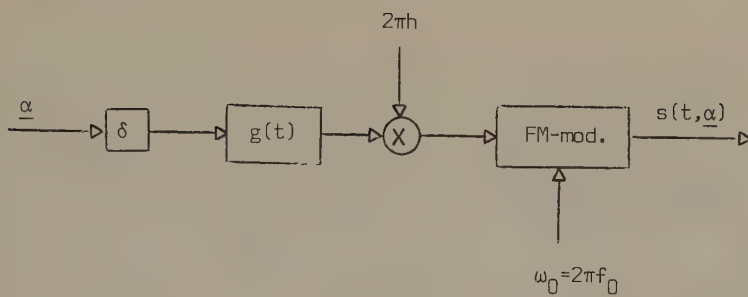


Figure 1. A conceptual modulator for Continuous Phase Modulation.

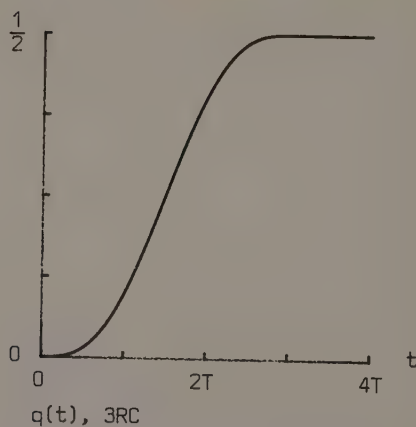
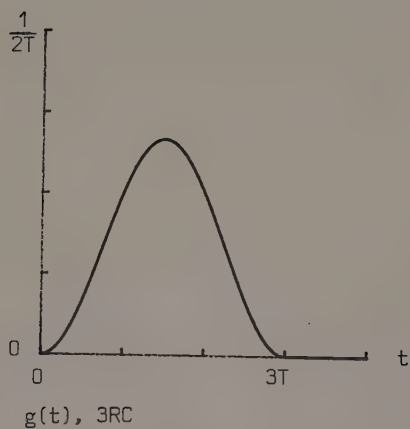
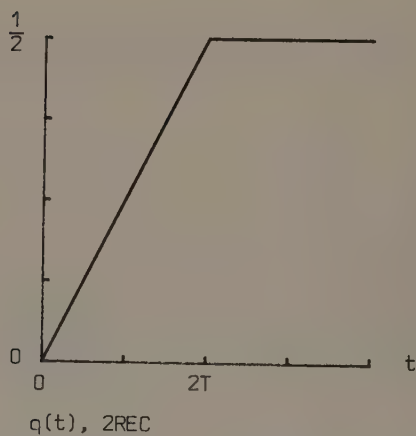
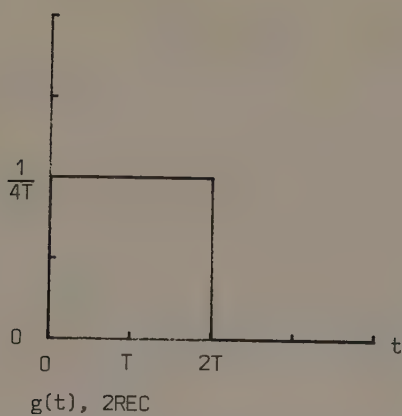
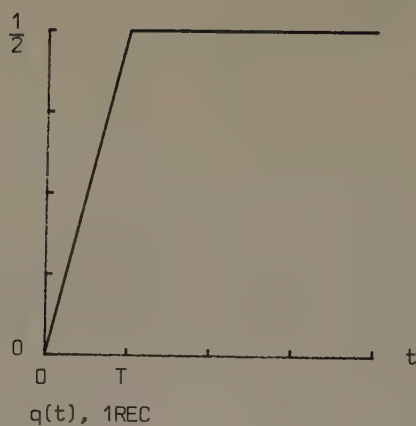
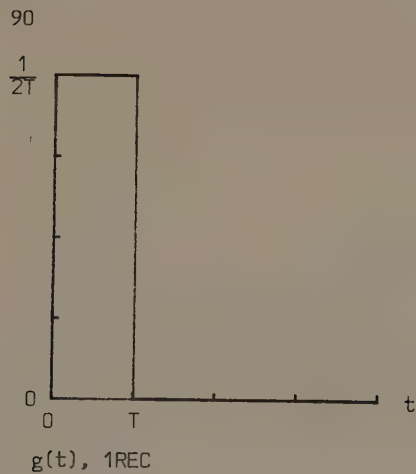


Figure 2. Pulse shapes $g(t)$ and phase responses $q(t)$ for the 1REC, 2REC and 3RC schemes.

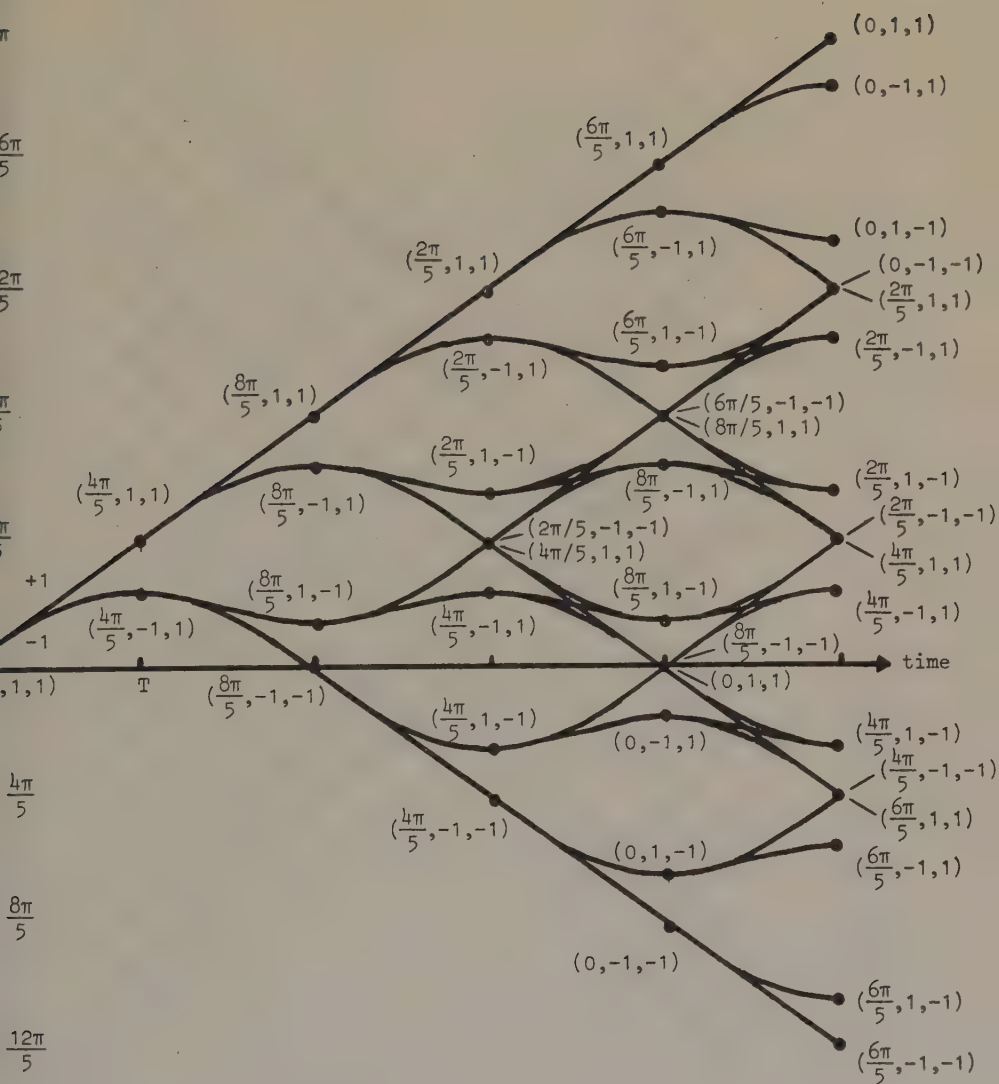


Figure 3. Phase tree for binary 3RC. The states for $h=4/5$ are also shown in the tree. Compare figure 4.

$$(\theta_n, \alpha_{n-1}, \alpha_{n-2}) =$$

$$(0, 1, 1)$$

$$(0, 1, -1)$$

$$(0, -1, 1)$$

$$(0, -1, -1)$$

$$(\frac{2\pi}{5}, 1, 1)$$

$$(\frac{2\pi}{5}, 1, -1)$$

$$(\frac{2\pi}{5}, -1, 1)$$

$$(\frac{2\pi}{5}, -1, -1)$$

$$(\frac{4\pi}{5}, 1, 1)$$

$$(\frac{4\pi}{5}, 1, -1)$$

$$(\frac{4\pi}{5}, -1, 1)$$

$$(\frac{4\pi}{5}, -1, -1)$$

$$(\frac{6\pi}{5}, 1, 1)$$

$$(\frac{6\pi}{5}, 1, -1)$$

$$(\frac{6\pi}{5}, -1, 1)$$

$$(\frac{6\pi}{5}, -1, -1)$$

$$(\frac{8\pi}{5}, 1, 1)$$

$$(\frac{8\pi}{5}, 1, -1)$$

$$(\frac{8\pi}{5}, -1, 1)$$

$$(\frac{8\pi}{5}, -1, -1)$$

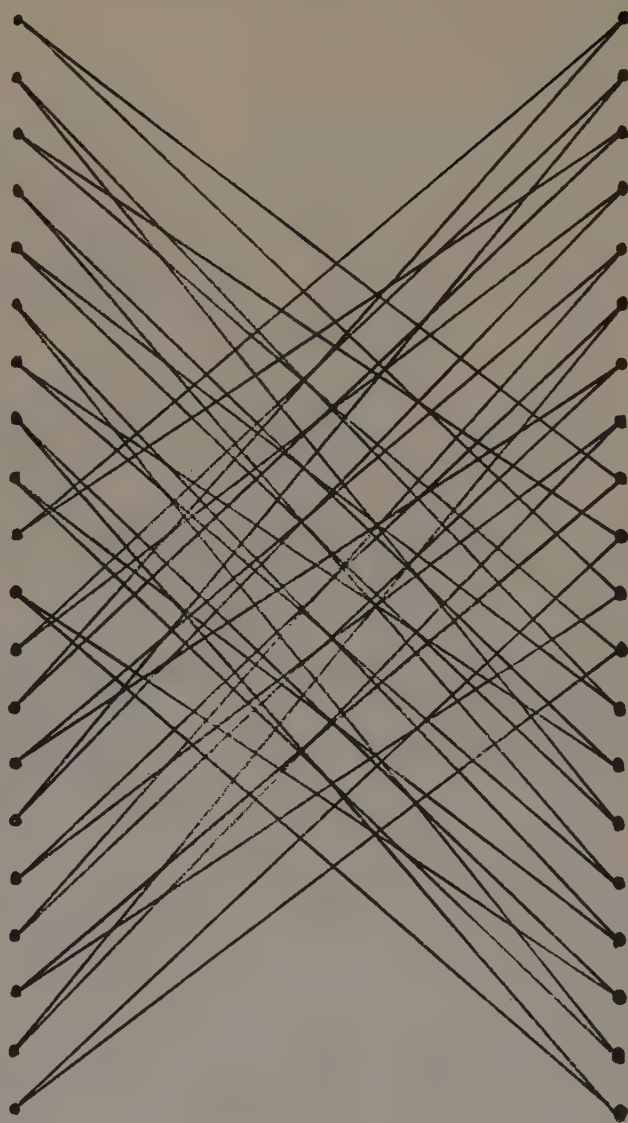


Figure 4. State trellis for binary 3RC with $h=4/5$.

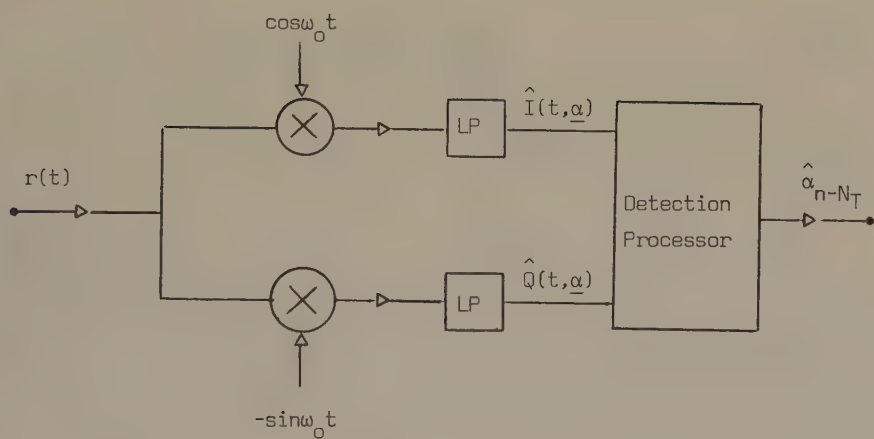


Figure 5. Basic receiver structure. The LP-filters only remove double carrier frequency.

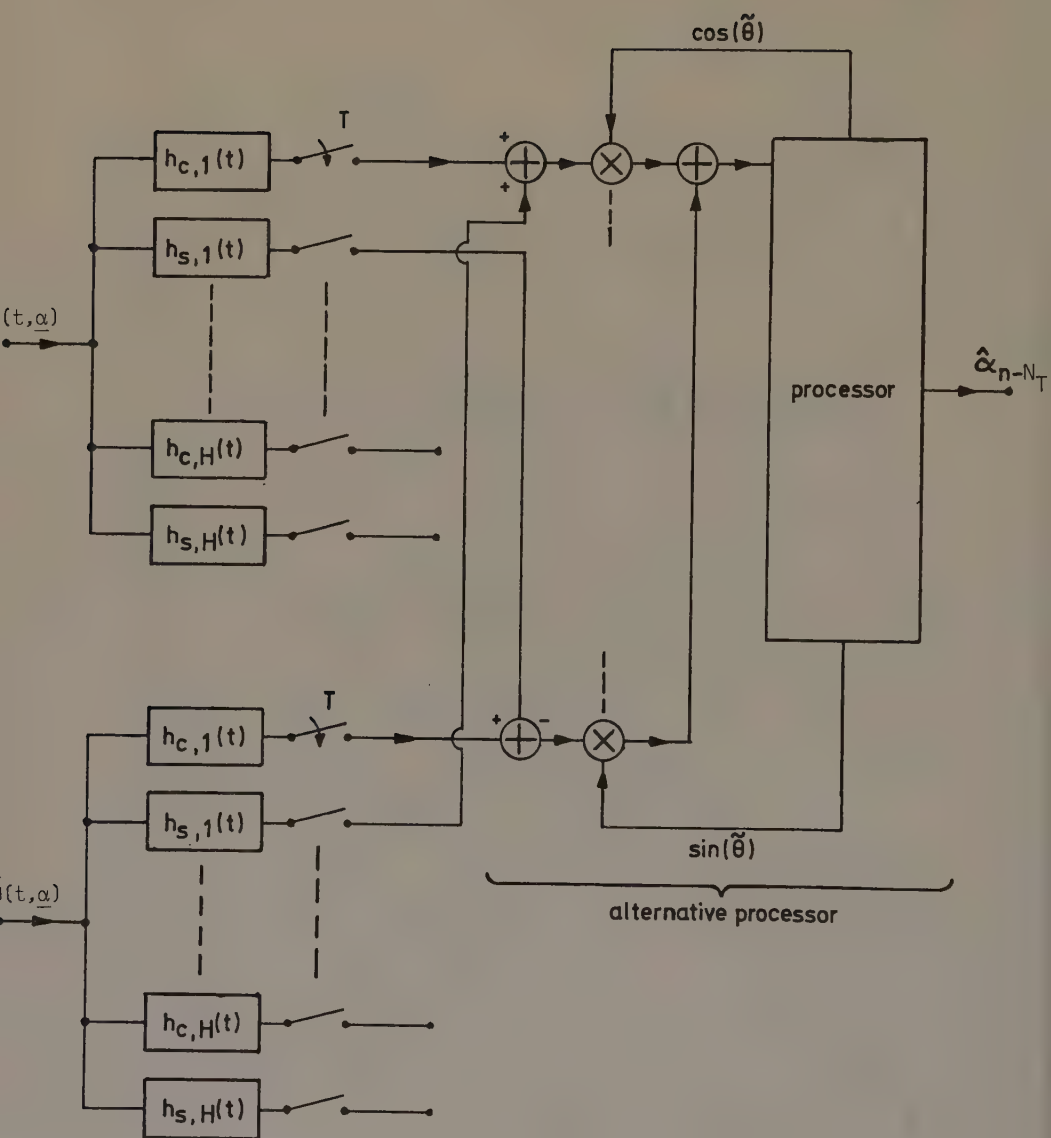


Figure 6. Matched filter receiver. $H = M^L/2$.

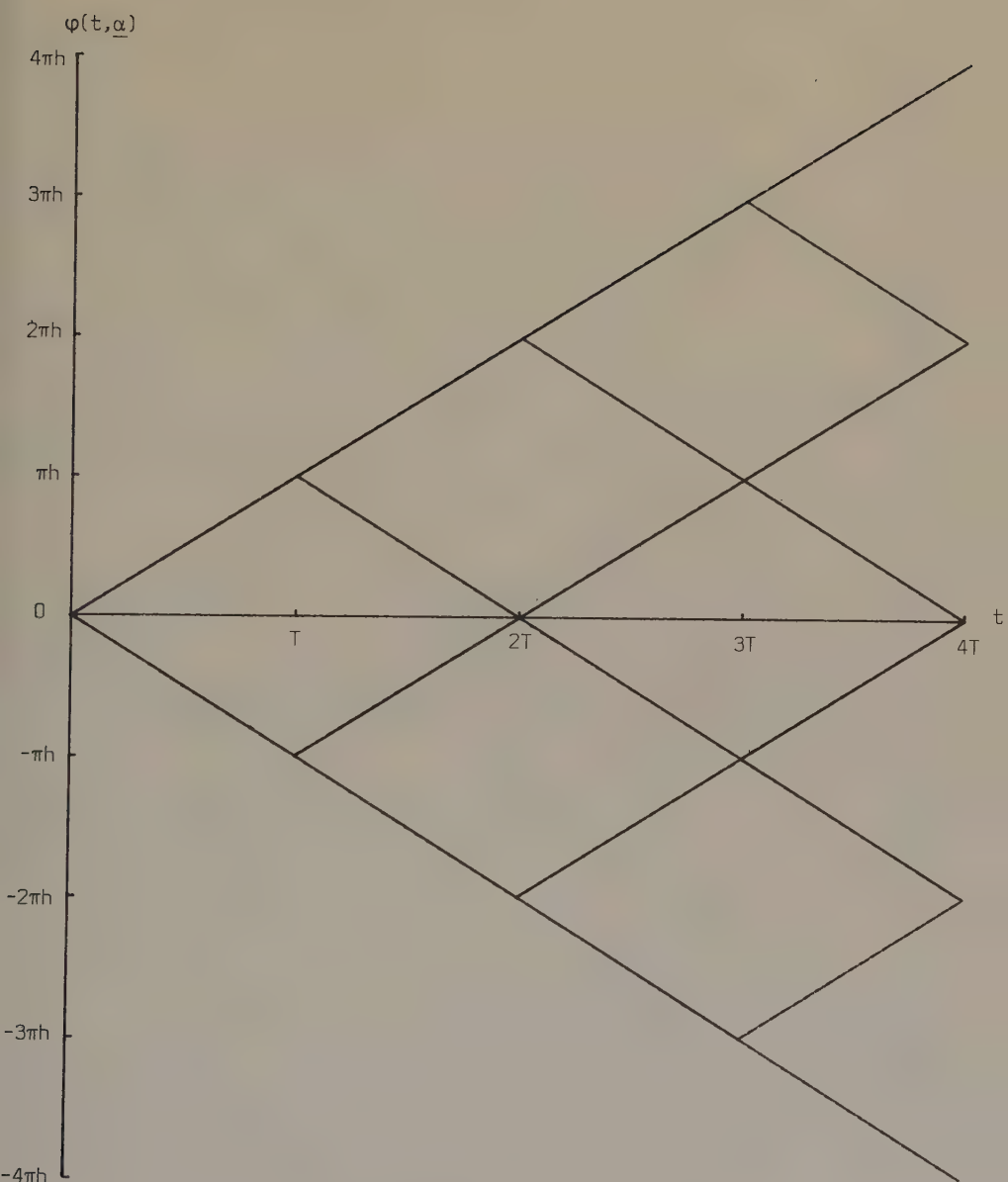


Figure 7. Phase tree for binary 1REC.

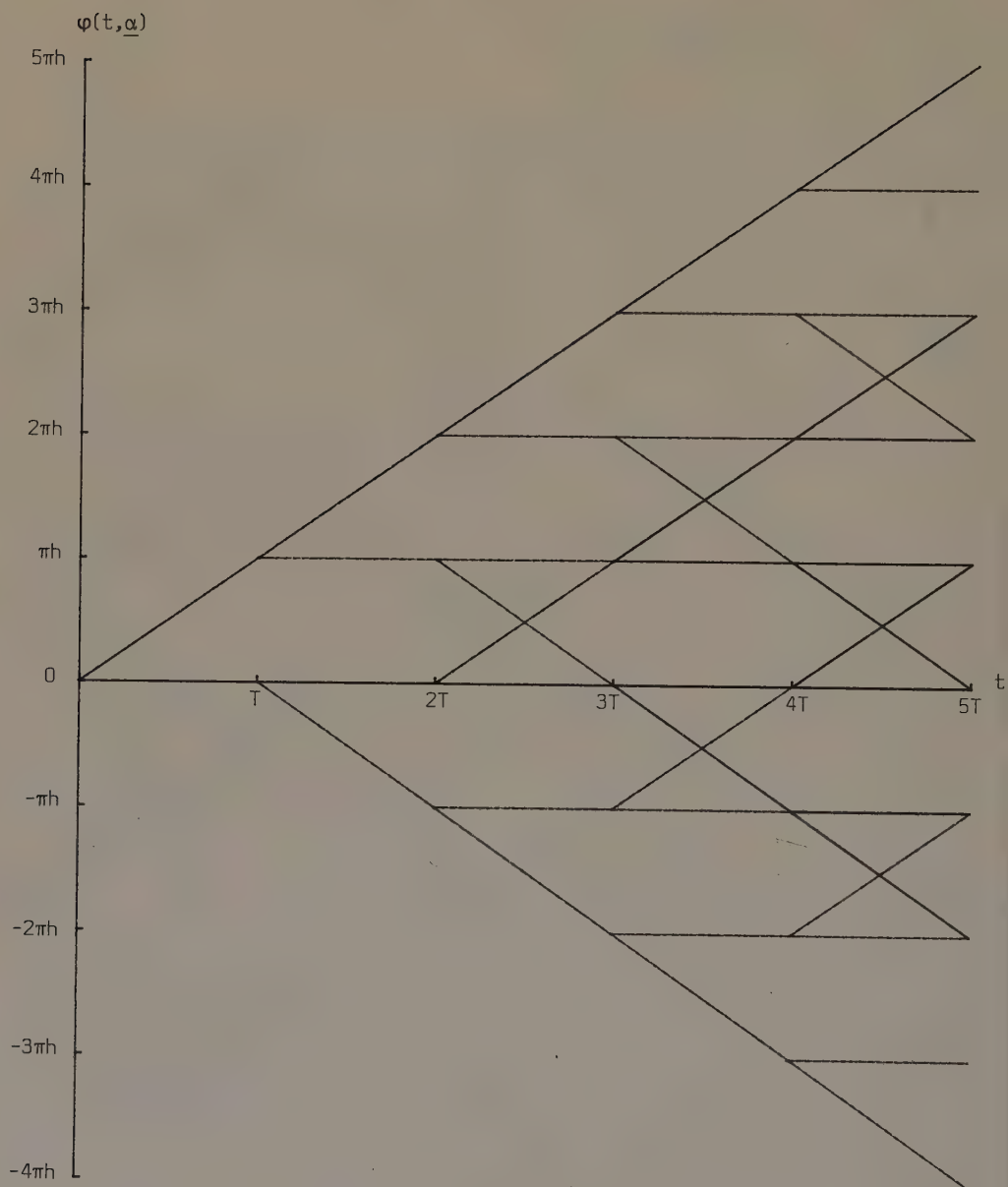


Figure 8. Phase tree for binary 2REC.

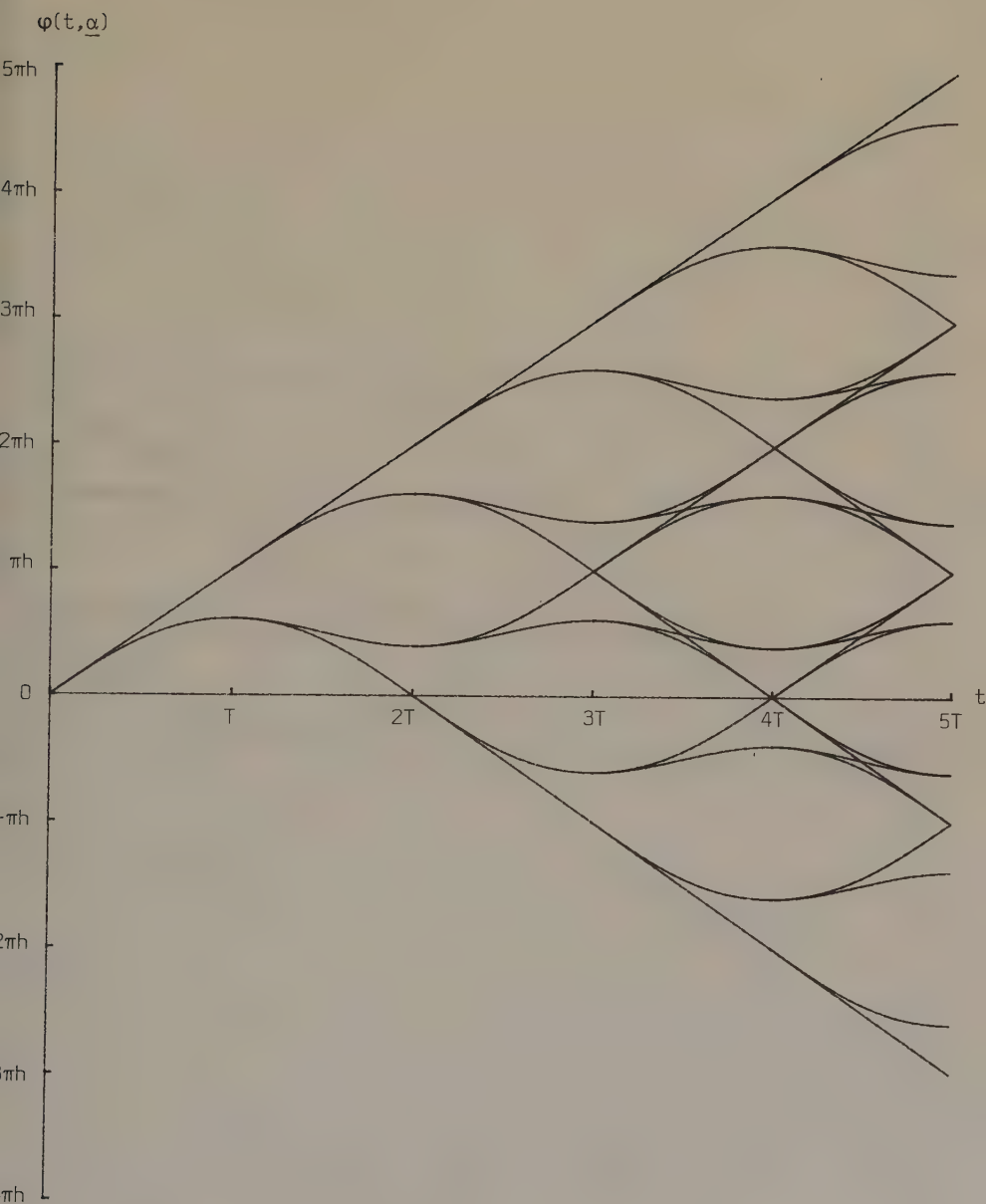


Figure 9. Phase tree for binary 3RC (also see figure 3).

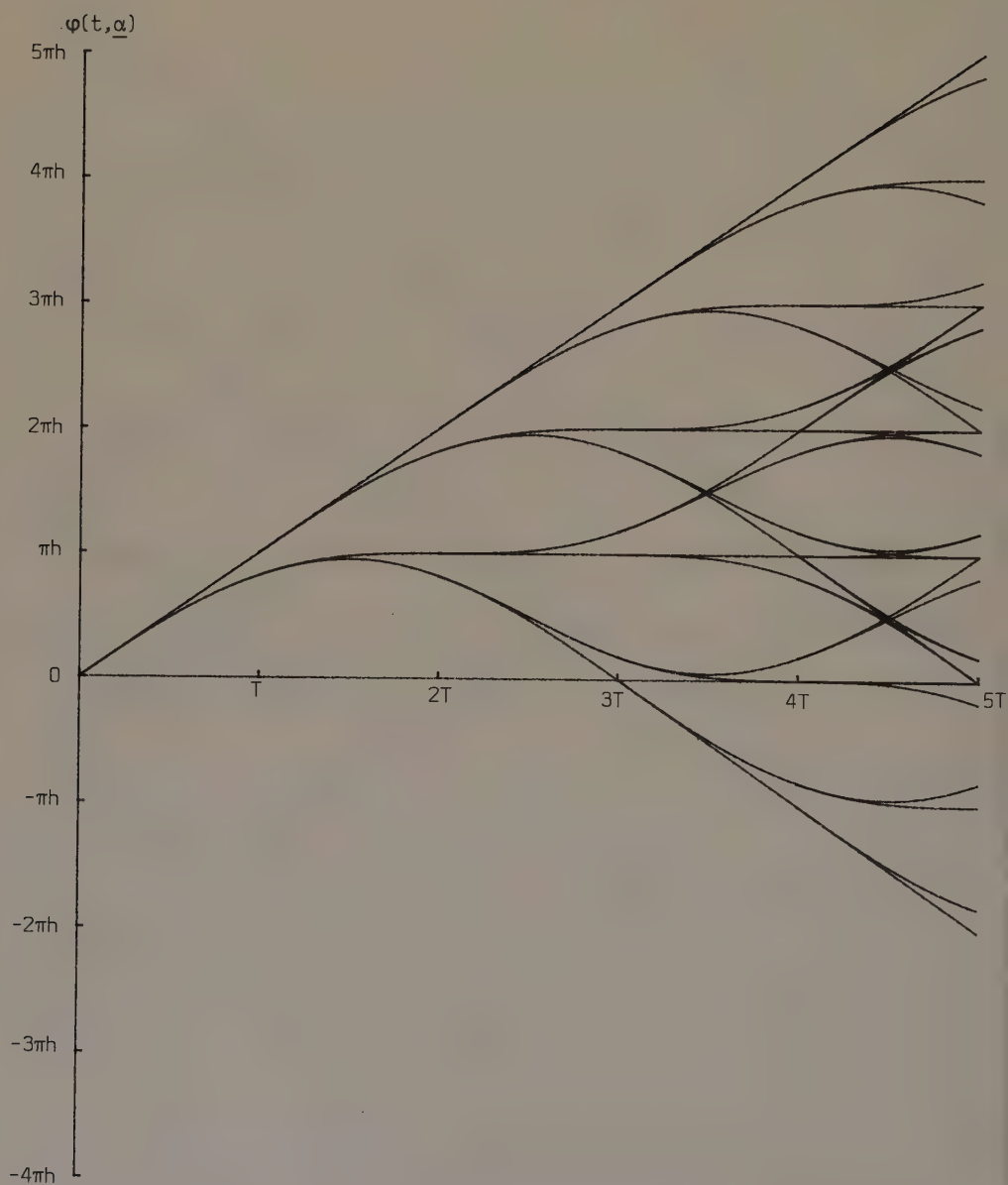


Figure 10. Phase tree for binary 4RC.

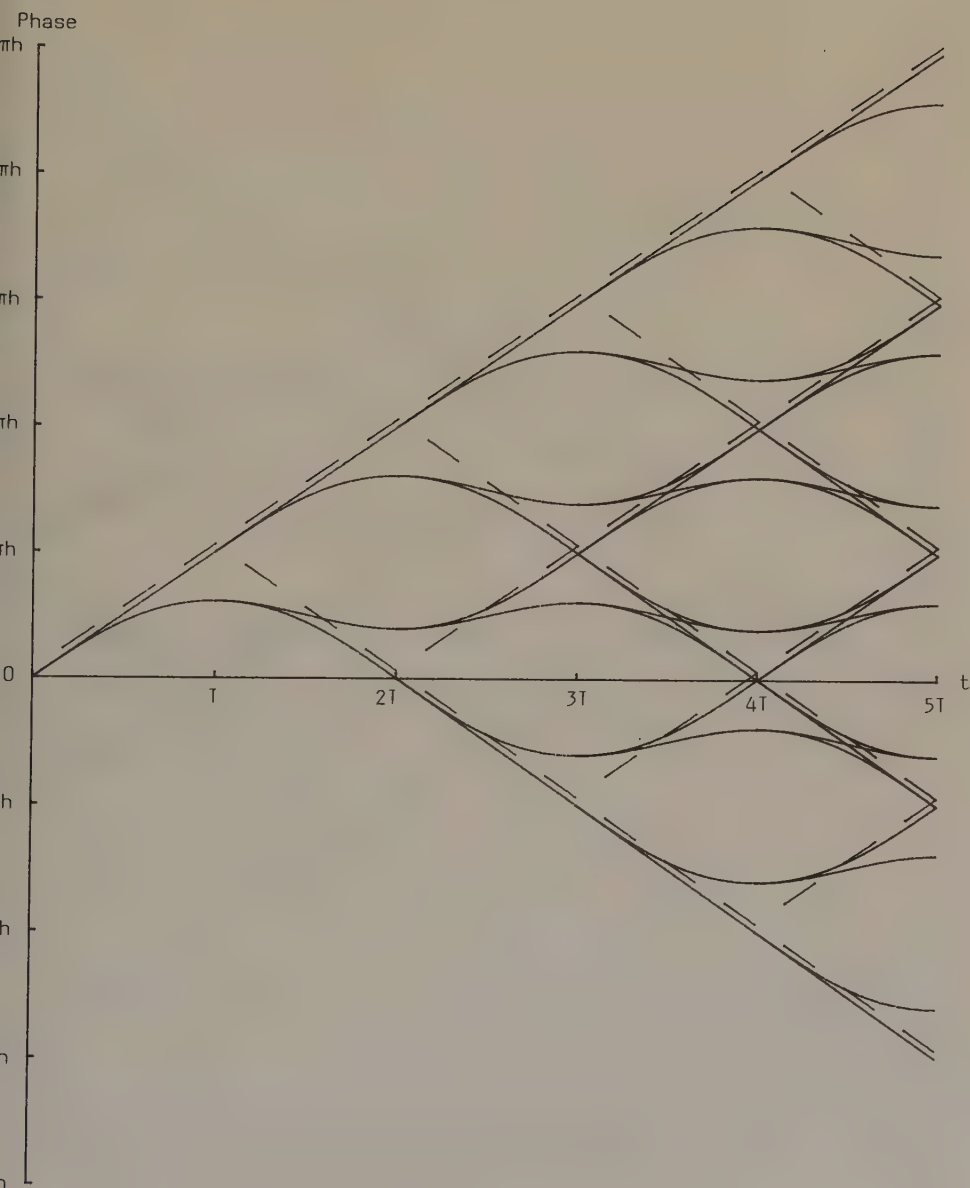


Figure 11. Transmitter phase tree (solid) based on 3RC received in receiver phase tree (dashed) based on 1REC. The phase trees are shown with correct timing and with a small phase offset.

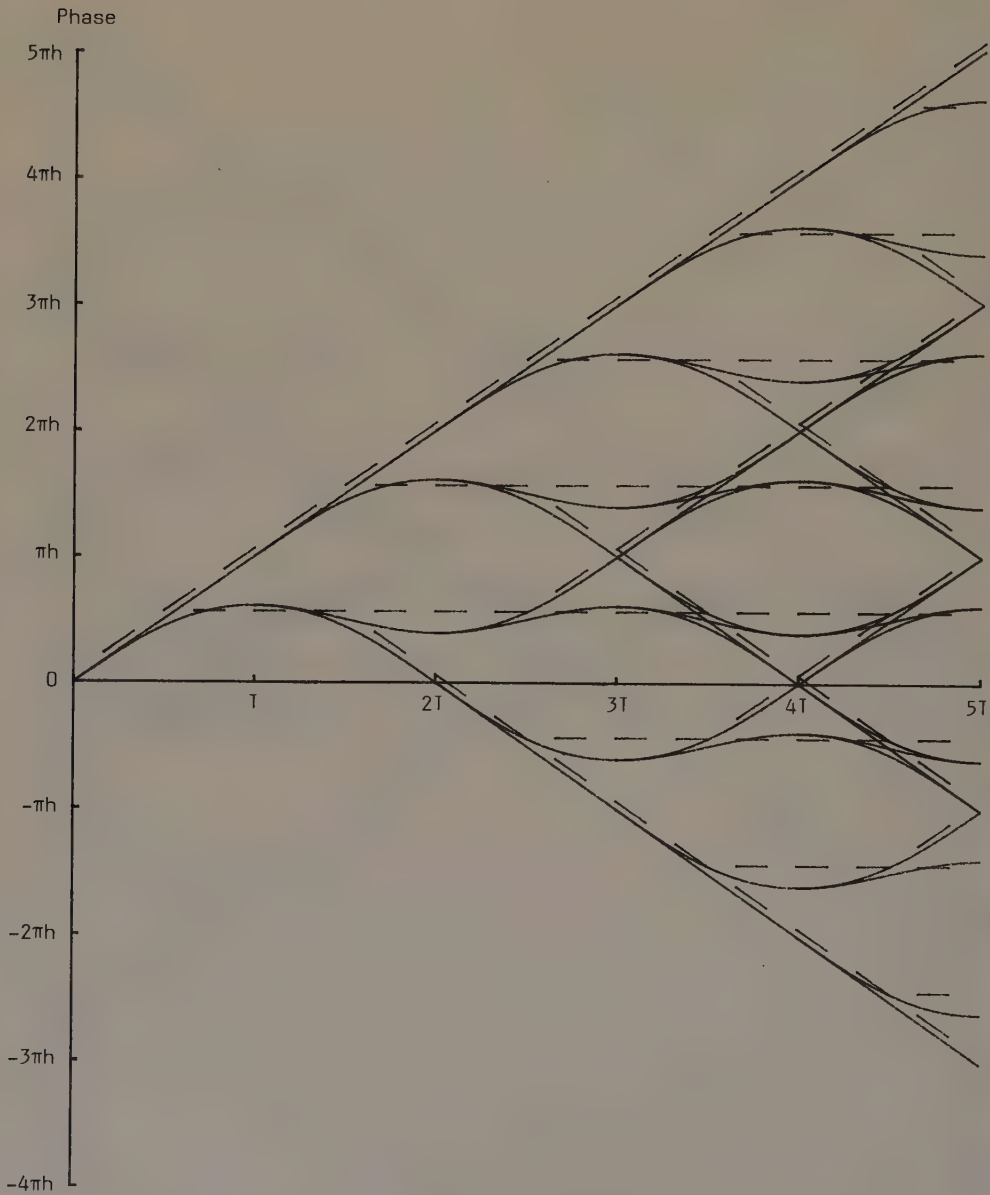


Figure 12. Transmitter phase tree (solid) for 3RC and receiver phase tree for 2REC. Correct timing, small phase offset.

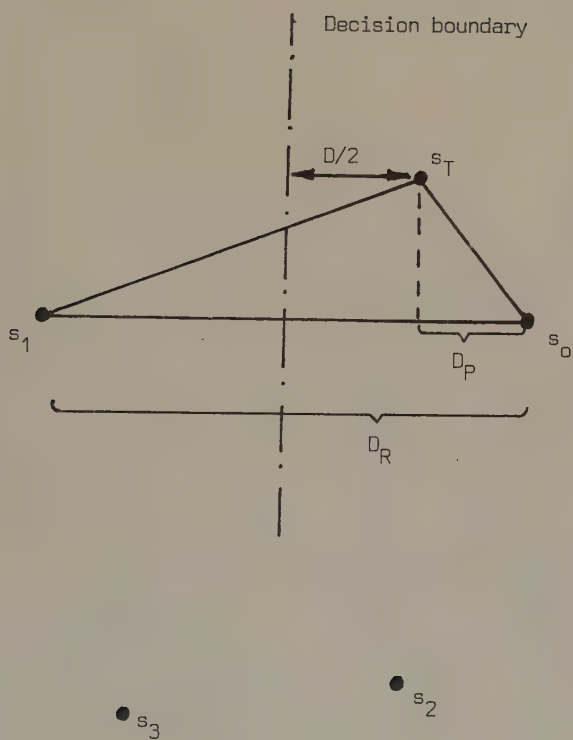


Figure 13. Calculated distances in the signal space.

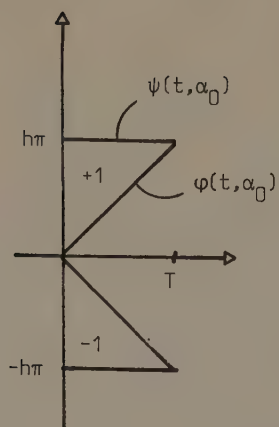


Figure 14a.

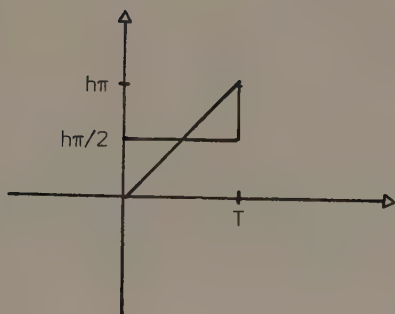
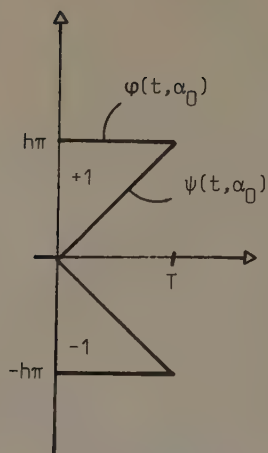


Figure 14b.

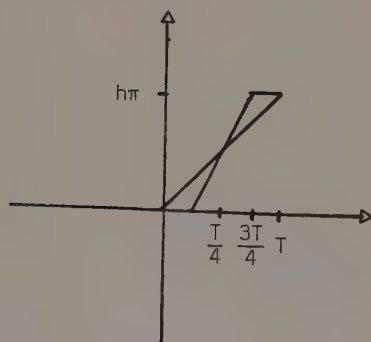


Figure 14c.

Figure 14. Example of transmitter and receiver phase.

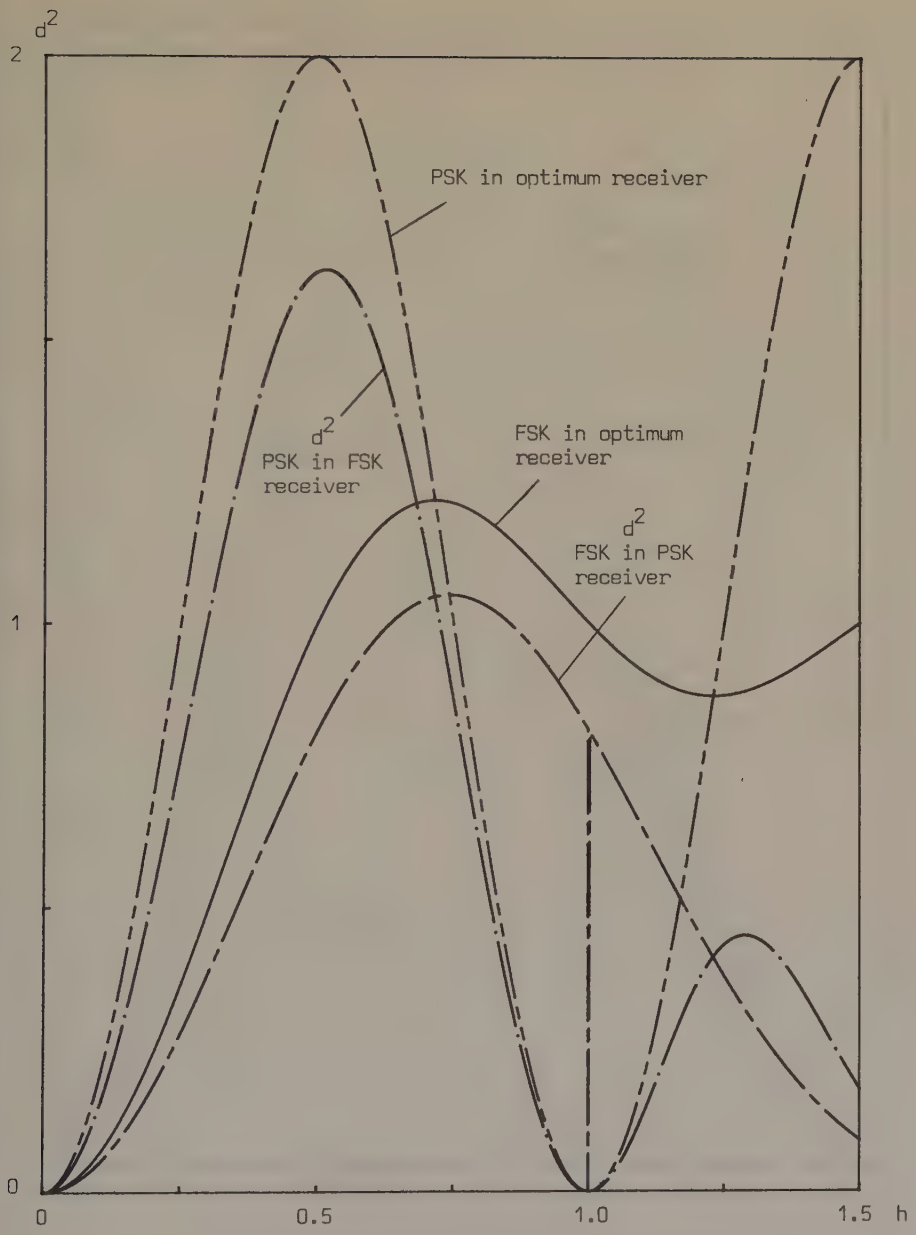


Figure 15. Euclidean distance d^2 between mismatched signal sets for transmitted FSK and PSK received in various receivers with $N=1$.

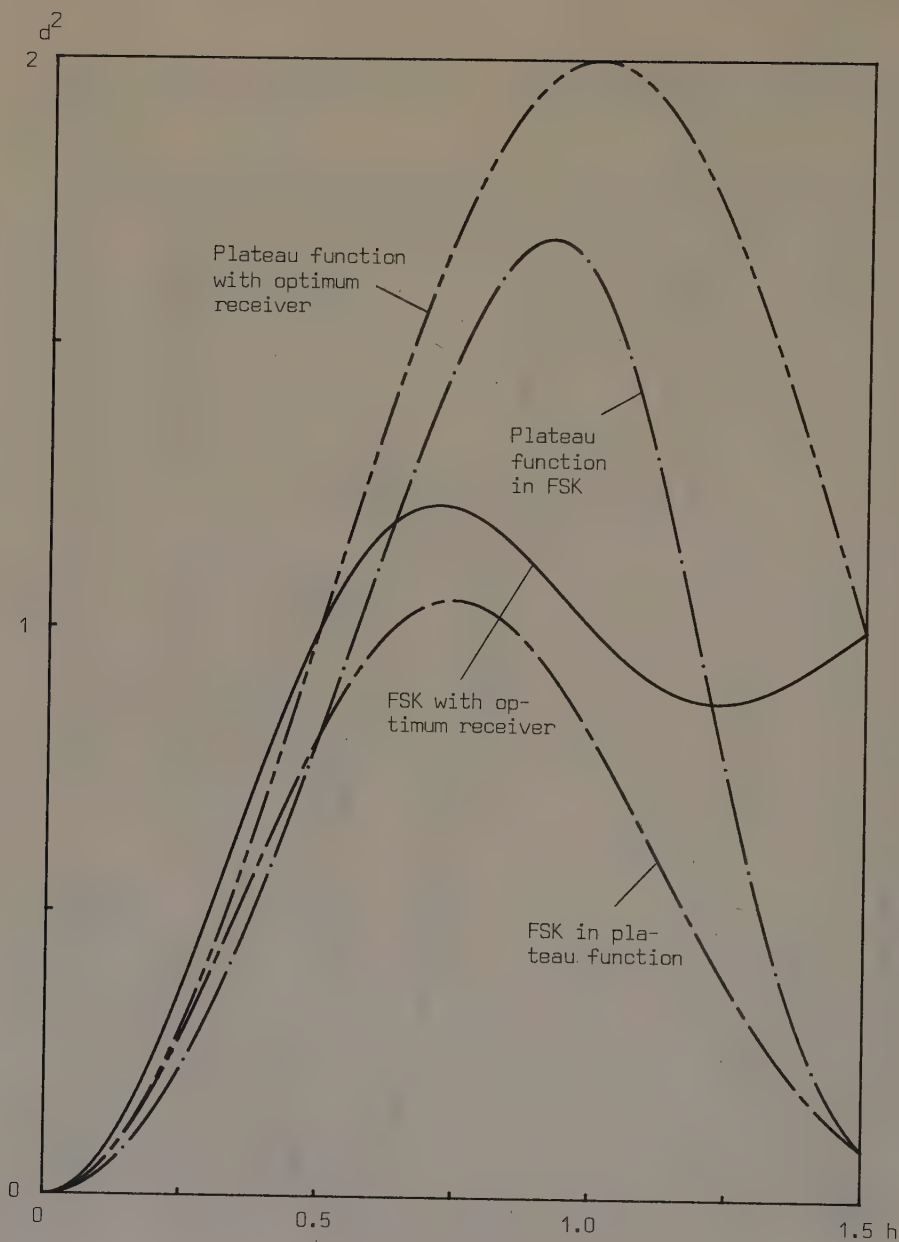


Figure 16. Distance d^2 for transmitted FSK and plateau function, see figure 15b, received in various receivers with $N=1$.

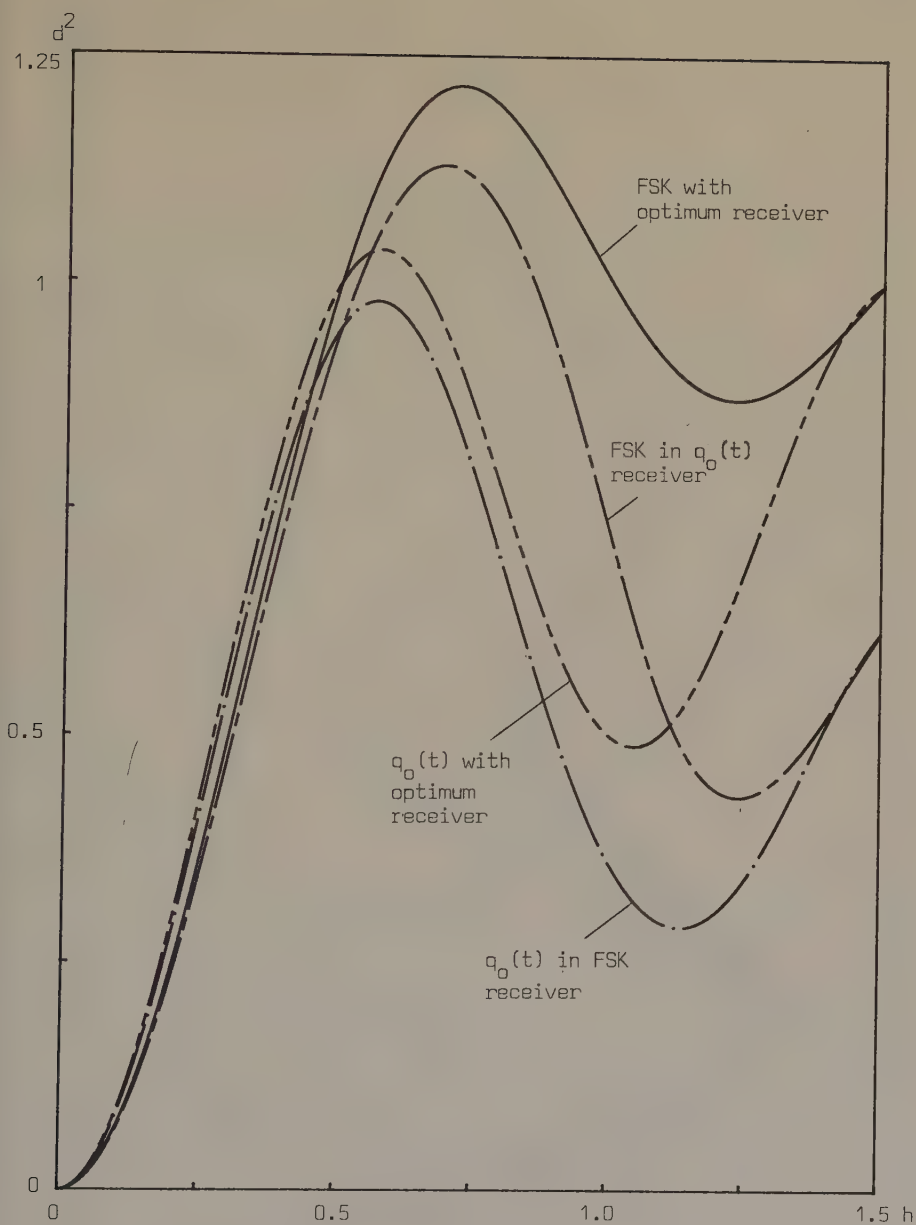


Figure 17. Distance d^2 for transmitted FSK and $q_0(t)$ defined by equation (96) received in various receivers with $N=1$.

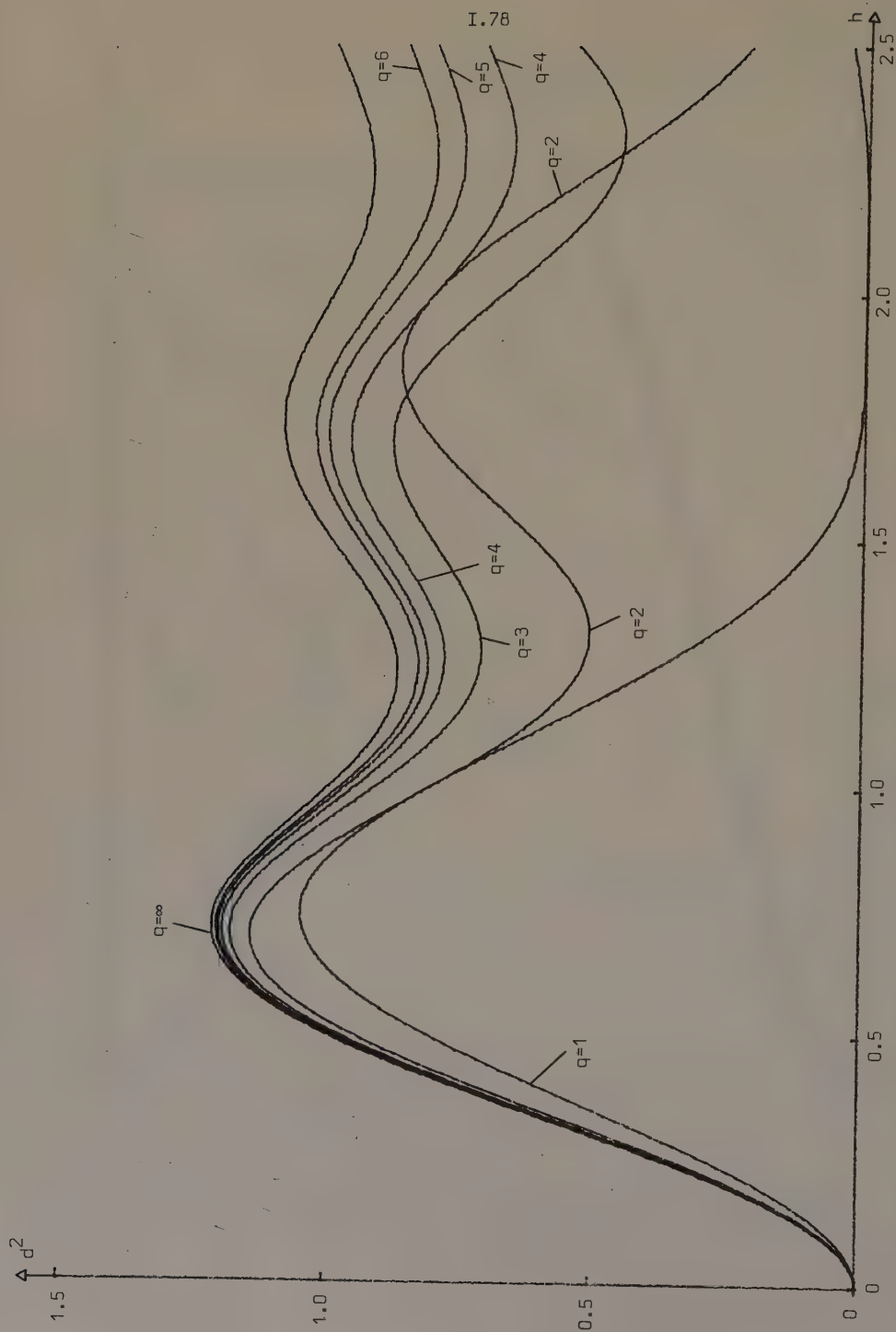


Figure 18. Minimum squared Euclidean distances for a suboptimum full response CPFSK receiver, $M=2$, $N=1$ observed symbol intervals.

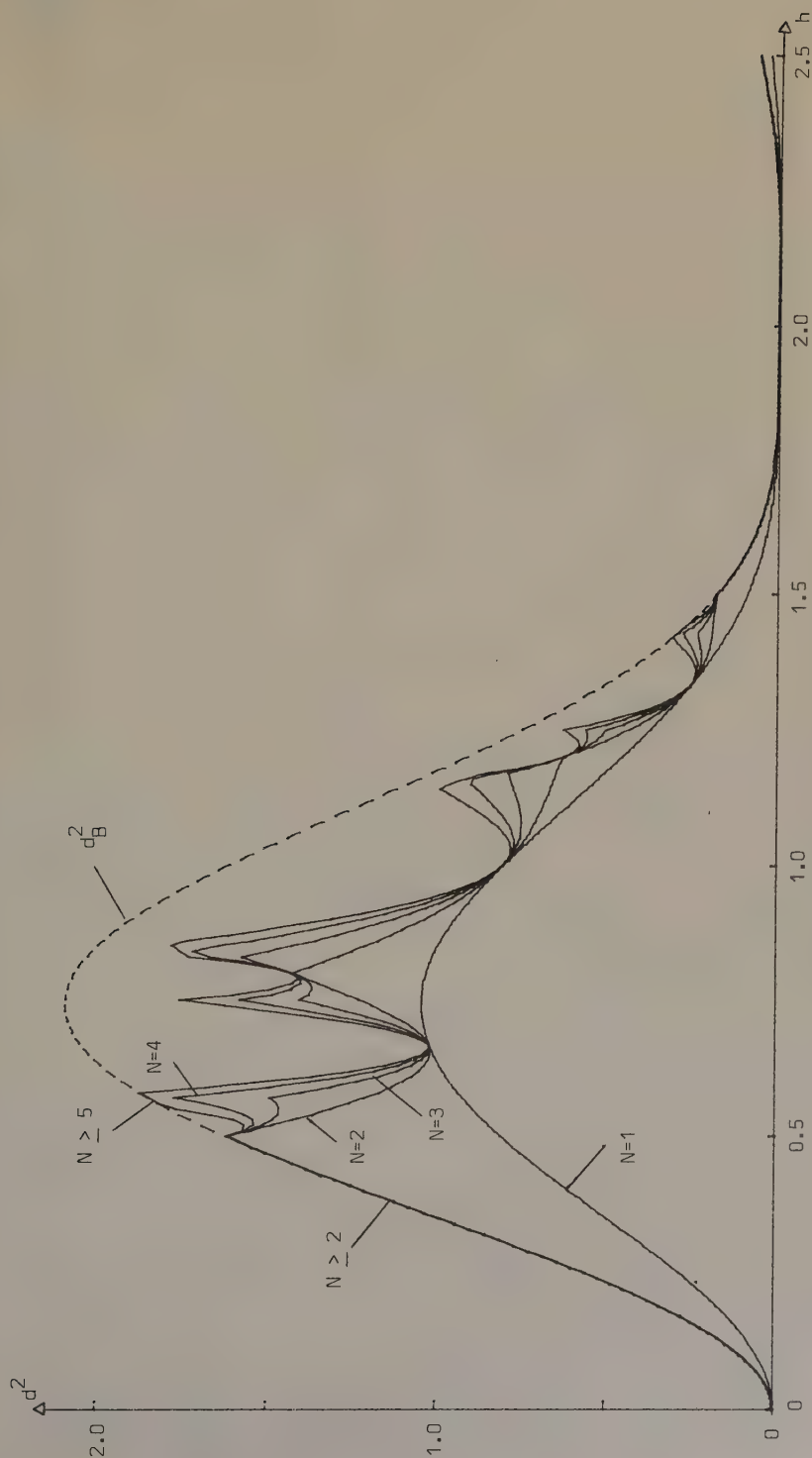


Figure 19. Minimum squared Euclidean distances for a suboptimum full response CPFSK receiver. $N = 1, 2, 3, 4$ and 5 observed bit intervals. The upper bound d_B^2 is shown dashed.

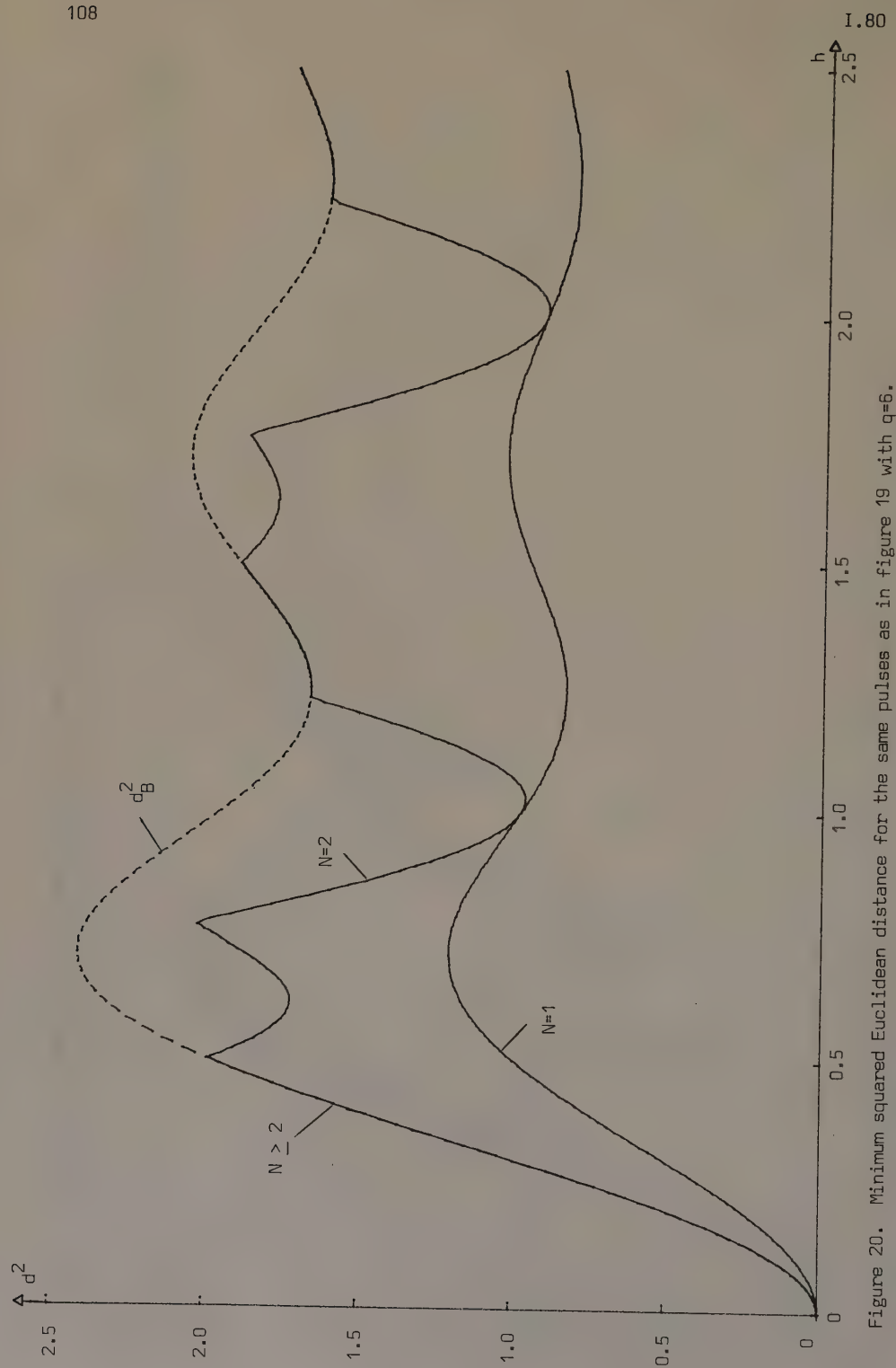


Figure 20. Minimum squared Euclidean distance for the same pulses as in figure 19 with $q=6$.

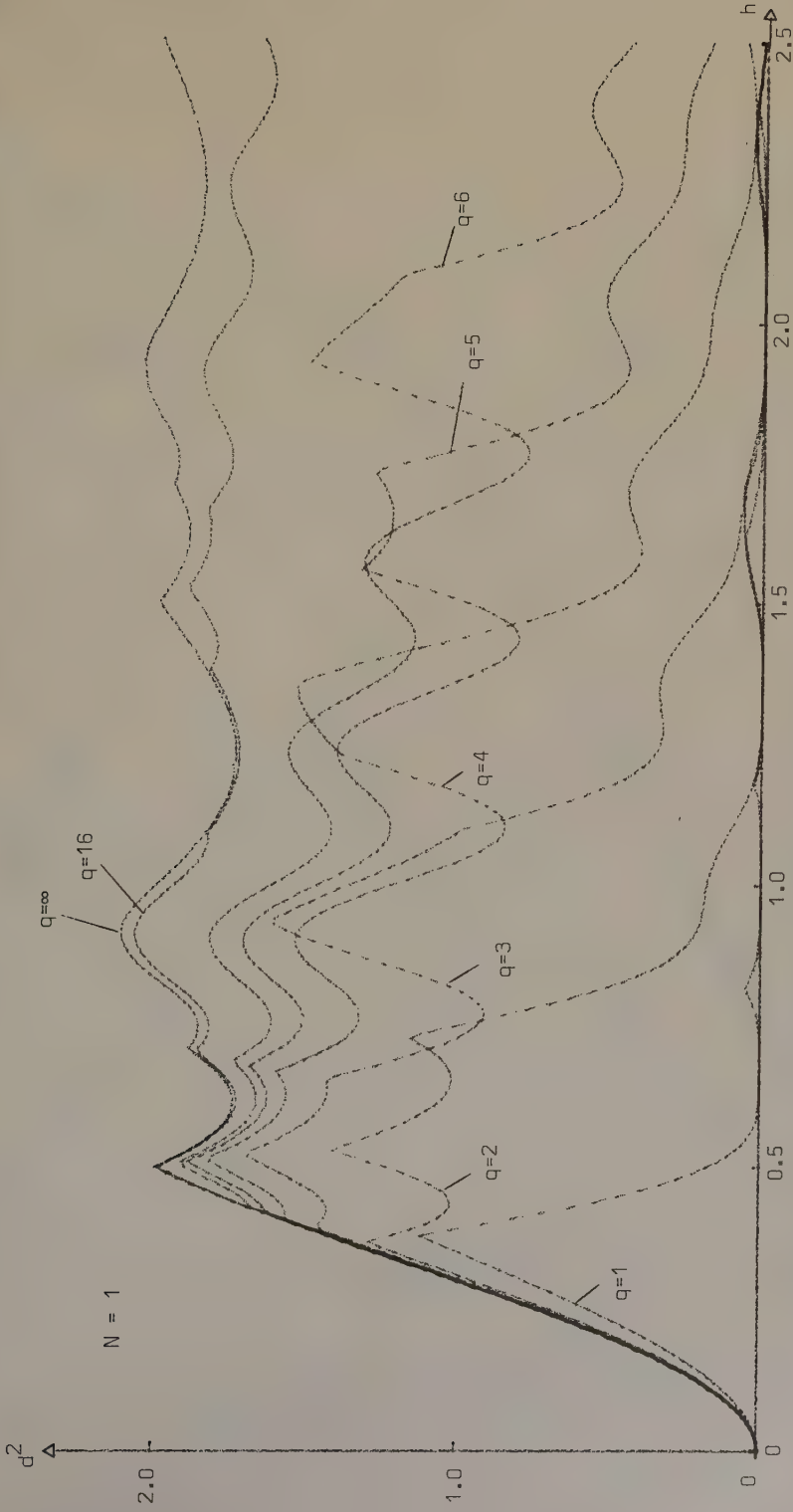


Figure 21. Minimum normalized squared Euclidean distances for a quaternary CPFSK scheme with different suboptimum receivers. $N=1$ observed symbol interval. The normalization is in E_b .

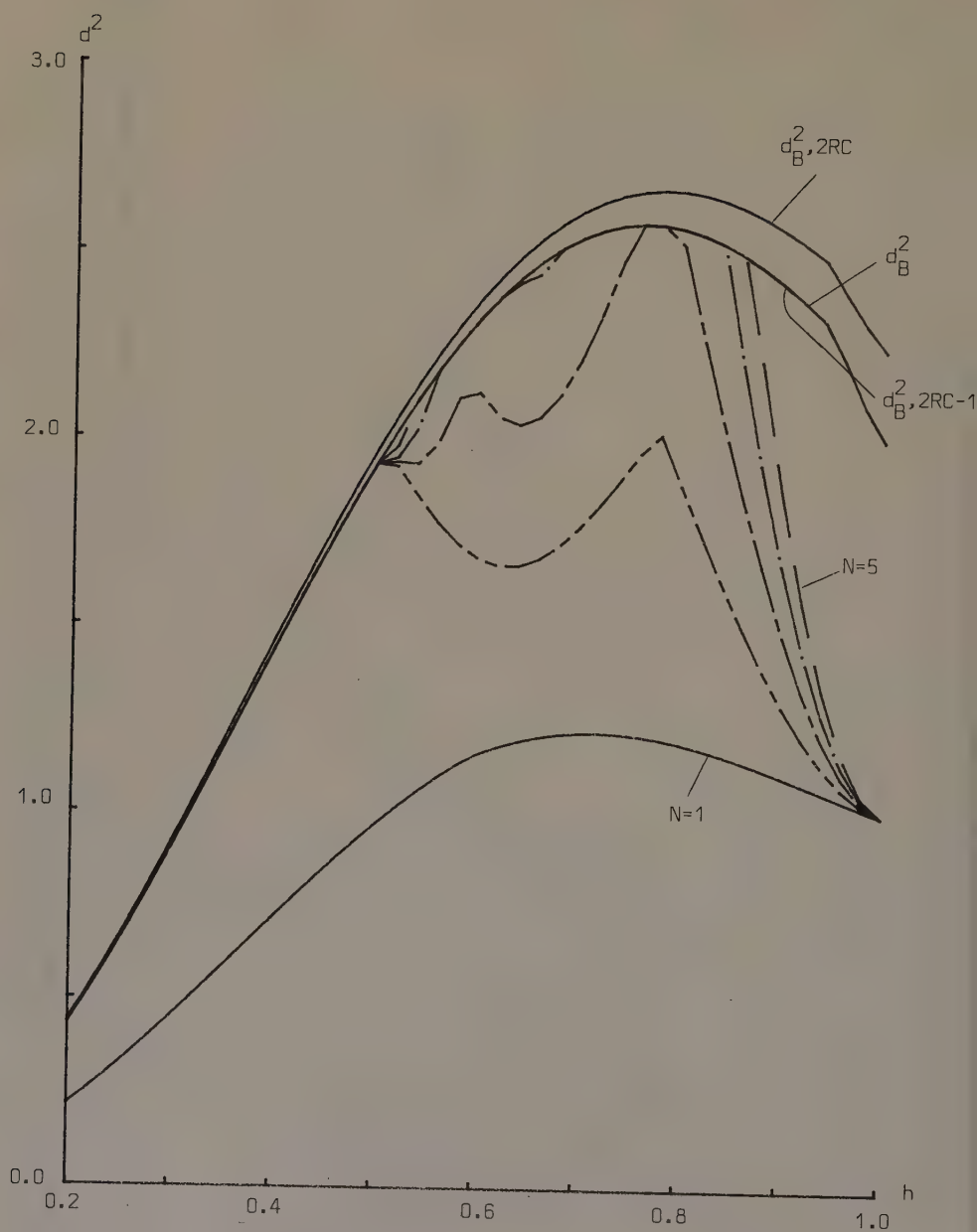


Figure 22. Minimum distance for binary 2RC-1T0.50.

Note that this receiver is optimum only for $h=1/2$.

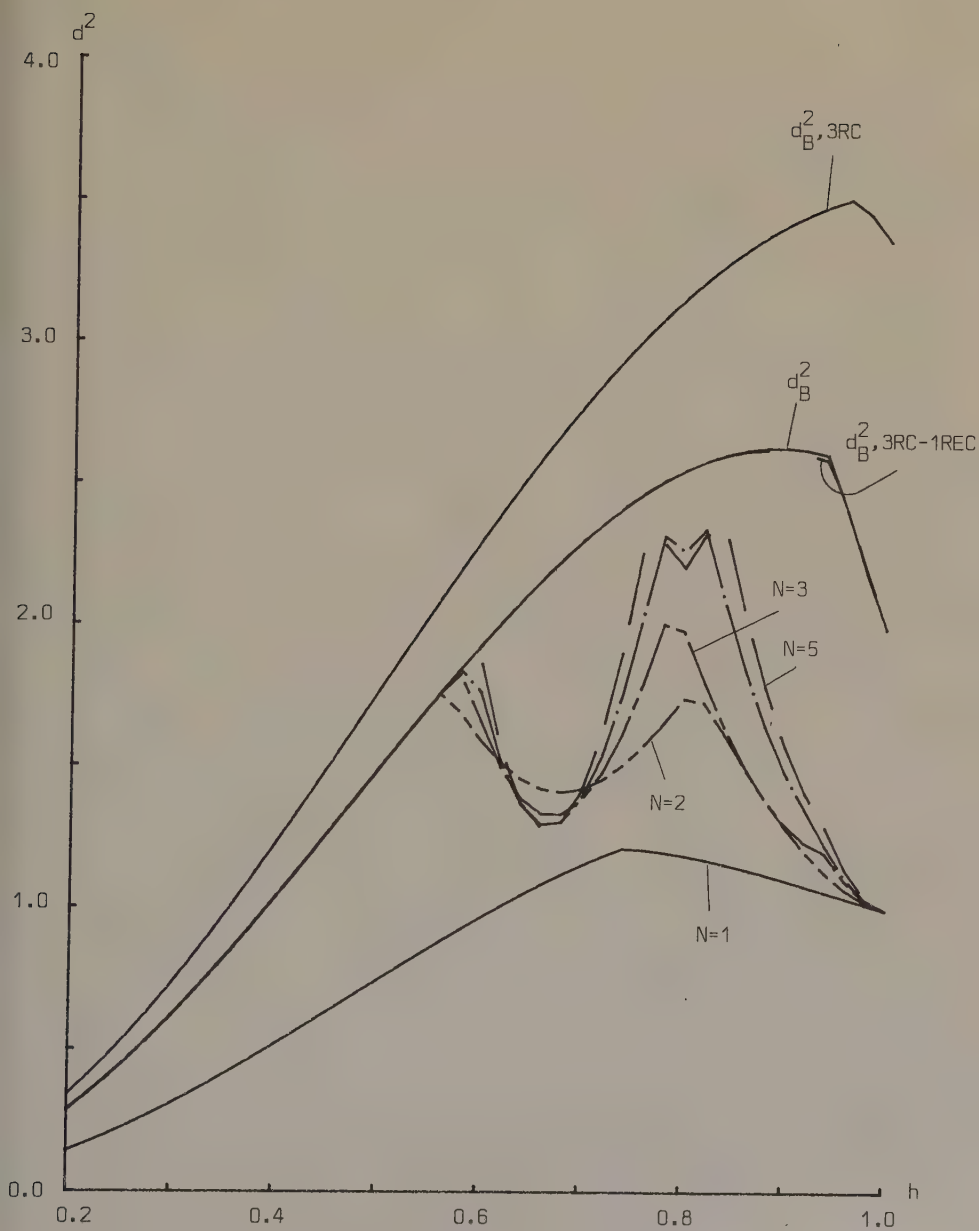


Figure 23. Minimum distance for binary 3RC-1T0.50.

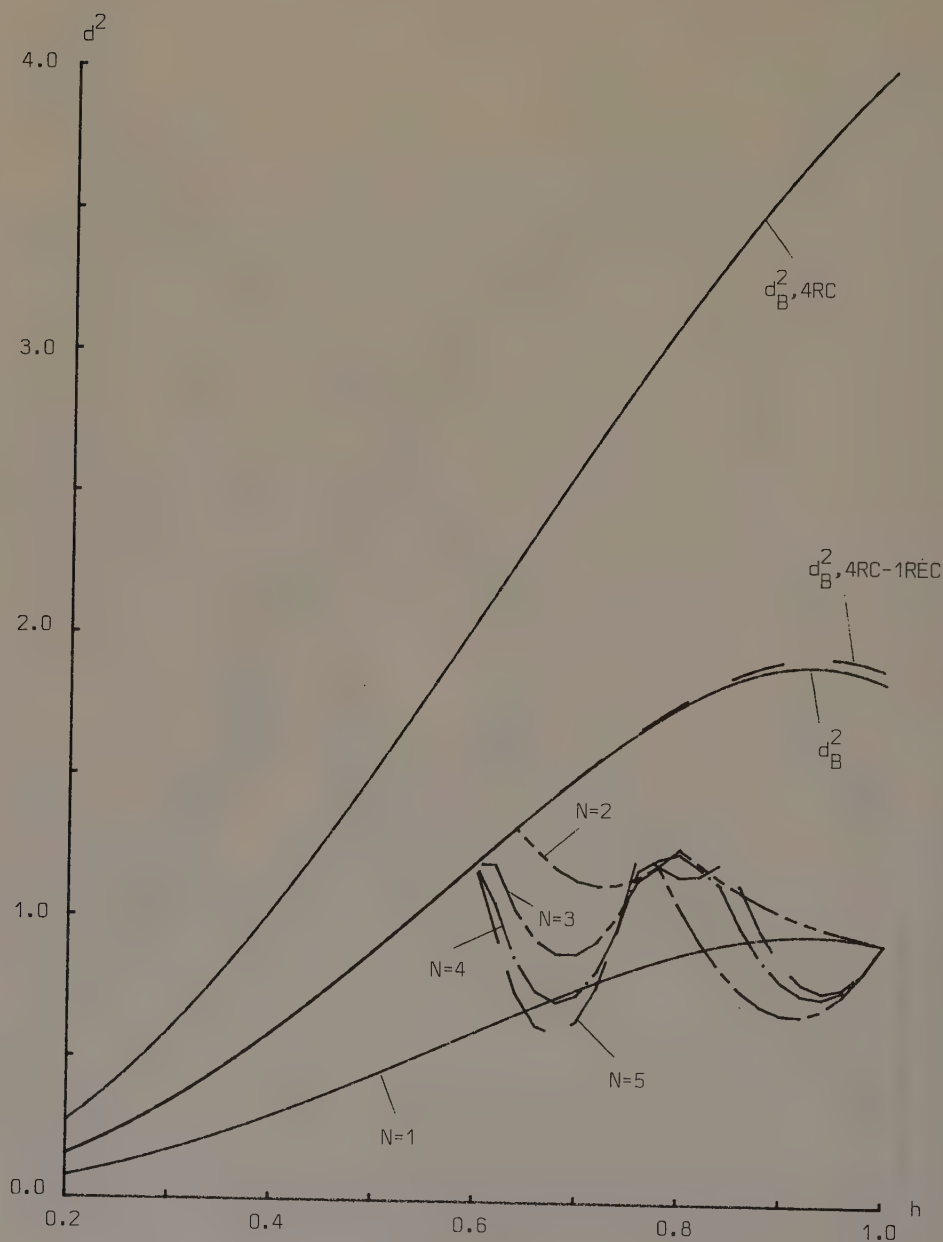


Figure 24. Minimum distance for binary 4RC-1T0.50.

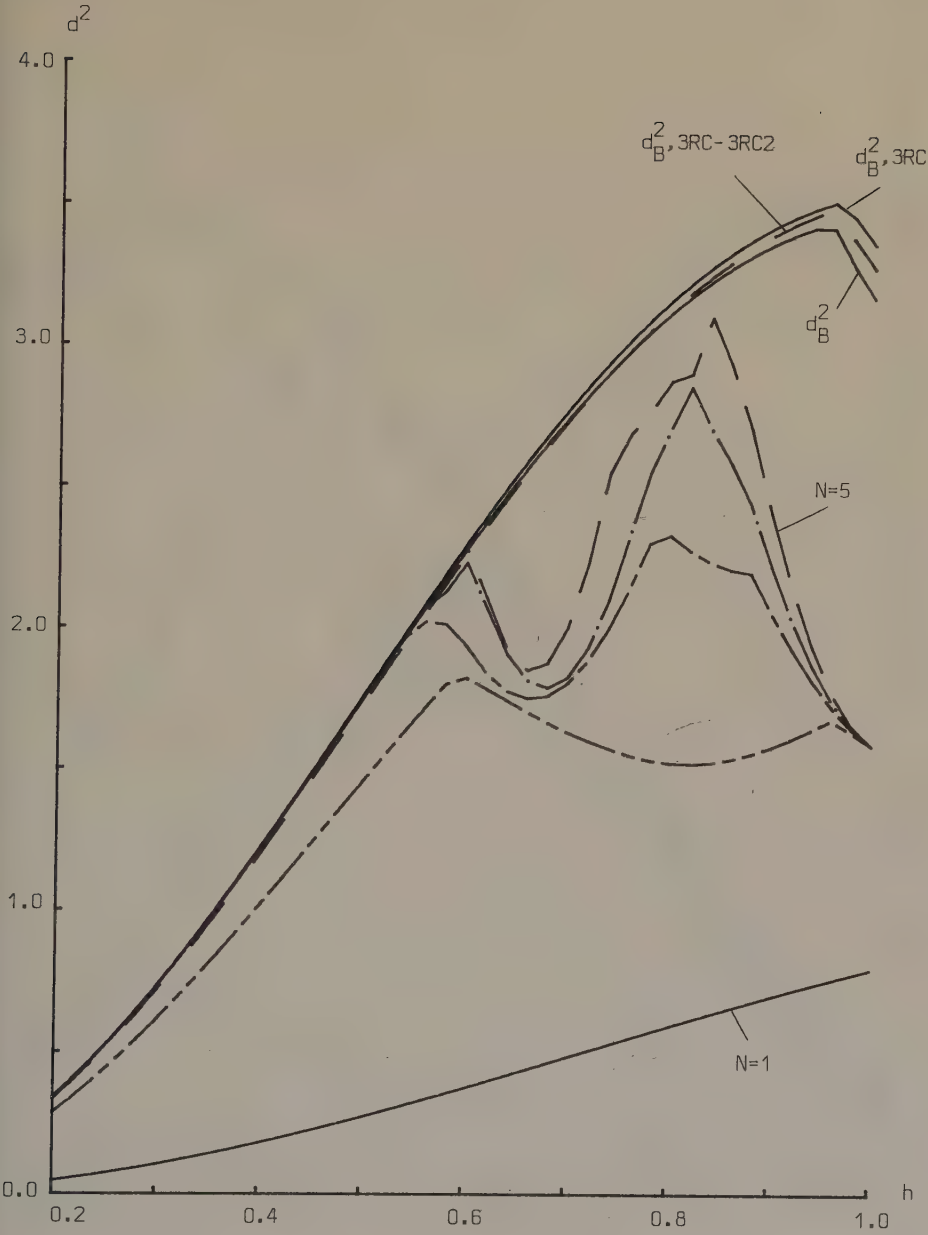


Figure 25. Minimum distance for binary 3RC-2T0.25.

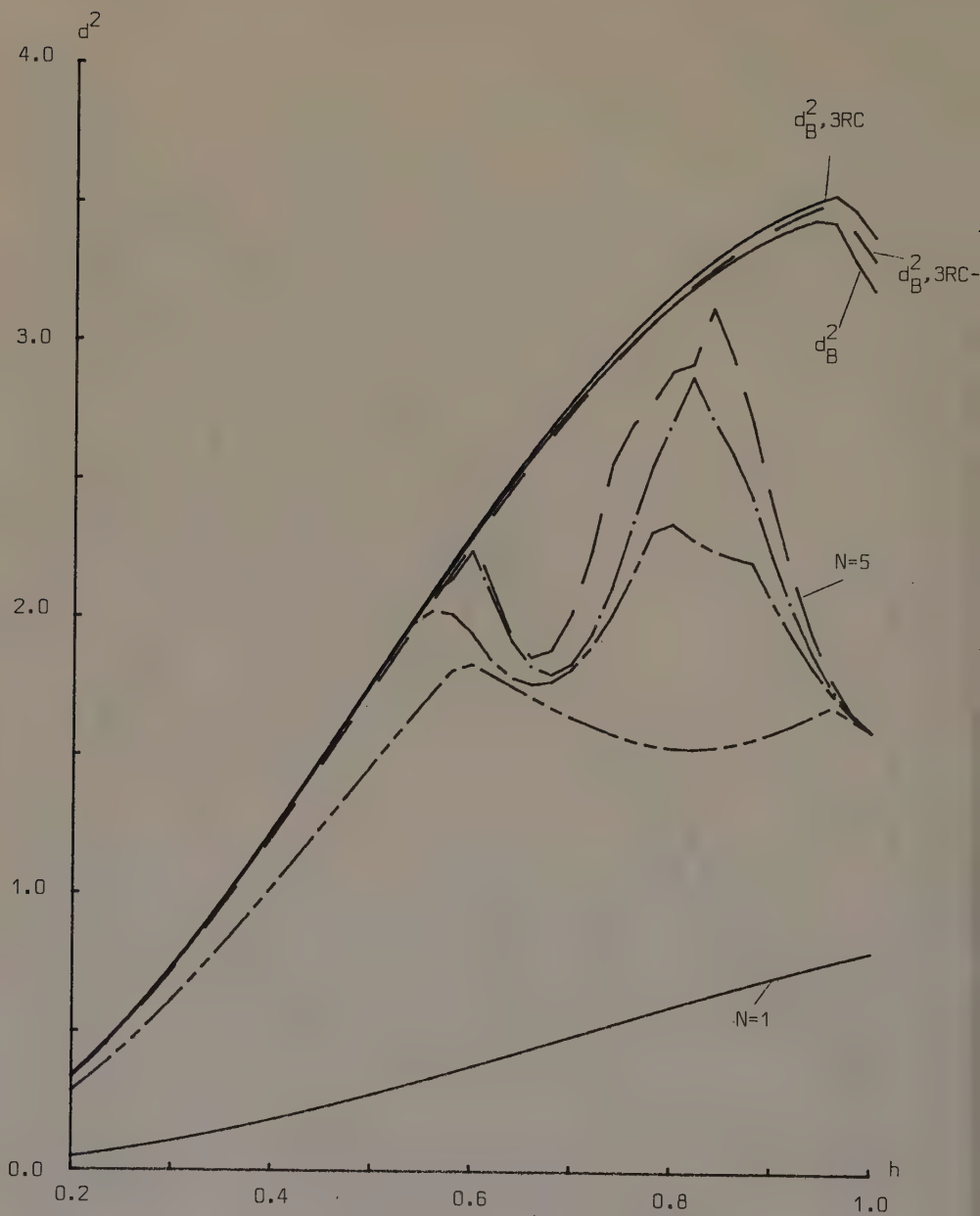


Figure 26. Minimum distance for binary 3RC-2T0.50.

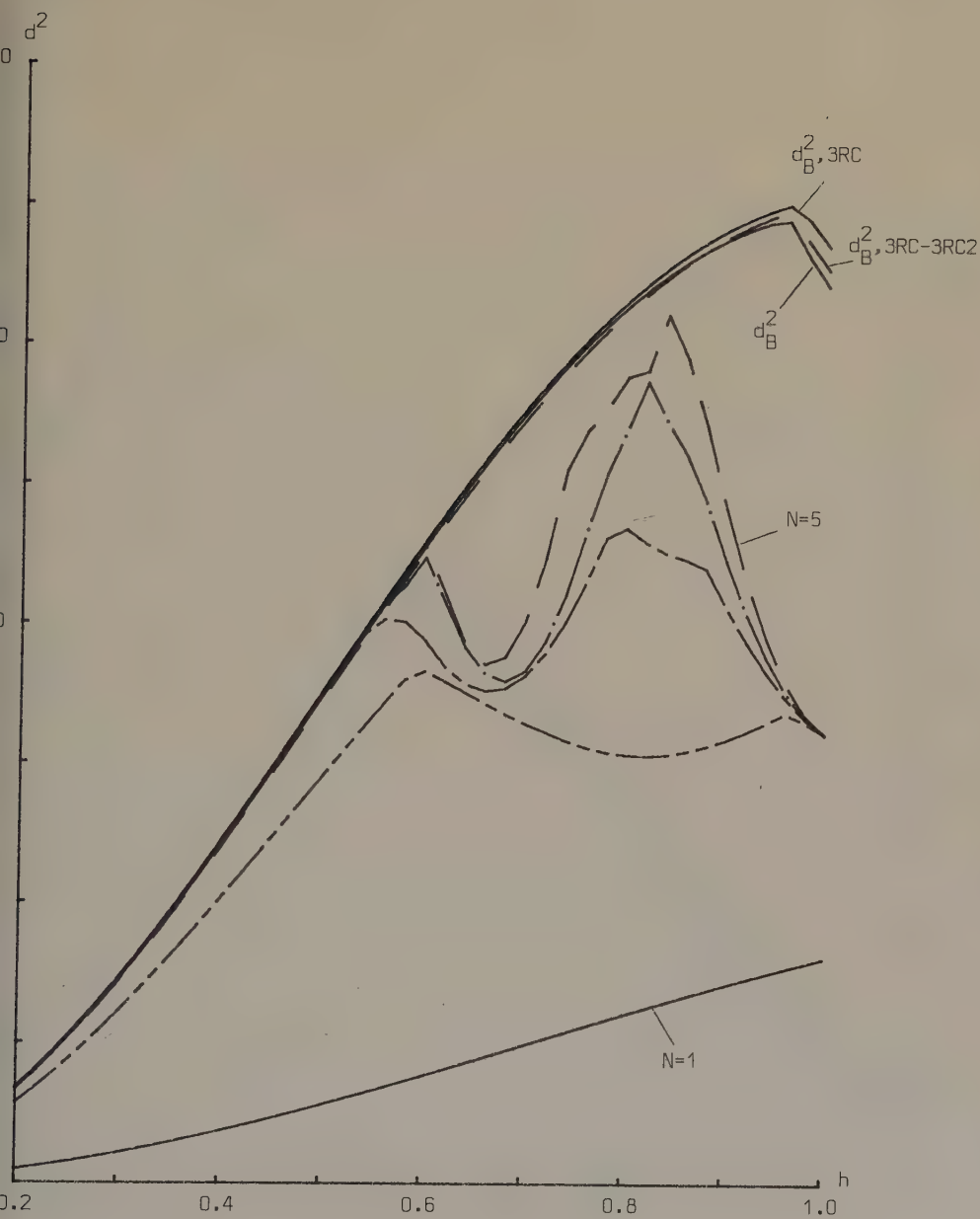


Figure 27. Minimum distance for binary 3RC-2T0.75.

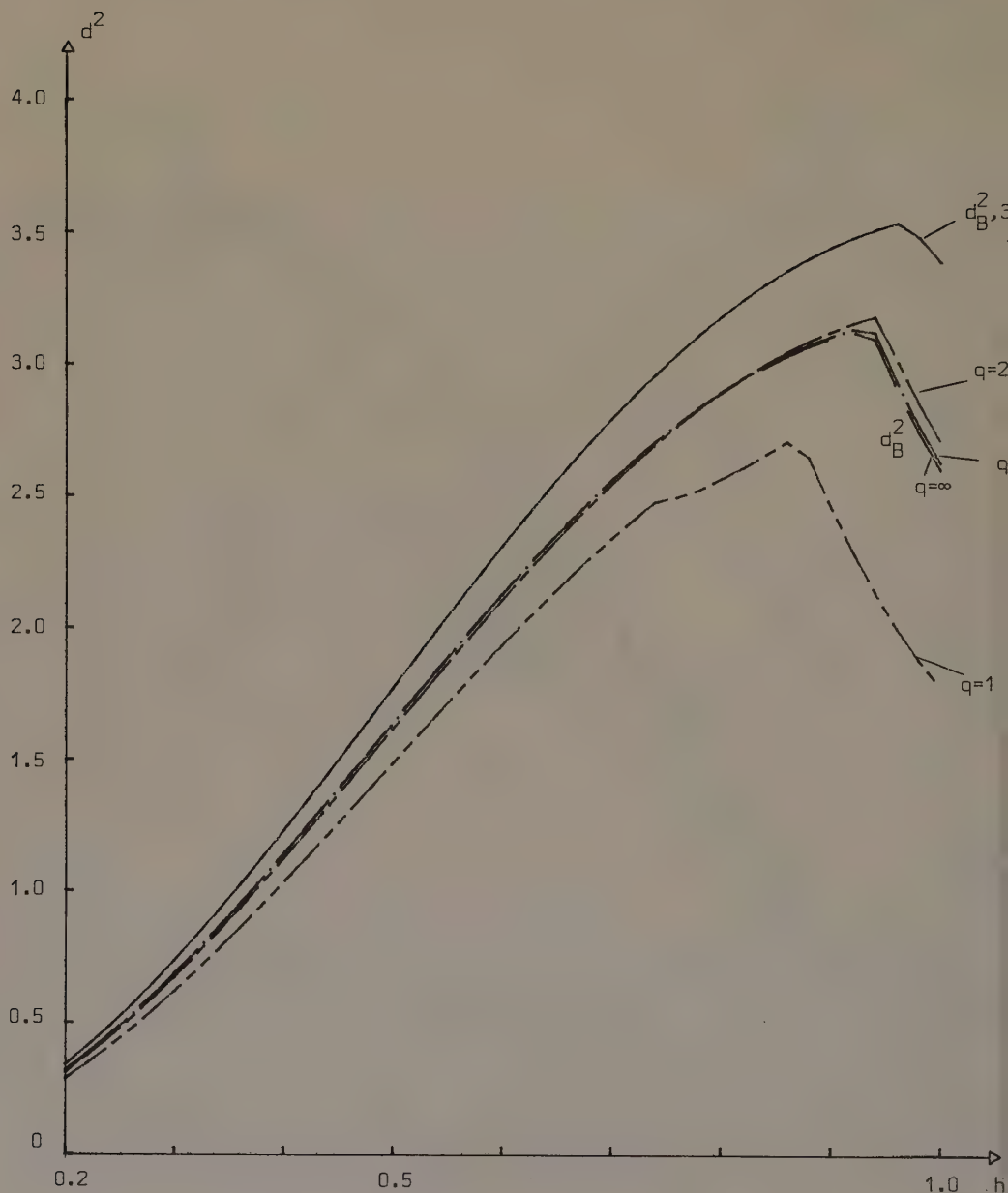


Figure 28. The upper bound on the minimum normalized squared Euclidean distance for binary 3RC with a sampled 2REC receiver, sampled with $q=1$, $q=2$, $q=4$ and $q=\infty$. The upper bound for the distance for the optimum 3RC receiver is also shown for comparison.

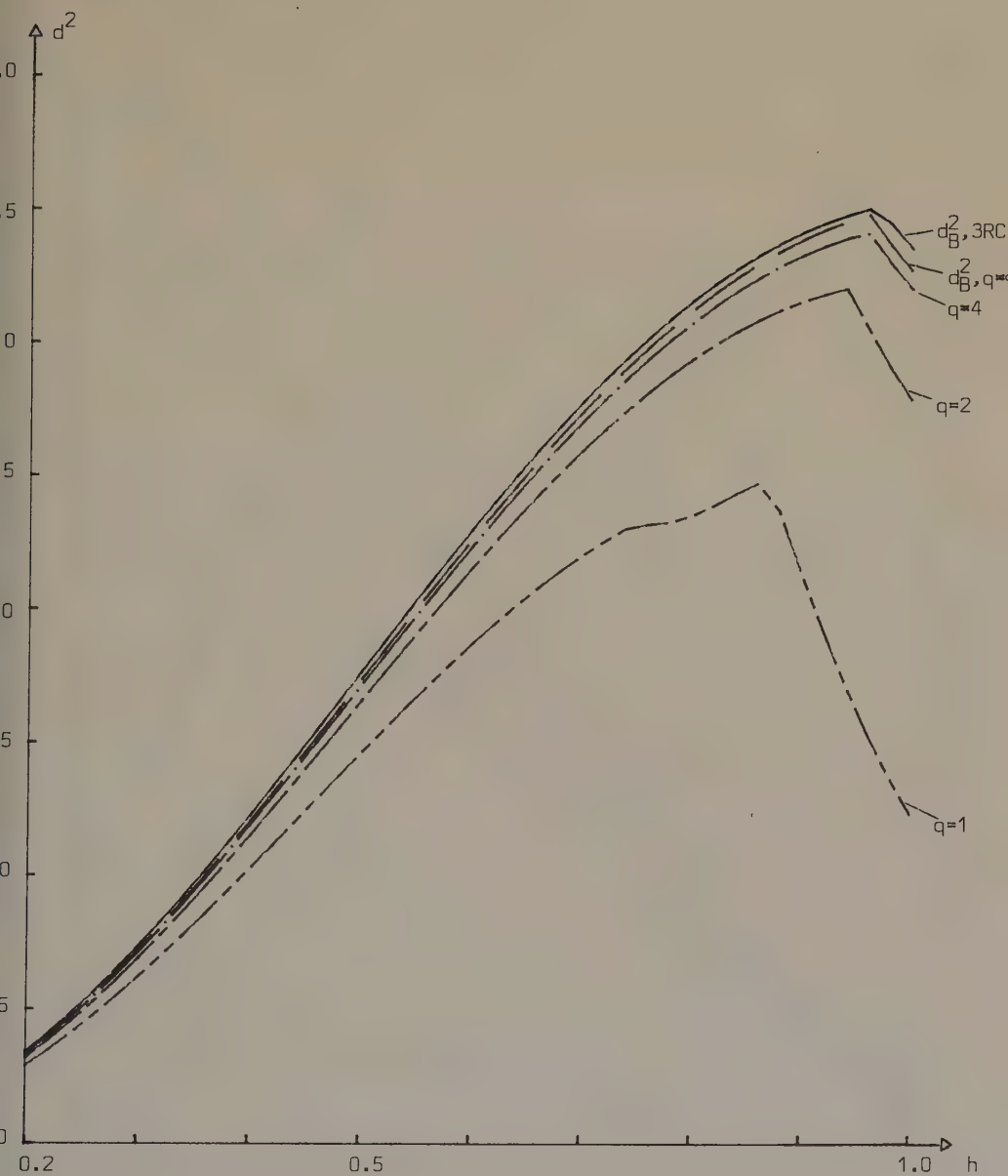


Figure 29. The upper bound on the minimum normalized squared Euclidean distance for binary 3RC with a 3RC2 receiver, sampled with $q=1$, $q=2$, $q=4$ and $q=\infty$, as a function of h . The upper bound for the optimum 3RC receiver is also shown for comparison.

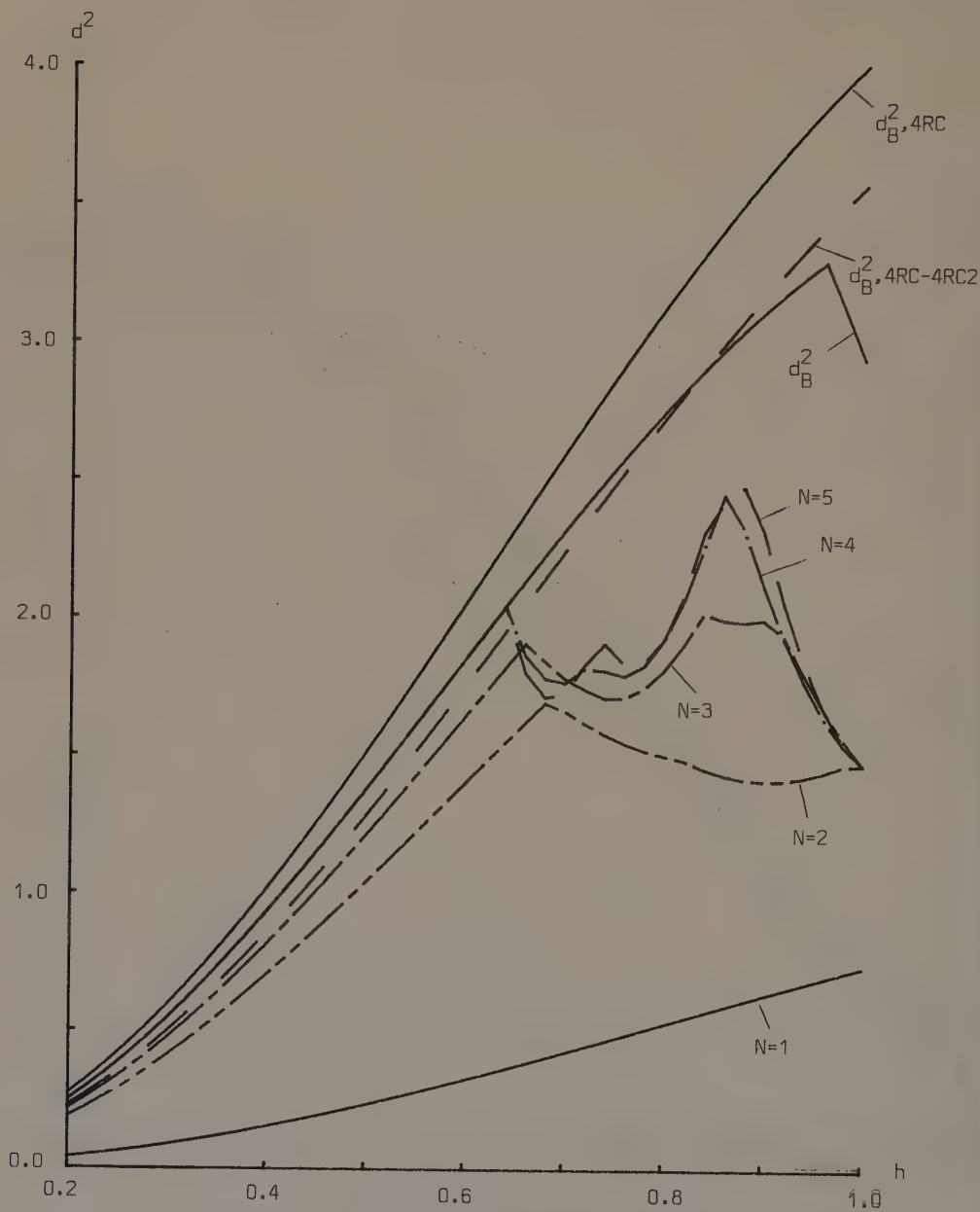


Figure 30. Minimum distance for binary 4RC-2T0.25.

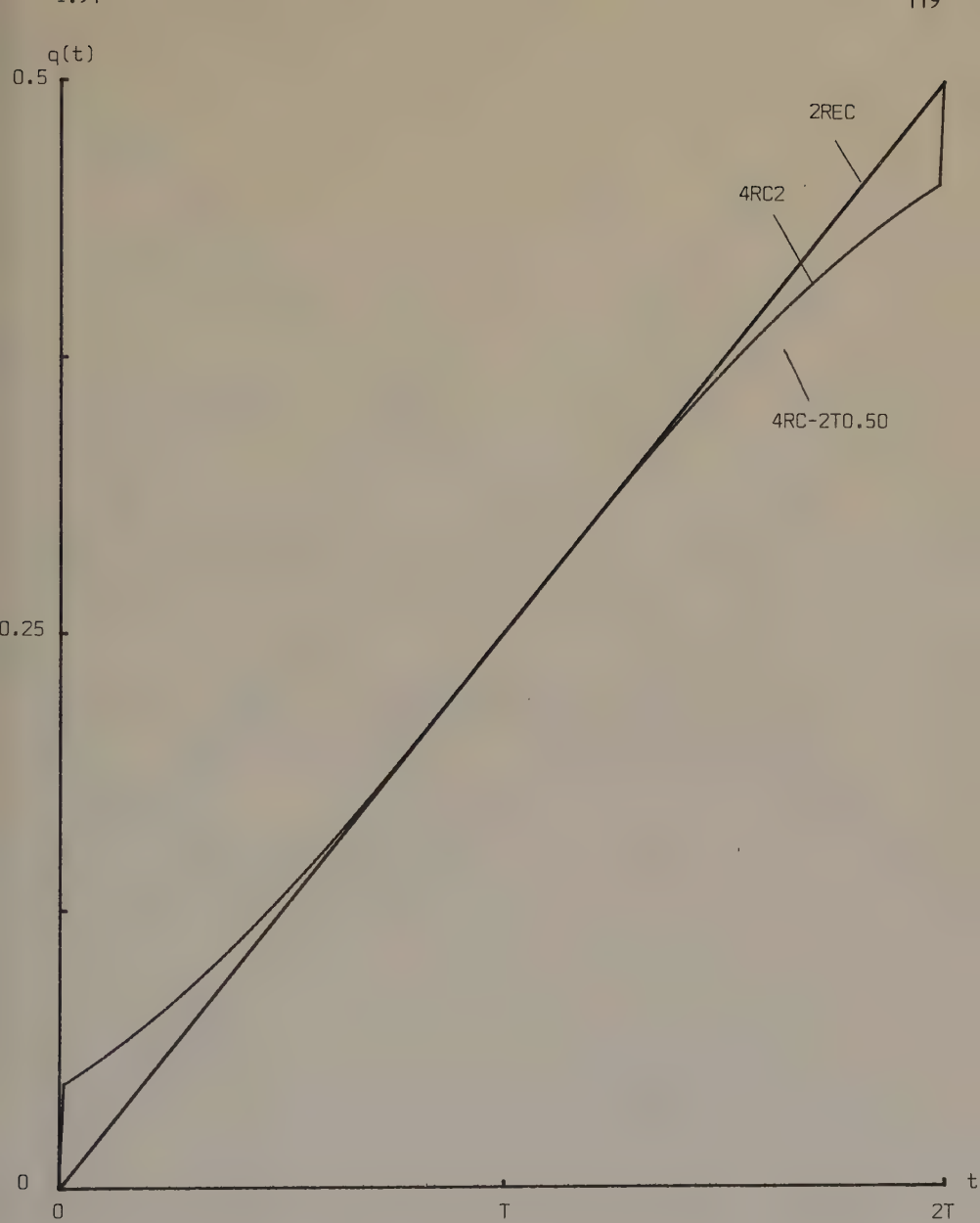


Figure 31. Phase response for binary 4RC-2T0.50.

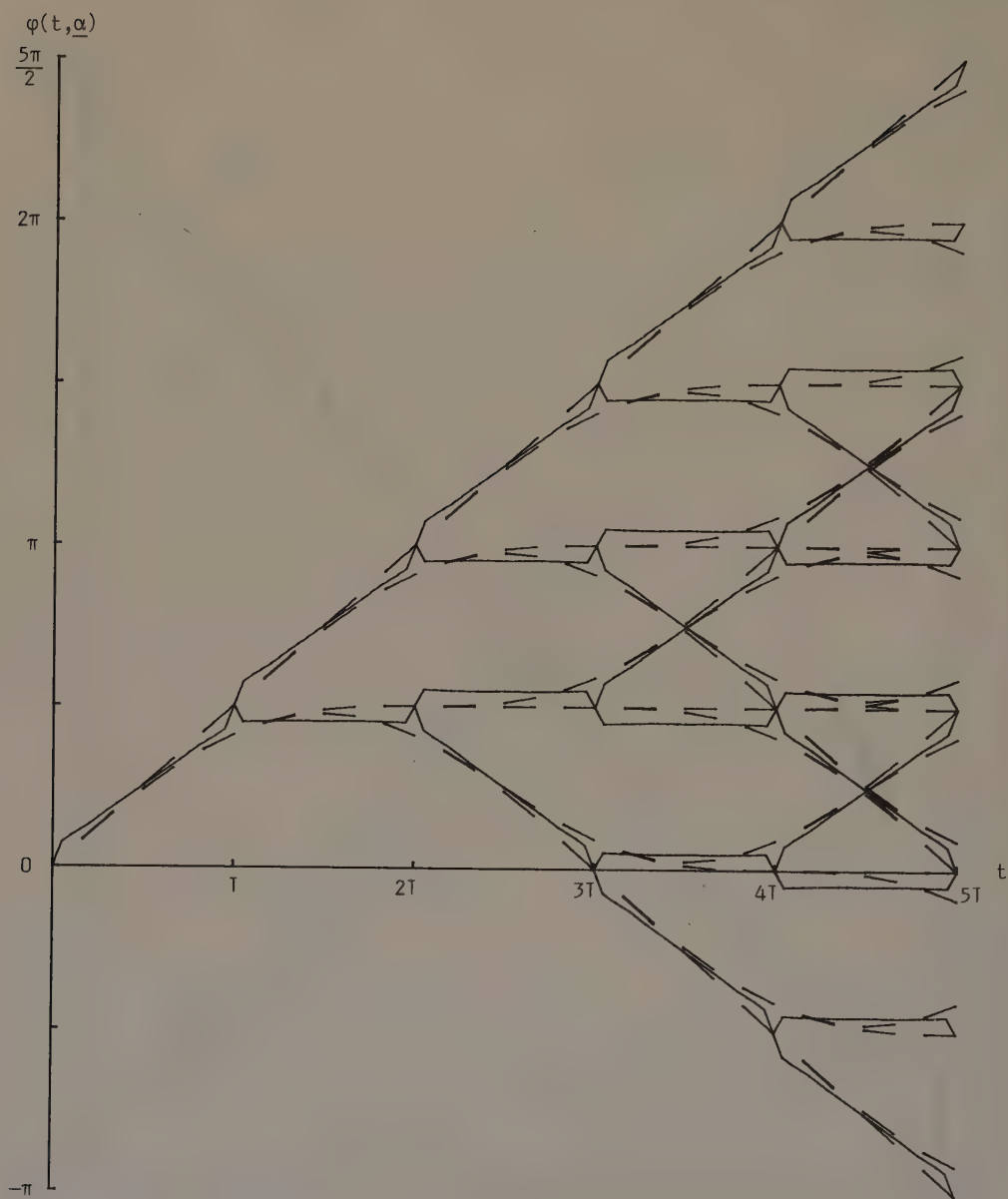


Figure 32. Phase tree for binary 4RC-2T0.50.

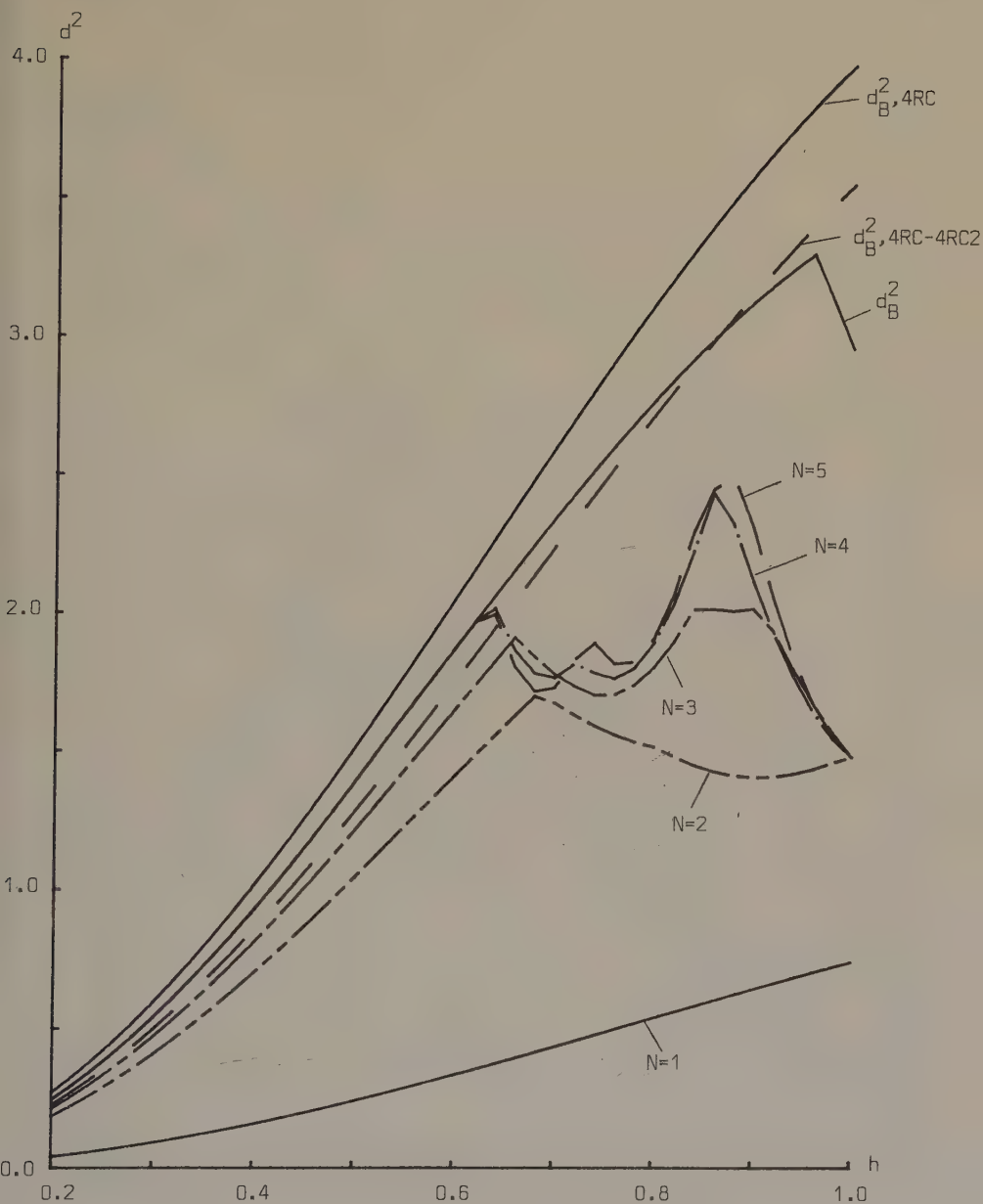


Figure 33. Minimum distance for binary 4RC-2T0.50.

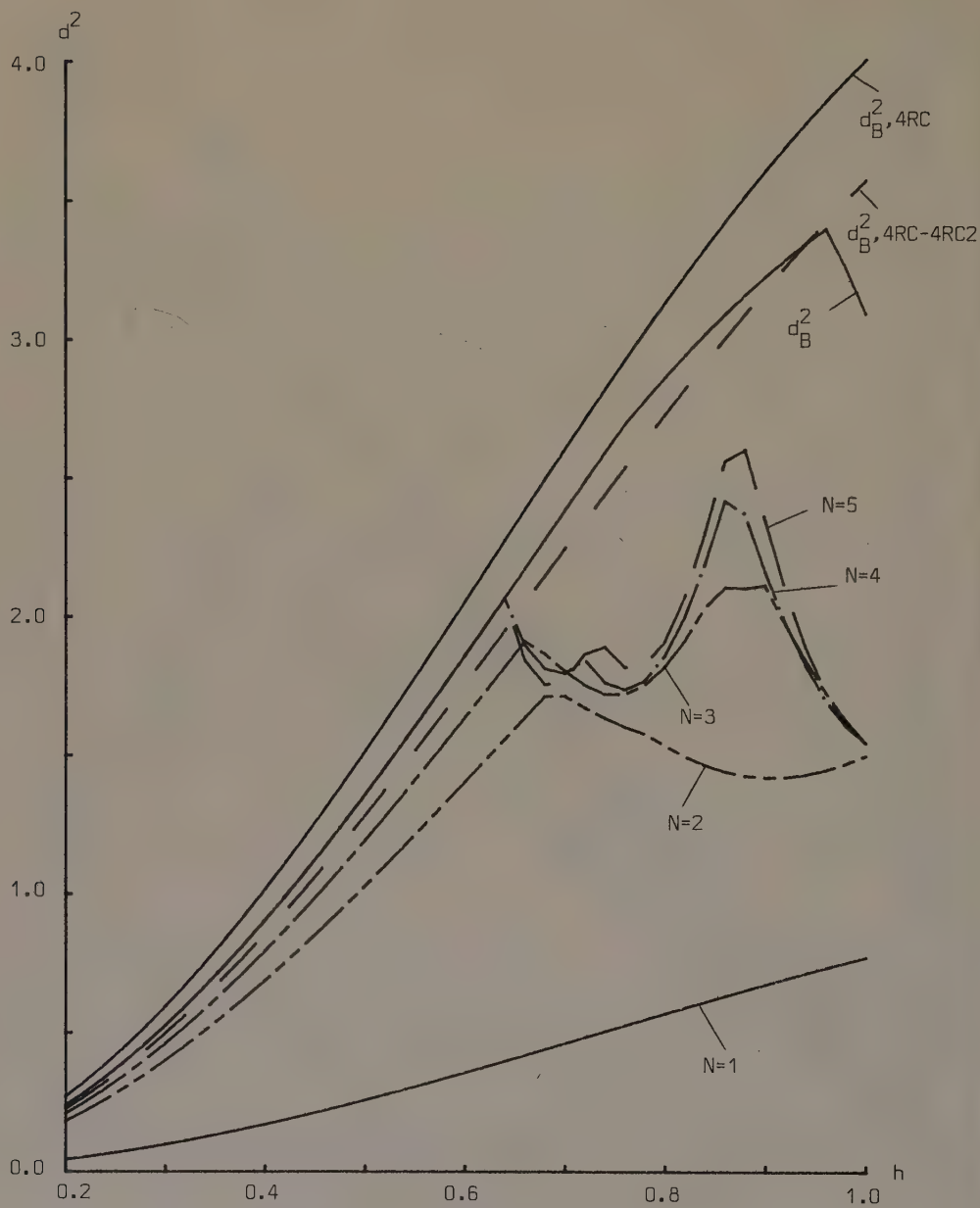


Figure 34. Minimum distance for binary 4RC-2T0.75.

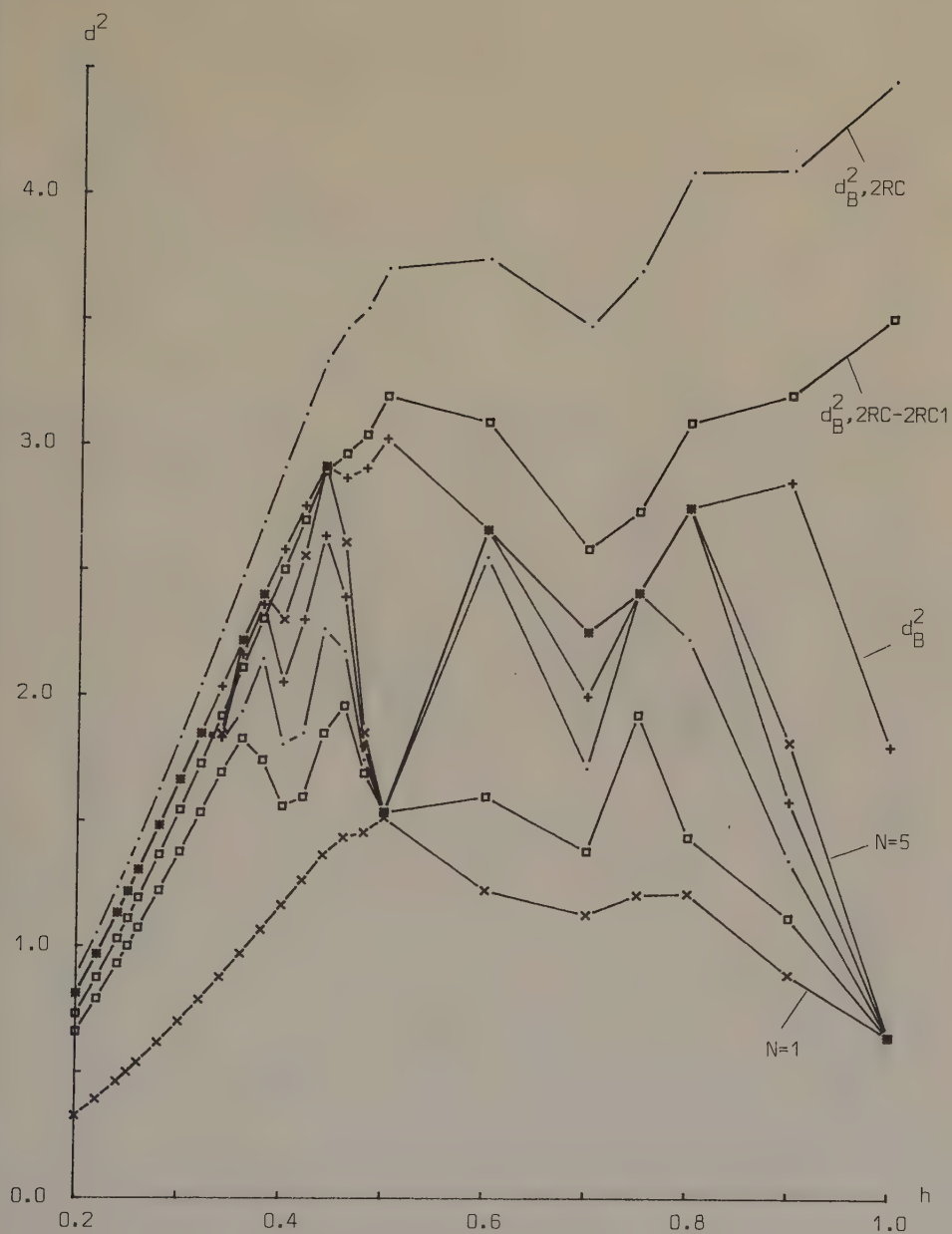


Figure 35. Minimum distance for quaternary 2RC-1T0.25.

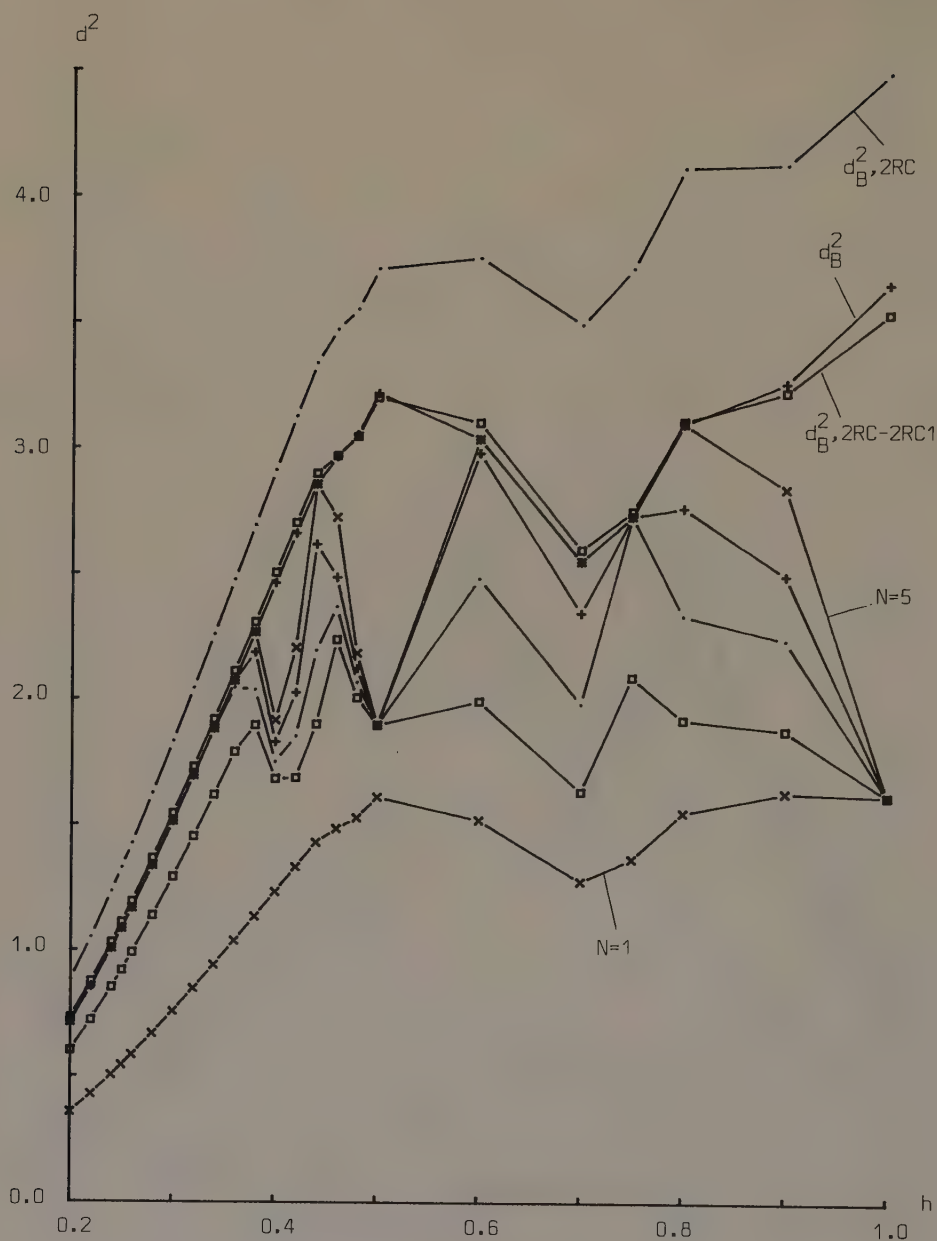


Figure 36. Minimum distance for quaternary 2RC-1T0.50.

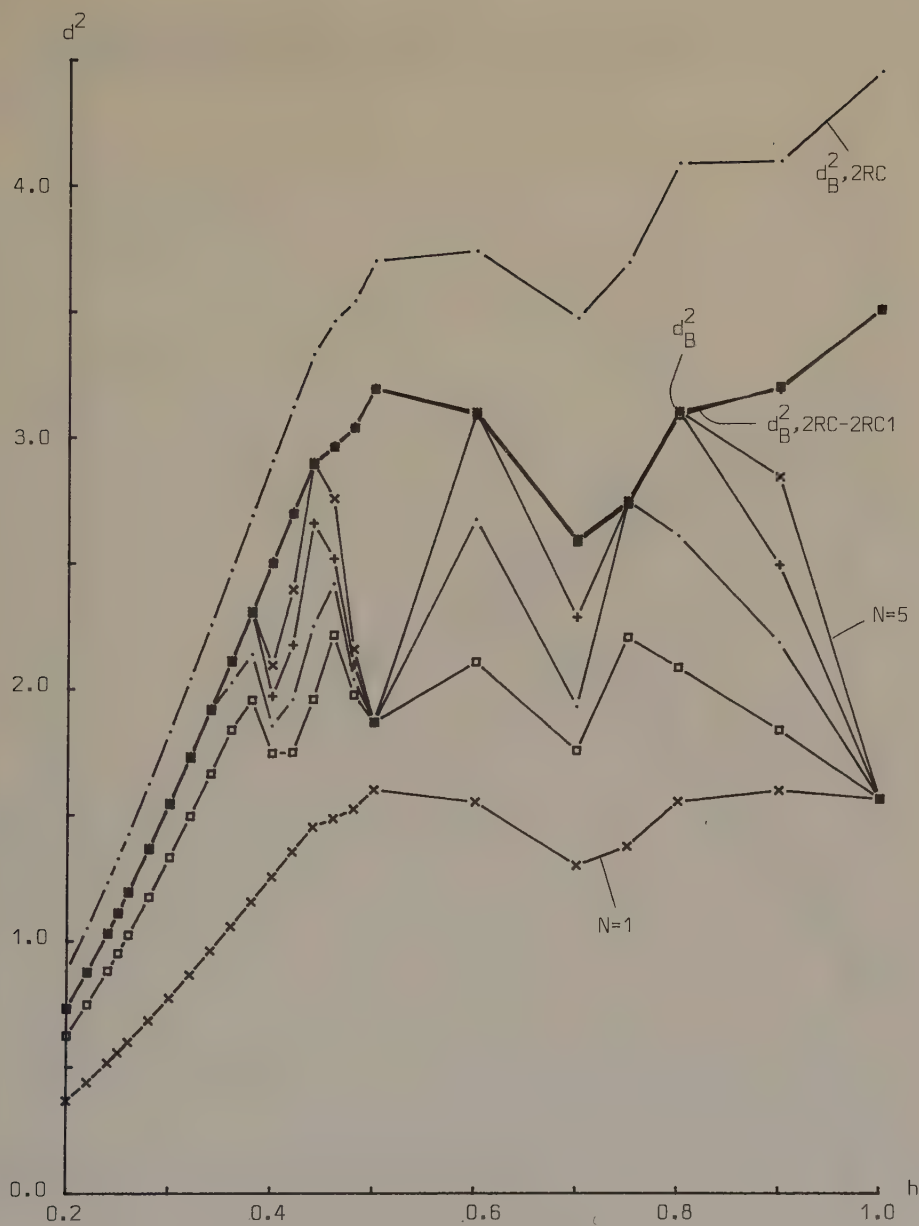


Figure 37. Minimum distance for quaternary 2RC-1T0.75.

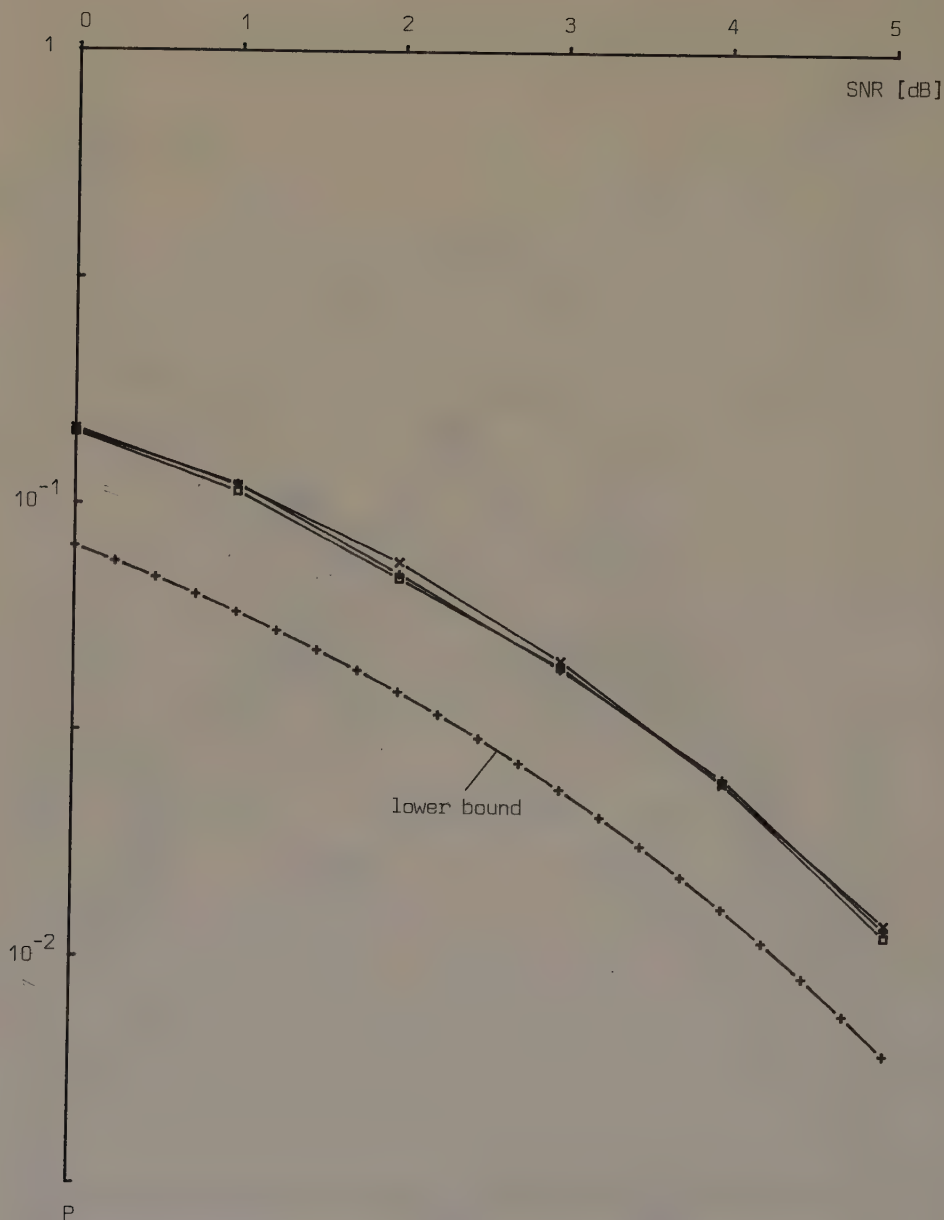


Figure 38. Simulated bit error probability for 2RC-1REC(+), 2RC-1T0.50(x) and optimum 2RC(□) for $h=1/2$. The lower bound for the optimum receiver is also shown. The results are based on 2500 errors.

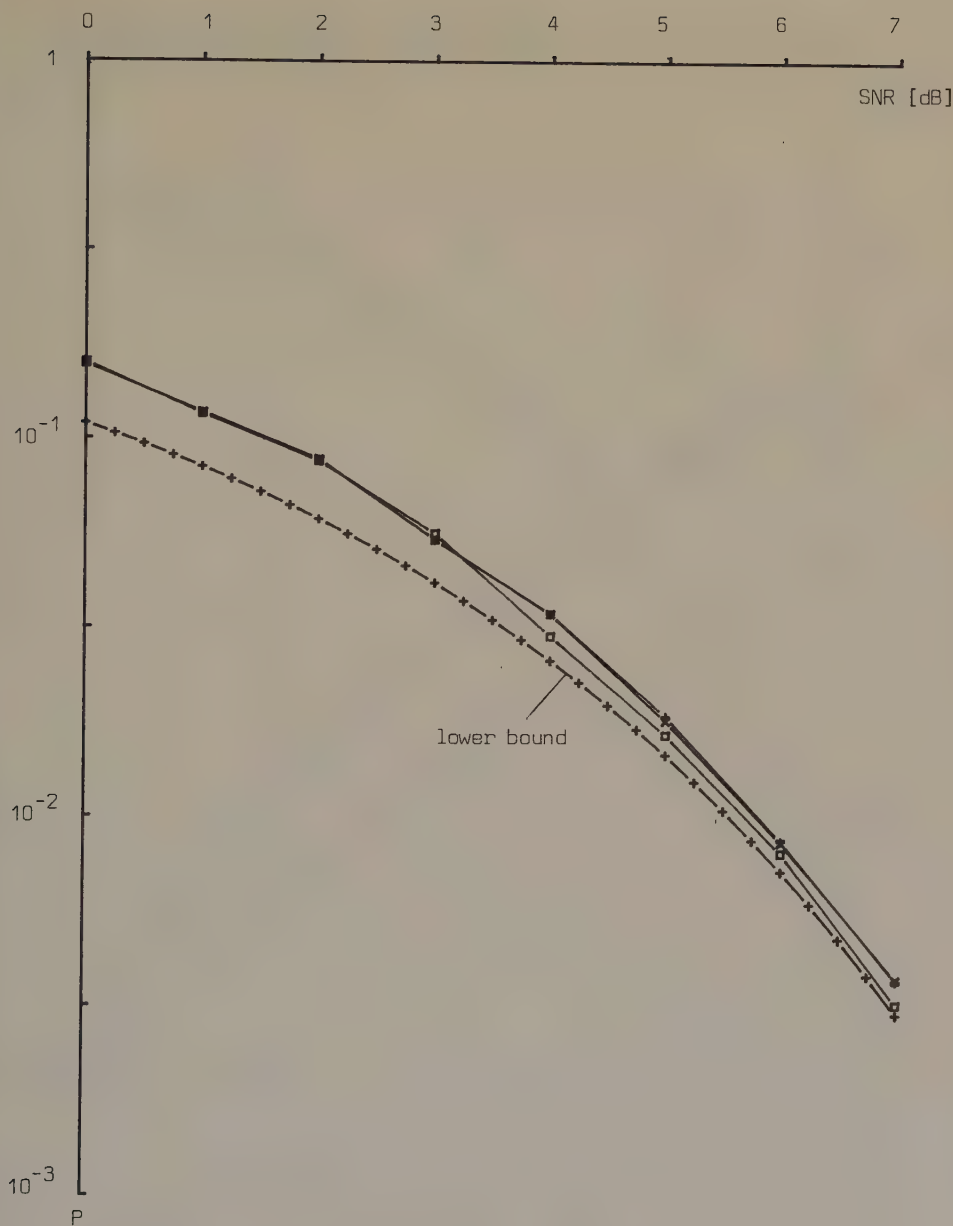


Figure 39. Simulated bit error probability for 4RC-4RC2(+), 4RC-2T0.50(x) and optimum 4RC(o) for $h=1/2$. The lower bound for the optimum receiver is also shown. The results are based on 2500 errors.

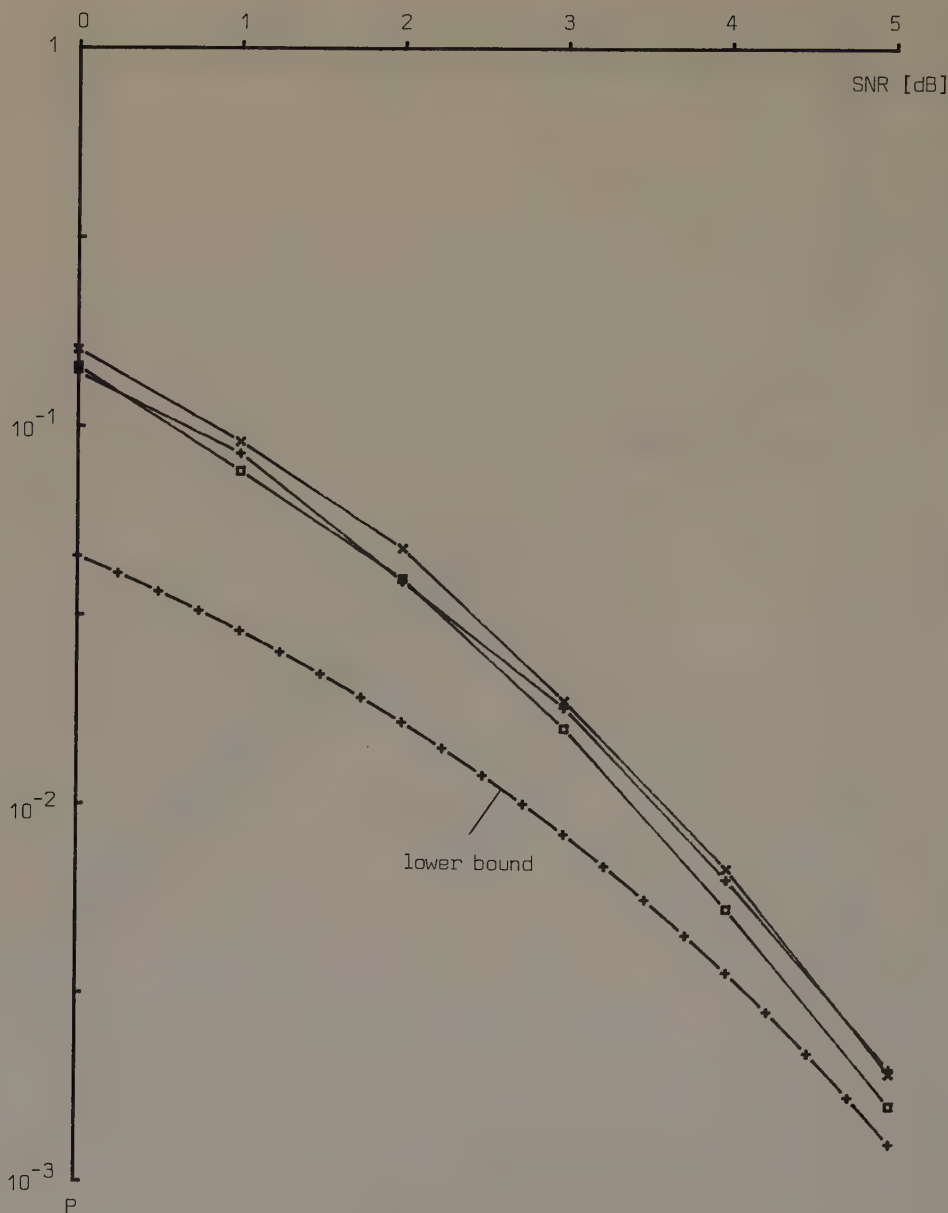


Figure 40. Simulated bit error probability for 4RC-4RC2(+), 4RC-2T0.75(x) and optimum 4RC(□) for $h=3/4$. The lower bound for the optimum receiver is also shown. The results are based on 2500 errors.

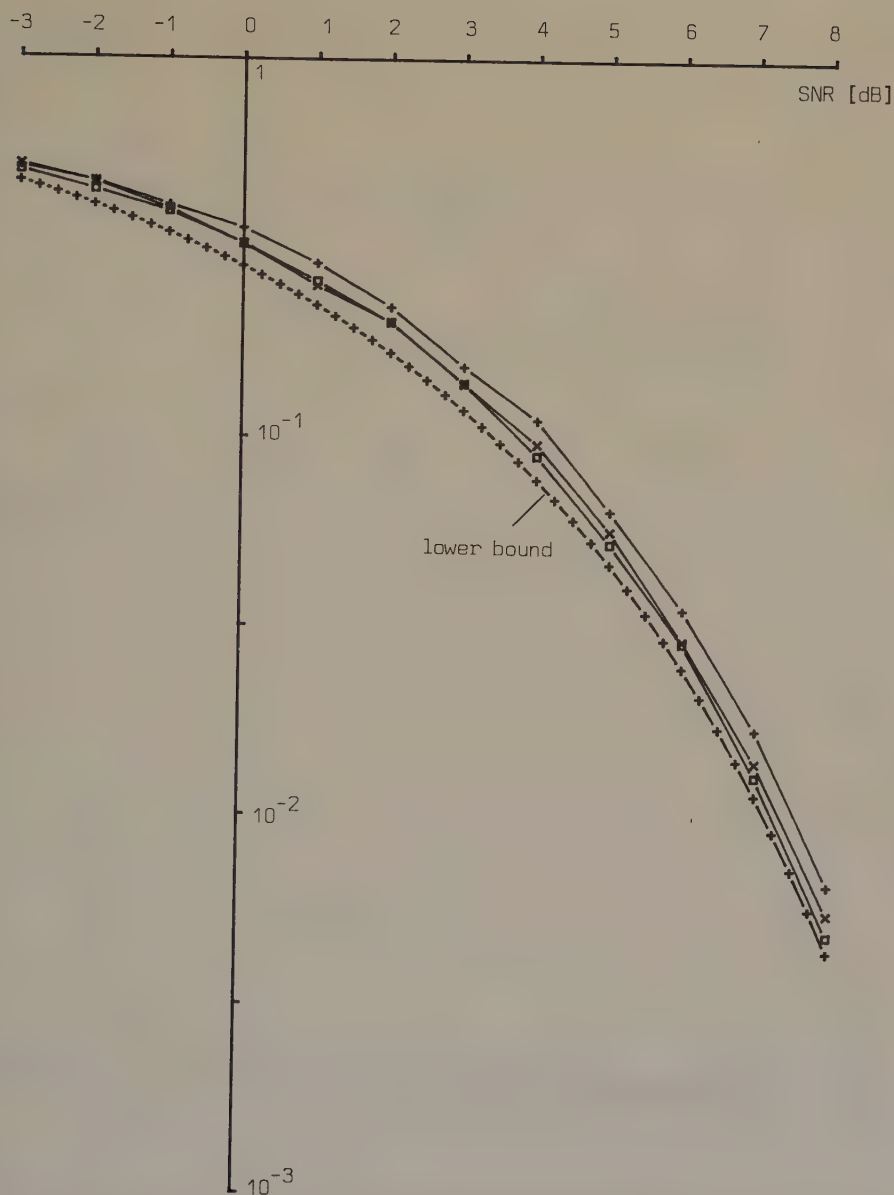


Figure 41. Simulated symbol error probability for quaternary 2RC-2RC1(+), 2RC-1T0.25(x) and optimum 2RC(□) for $h=1/4$. The lower bound for the optimum receiver is also shown. The results are based on 2500 errors.

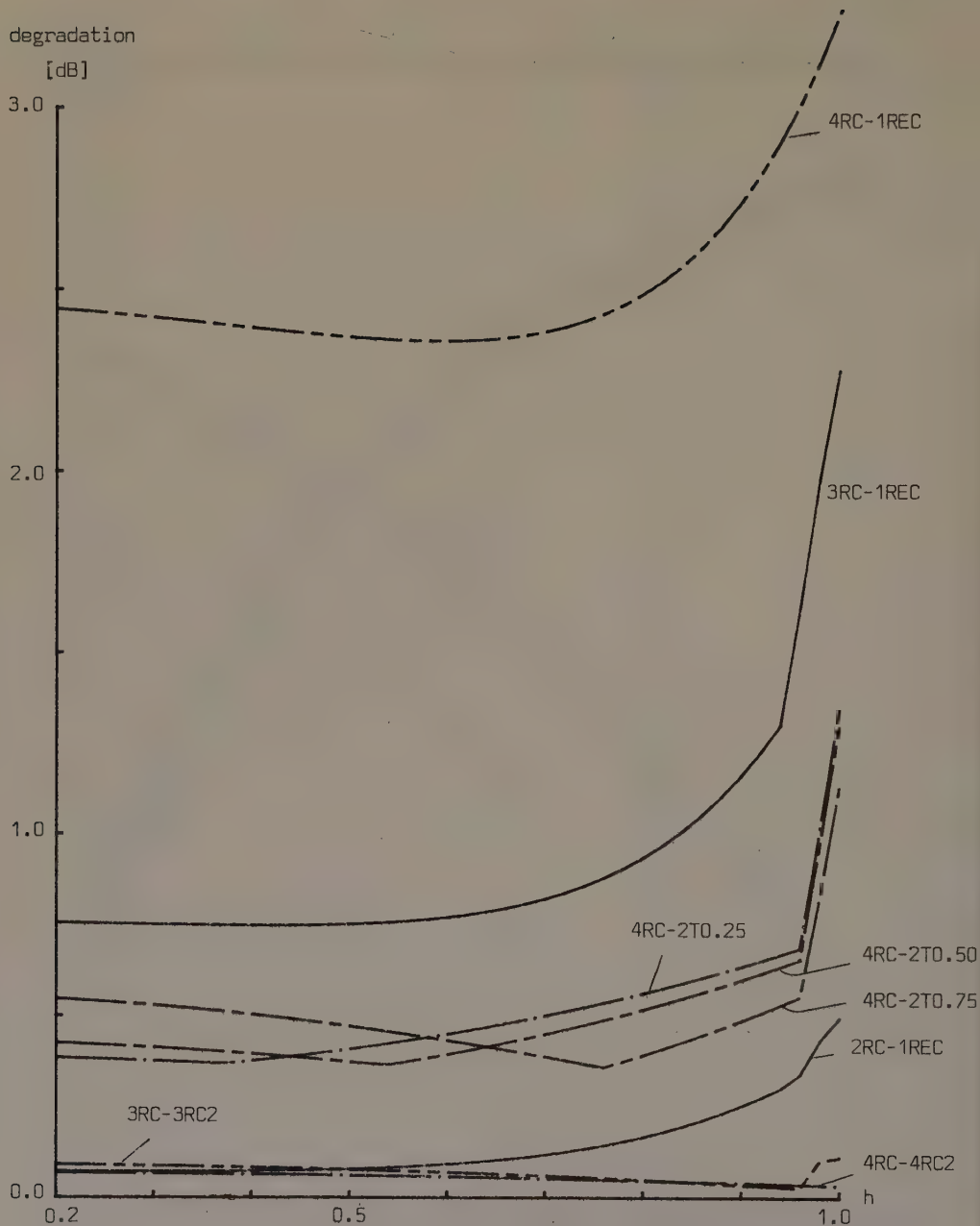


Figure 42. The degradation in asymptotic error probability compared to the optimum receiver for various binary schemes.

degradation

[dB]

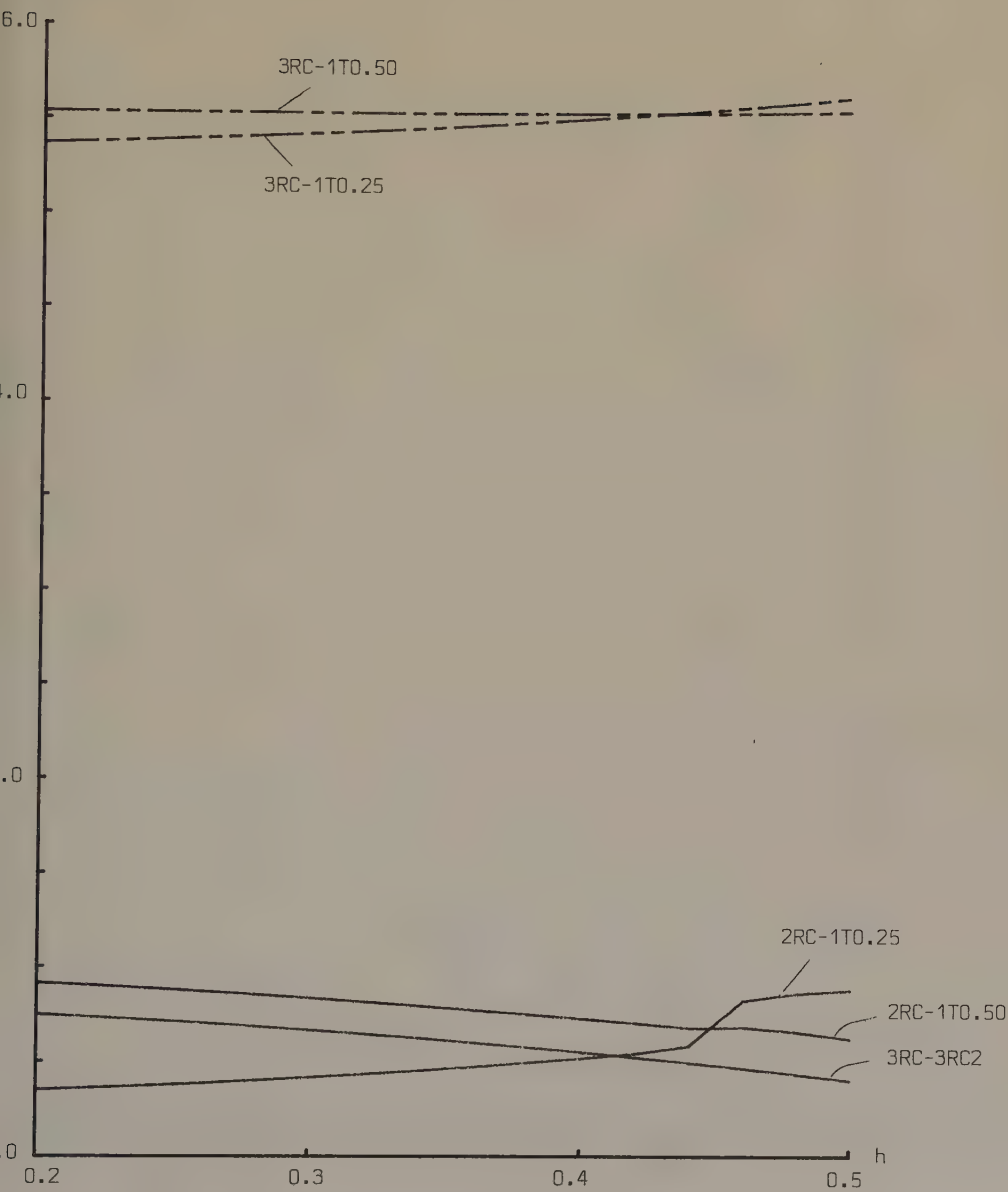


Figure 43. The degradation in asymptotic error probability compared to the optimum receiver for various quaternary schemes.

degradation
[dB]

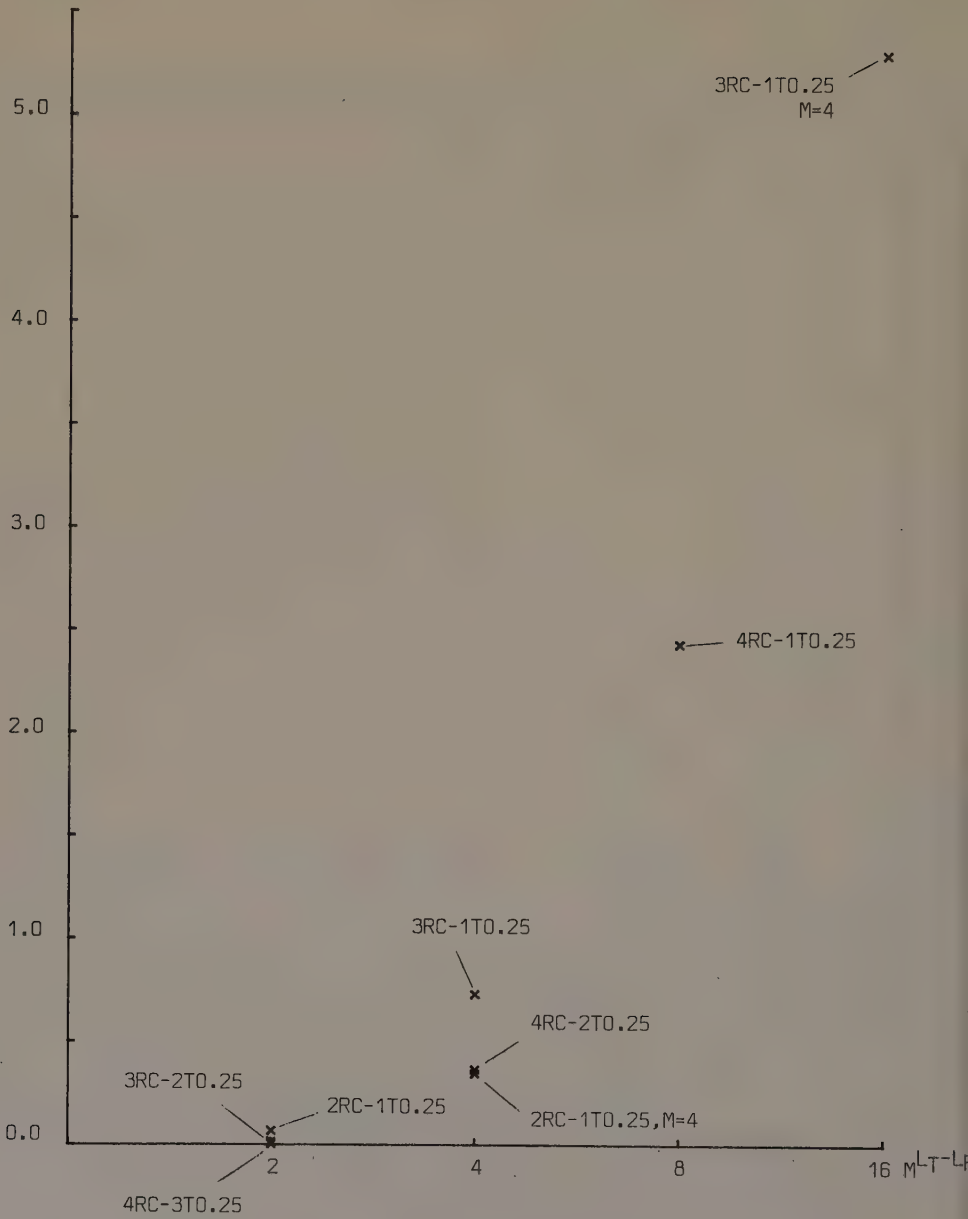


Figure 44. Power degradation versus complexity reduction for the optimum reduced-complexity receiver compared to the optimum receiver for $h=1/4$.

degradation
[dB]

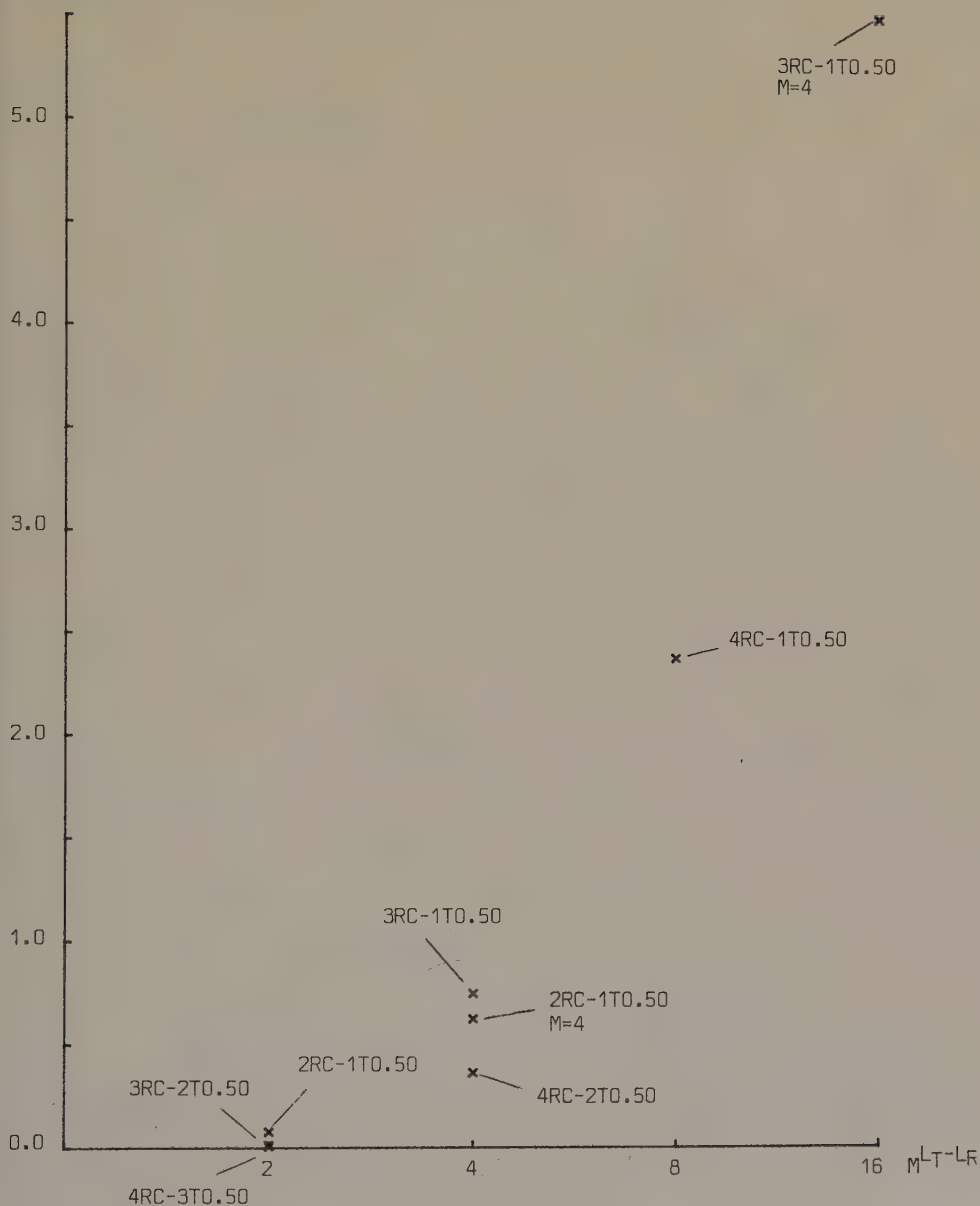


Figure 45. Power degradation versus complexity reduction for the optimum reduced-complexity receiver compared to the optimum receiver for $h=1/2$.

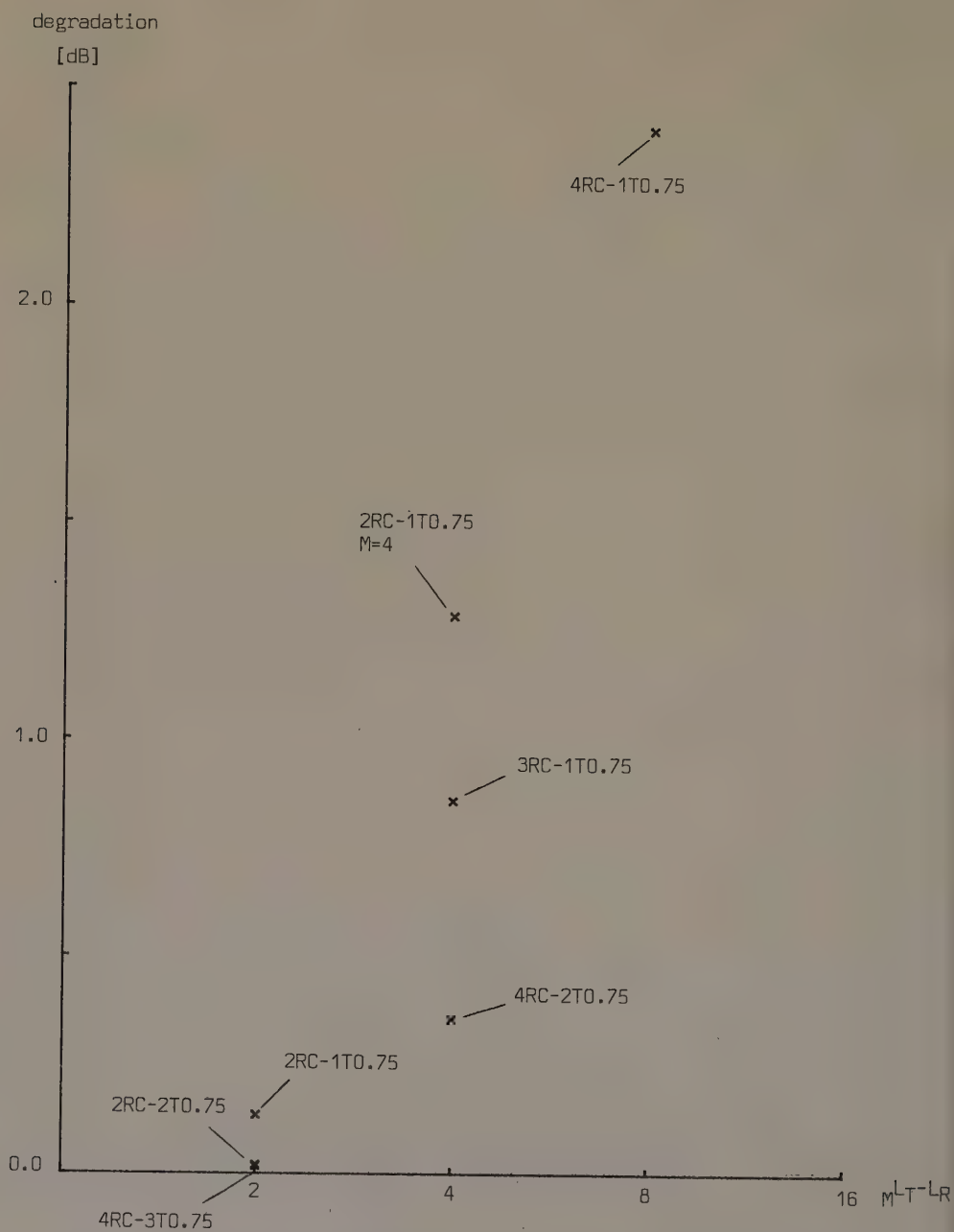


Figure 46. Power degradation versus complexity reduction for the optimum reduced-complexity receiver compared to the optimum receiver for $h=3/4$.

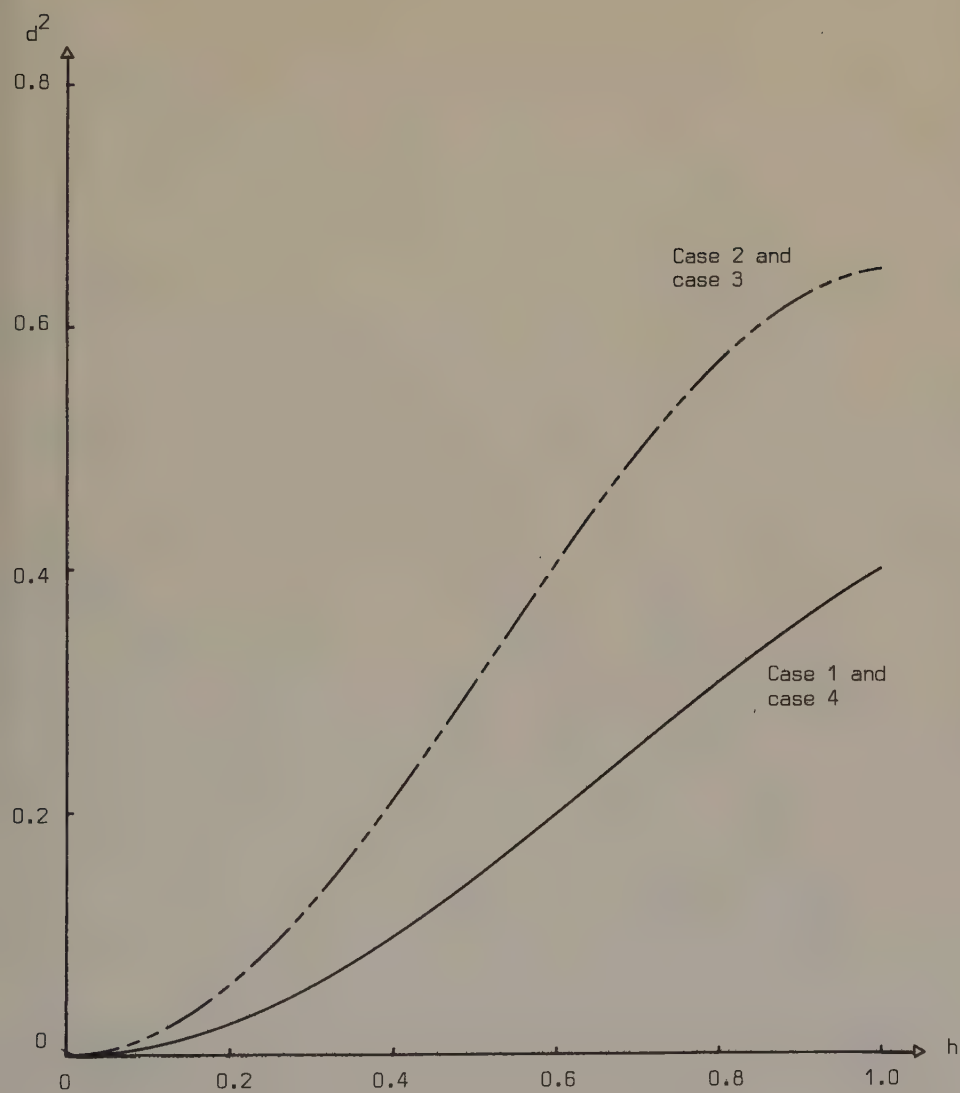


Figure A5. Summary of distance results for $N=1$.

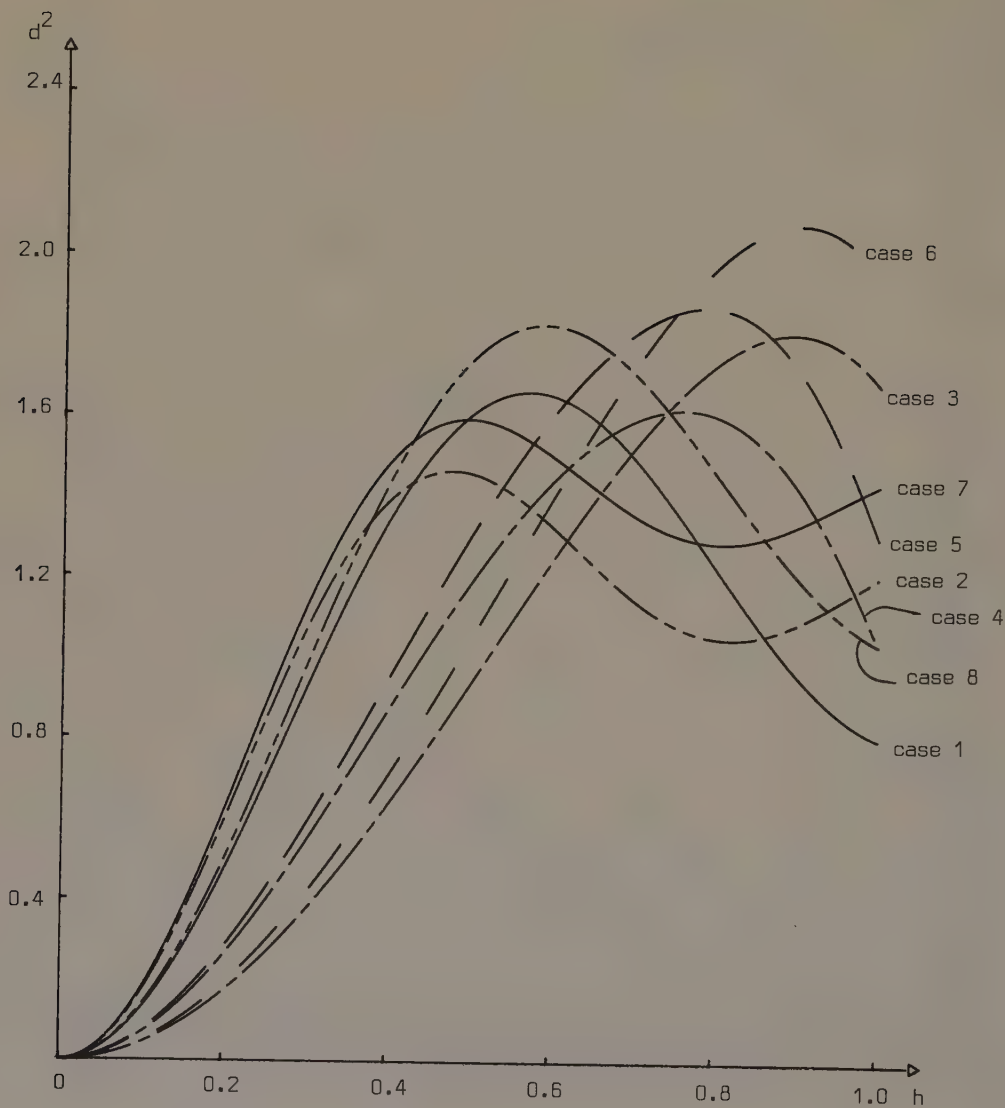


Figure A14. Summary of distance results for $N=2$.

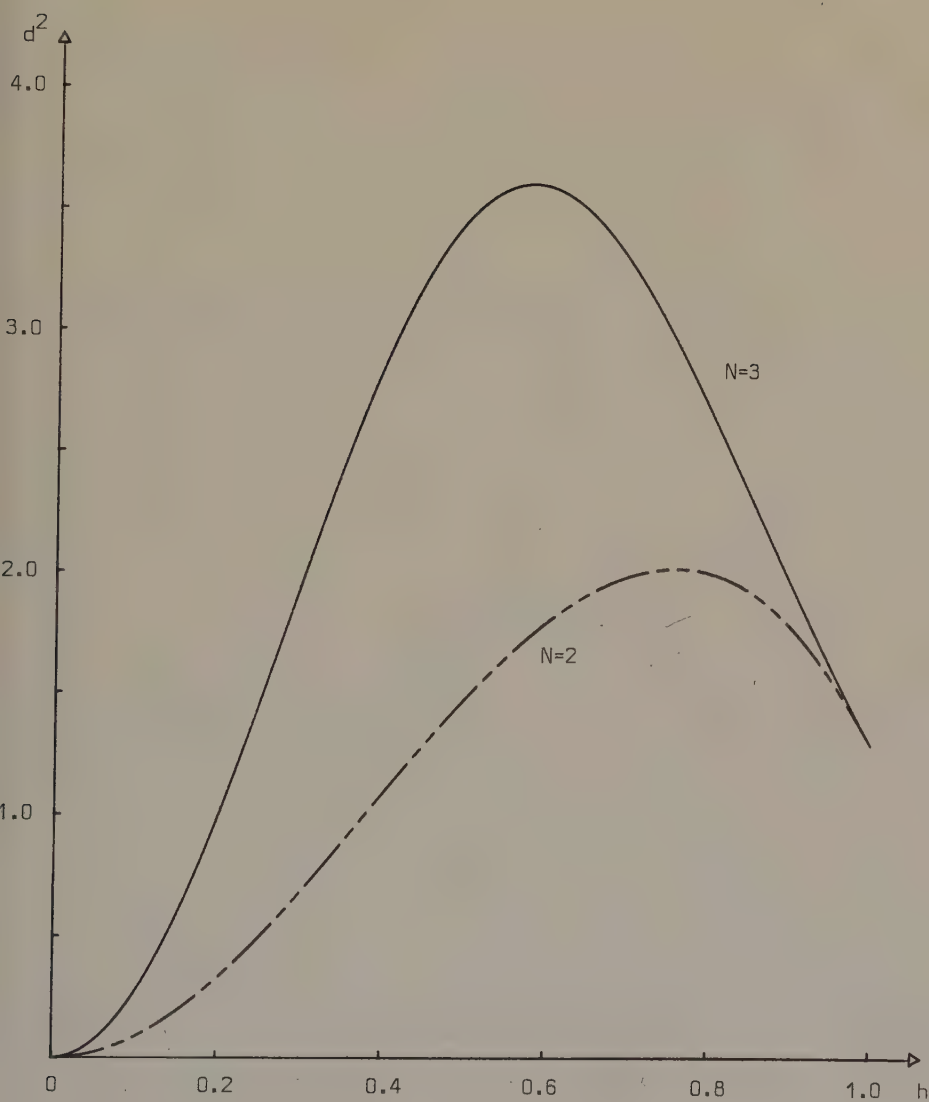


Figure A16. Distance for the case $N=3$, $\underline{\alpha} = -1, -1, 1, 1$ and $\underline{\beta} = -1, 1, 1, 1$. For comparison the corresponding $N=2$ case. For $N=1$ see figure A14.

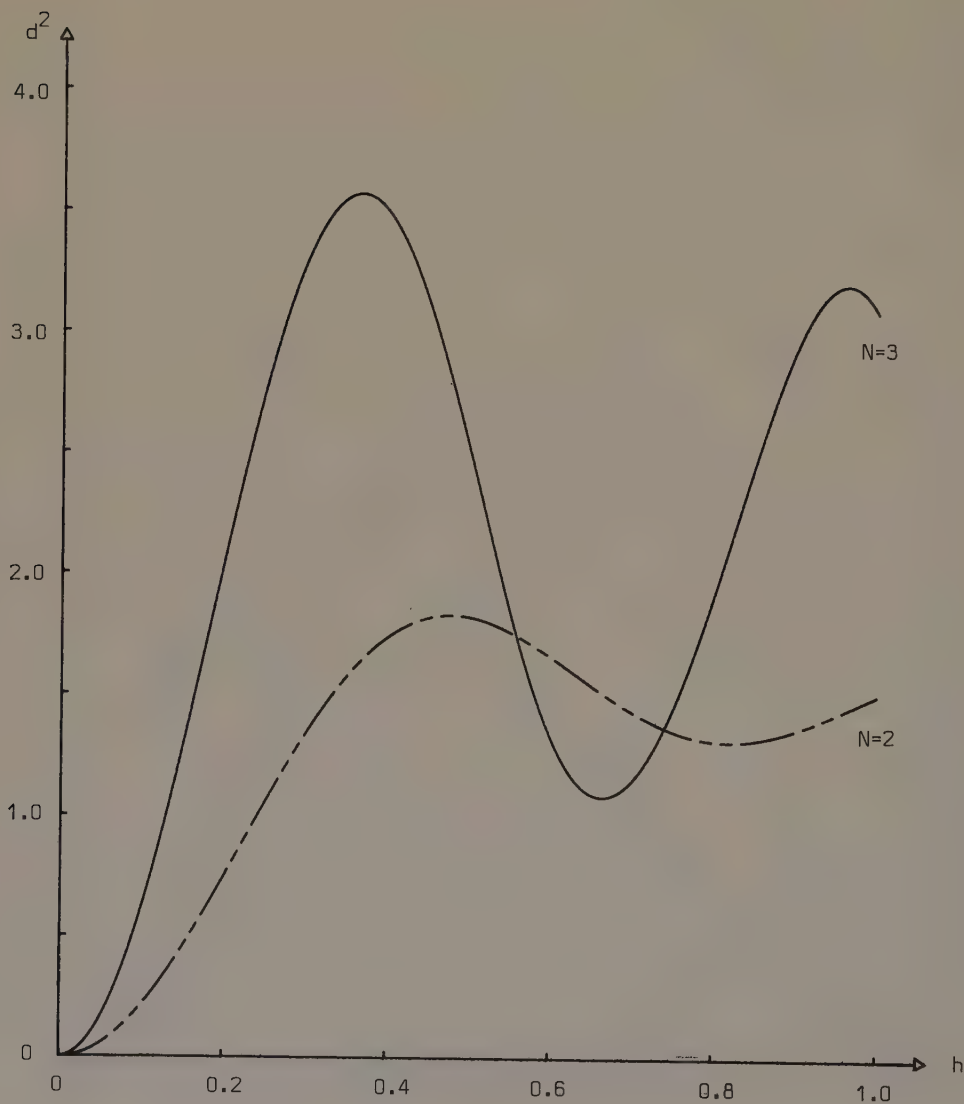


Figure A18. Distance for the case $N=3$, $\underline{\alpha} = -1, -1, -1, 1$ and $\underline{\beta} = -1, 1, 1, -1$. The corresponding $N=2$ case for comparison. Note the decrease of d^2 with N for certain h -values!

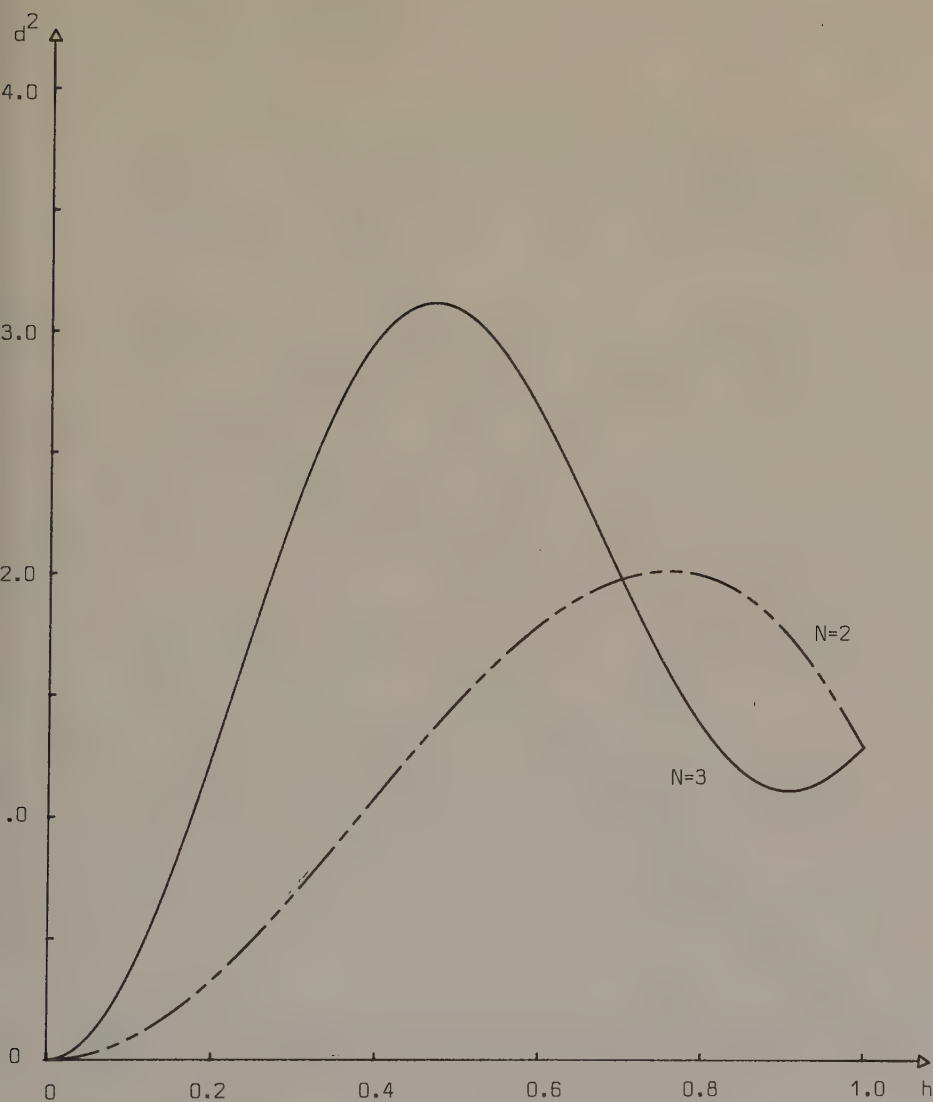


Figure A21. Distance for the case $N=3$, $\underline{\alpha} = -1, -1, 1, -1$ and $\underline{\beta} = -1, 1, 1, -1$.
For comparison the corresponding $N=2$ case.

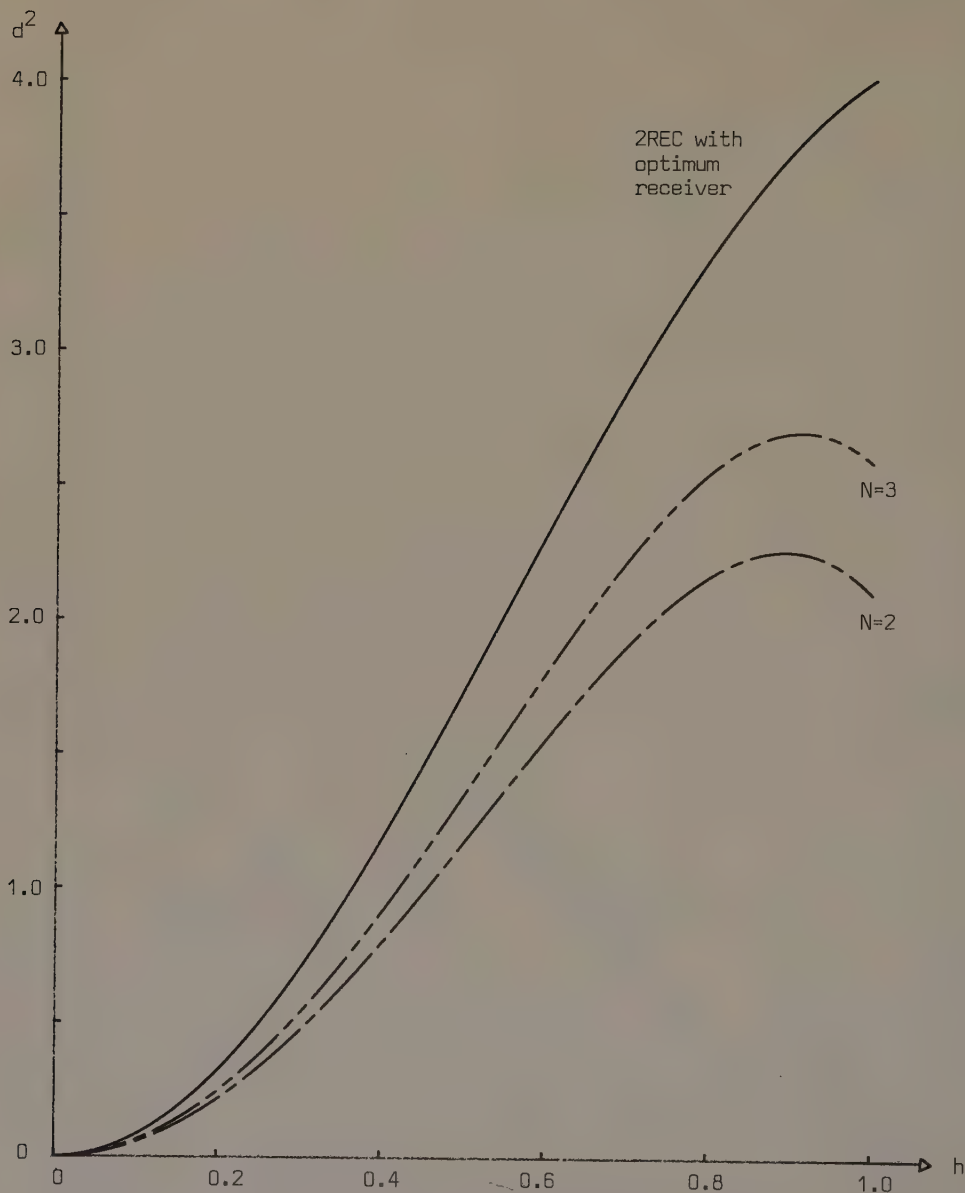


Figure A24. The $N=3$ curve give the upper bound on the minimum squared Euclidean distance for low h -values for 2REC received in a 1REC receiver. For comparison, the minimum squared distance for the optimum receiver for 2REC is also shown, [2]. The $N=3$ case shown are the sequences $\underline{\alpha} = -1, -1, 1, 1$ and $\underline{\beta} = -1, 1, -1, 1$, see figure A23, page I.60.

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RECEIVERS FOR CPM

PART II

PARALLEL MSK-TYPE RECEIVERS

ABSTRACT

The optimum receiver for Continuous Phase Modulation (CPM) schemes is sometimes complex. In this part a parallel MSK-type receiver is studied. It is useful for binary schemes with modulation index $h=1/2$. We consider coherent detection of signals transmitted over an additive white Gaussian noise channel and also over a Rayleigh fading channel.

The MSK-type receiver can be implemented with only two filters and simple decision logic. A distance measure giving the asymptotic performance on the Gaussian channel is derived. Error probability formulas are given both for the nonfading and the fading channel. An algorithm giving the asymptotically optimum filter for the nonfading Gaussian channel will be presented and the optimum filter for low signal-to-noise ratios is derived. For intermediate signal-to-noise ratios numerical optimizations are done both for the Gaussian and the Rayleigh fading channels. The optimum receiver filters are given for a variety of schemes. It is shown that also for the more complex CPM-schemes, the loss compared to the optimum Viterbi receiver is moderate.

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I. INTRODUCTION

From part I it is known that schemes with very good detection properties can be found in the CPM family. Schemes with combined good spectral and detection properties can also be obtained in this family. The problem with these schemes is receiver complexity. They require a maximum likelihood sequence detector [13] implemented by means of a Viterbi detector. From part I it is known that sometimes a reduced-complexity Viterbi receiver can be successfully used. These receivers typically consist of a filterbank followed by a Viterbi processor, where sometimes the number of states is quite large. This means that the receiver is often complex.

For some cases of CPM it is not, however, necessary to use the Viterbi detector. Recently, much attention has also been paid to the MSK-type of receiver, [3]-[7], [9]-[12], [14], [19]-[22], [24]. This receiver has only two filters and only a small amount of processing. The receiver makes single symbol decisions. This simplified receiver is of course in general sub-optimum. The receiver works well for binary modulation with modulation index $h=1/2$ [3]-[7]. Examples of such schemes are given in [9]-[12], [14], [19]-[22], [24]. Various ideas in selecting the receiver filters are analysed in [3]-[5] and an optimum filter is derived in [9]-[12] for various correlative FSK (piecewise linear phase functions). In [24] a filter based on a minimum squared error approach is given and analysed for a TFM scheme. The performance for this receiver is almost equal to the optimum receiver for schemes with a moderate degree of smoothing, i.e. overlapping pulses of length up to, say 3-4 symbol intervals.

In this part the parallel MSK-type of receiver is analysed. A distance measure giving the performance for large signal-to-noise ratios on the non-fading Gaussian channel is introduced. The error probability in detecting the phase node is derived for the additive white Gaussian noise channel. The bit error probability can be calculated from the error probability in detecting the phase node. The error probability in detecting the phase node is also derived for the Rayleigh fading channel. Diversity schemes are also considered.

The optimum filters for various values on the signal-to-noise ratio on both types of channels are given for a variety of schemes. An algorithm giving the asymptotically optimum filter on the Gaussian channel is also presented. Comparisons with the receiver filter used in [11] are given.

Throughout part II perfect carrier recovery and synchronization are assumed.

This part is organized as follows. In section II the modulation system is defined and the performance measure is derived. The optimum filter for low signal-to-noise ratios is derived in section III, both for the nonfading and the fading channel. An algorithm giving the asymptotically (large signal-to-noise ratios) optimum filter on the nonfading channel is also presented. A numerical minimization routine for intermediate signal-to-noise ratios is also presented. In section IV some numerical results are presented. Both the optimum filters and the average error probabilities with these filters are given. Finally section V is a discussion and conclusion section.

II. RECEIVER PRINCIPLES AND RECEIVER PERFORMANCE

In this part we consider one approach to the problem of coherent detection of CPM-signals, with simple receivers. For modulation index $h=1/2$ the received signal has attractive properties, which can be used by a simple receiver. This part deals only with binary signaling schemes with modulation index $h=1/2$ and perfect timing in the receiver is assumed.

First a brief introduction to the CPM signaling scheme. Let $\underline{\alpha} = \dots \alpha_{n-2}, \alpha_{n-1}, \alpha_n, \alpha_{n+1}, \alpha_{n+2}, \dots$ be an infinitely long sequence of statistically independent binary data symbols, taking the values ± 1 with equal probability. The transmitted modulated constant envelope signal is given by

$$s(t, \underline{\alpha}) = \sqrt{\frac{2E}{T}} \cos(2\pi f_0 t + \varphi(t, \underline{\alpha}) + \varphi_0) \quad (1)$$

where the information carrying phase is

$$\varphi(t, \underline{\alpha}) = 2\pi h \sum_{i=-\infty}^{\infty} \alpha_i \cdot q(t-iT) \quad (2)$$

The symbol energy is denoted E and is for these binary schemes equal to the bit energy E_b . T is the symbol time and f_0 the carrier frequency. h is the modulation index. φ_0 is an arbitrary constant phase shift, which can be set to zero without loss of generality. The phase response $q(t)$ is defined by

$$q(t) = \int_{-\infty}^t g(\tau) d\tau \quad (3)$$

where $g(t)$ is the frequency response. For details see [1], [2].

Only symmetric pulses $g(t)$ will be considered in this paper. Schemes based on various frequency pulses will be compared. The Raised Cosine pulse of length L symbol intervals, denoted LRC, is given by

$$g(t) = \begin{cases} \frac{1}{2LT} \left[1 - \cos \left(\frac{2\pi t}{LT} \right) \right] & ; \quad 0 \leq t \leq LT \\ 0 & ; \quad \text{otherwise} \end{cases} \quad (4)$$

The Tamed Frequency Modulation scheme (TFM) is given by

$$g(t) = \frac{1}{8} [g_0(t-T) + 2g_0(t) + g_0(t+T)] \quad (5)$$

where

$$g_0(t) \approx \frac{1}{T} \left[\frac{\sin\left(\frac{\pi t}{T}\right)}{\frac{\pi t}{T}} - \frac{\pi^2}{24} \cdot \frac{2\sin\left(\frac{\pi t}{T}\right) - \frac{2\pi t}{T} \cos\left(\frac{\pi t}{T}\right) - \left(\frac{\pi t}{T}\right)^2 \sin\left(\frac{\pi t}{T}\right)}{\left(\frac{\pi t}{T}\right)^3} \right] \quad (6)$$

Next, the pulse Spectral Raised Cosine with roll-off factor β and main-lobe width L symbol intervals (LSRC) is given by

$$g(t) = \frac{1}{LT} \frac{\sin\left(\frac{2\pi t}{LT}\right)}{\frac{2\pi t}{LT}} \cdot \frac{\cos\left(\beta \cdot \frac{2\pi t}{LT}\right)}{1 - \left(\frac{4\beta}{LT} \cdot t\right)^2} ; \quad 0 \leq \beta \leq 1 \quad (7)$$

In the frequency domain this pulse is defined by

$$G(\omega) = \begin{cases} 1 & ; |\omega| < \frac{2\pi}{LT} (1-\beta) \\ \frac{1}{2} \left(1 - \sin \left(\frac{LT}{4\beta} \left(\omega - \frac{2\pi}{LT} \right) \right) \right) & ; \frac{2\pi}{LT} (1-\beta) \leq |\omega| < \frac{2\pi}{LT} (1+\beta) \\ 0 & ; \text{otherwise} \end{cases} \quad (8)$$

Another pulse is the Gaussian MSK pulse with factor $B_b T$ [4], [19] defined by

$$g(t) = h(t) * \text{rect}\left(\frac{t}{T}\right) \quad (9)$$

where the function $\text{rect}(\cdot)$ by definition is

$$\text{rect}\left(\frac{t}{T}\right) = \begin{cases} 1 & |t| < \frac{T}{2} \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

and $*$ means convolution.

The pulse $h(t)$ is the Gaussian pulse defined as

$$h(t) = \frac{1}{2T^2} \cdot \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2(\sigma T)^2}} \quad (11)$$

The 3 dB bandwidth of the Fourier spectrum of $h(t)$ is defined to be B_b and it is easily verified that

$$B_b \cdot T = \frac{\sqrt{2n2}}{2\pi\sigma} \quad (12)$$

Both the TFM pulse, the SRC pulse and GMSK pulse are not time limited, therefore they have been truncated symmetrically around $t=0$, to the total length of L_T symbol intervals. The truncated pulse is normalized to have

$$\int_{-\infty}^{\infty} g(\tau) d\tau = \frac{1}{2} \quad (13)$$

by definition of the CPM scheme [1], [2]. These truncated schemes are denoted TFML_T , LSRCL_T and GMSKL_T respectively.

Finally the Rectangular pulse of length L symbol intervals (LREC) is given by

$$g(t) = \begin{cases} \frac{1}{2LT} & ; \quad 0 < t < LT \\ 0 & ; \quad \text{otherwise} \end{cases} \quad (14)$$

In particular the full response scheme 1REC ($L=1$) with $h=1/2$ is identical to Minimum Shift Keying, MSK.

Power spectra for a variety of the above schemes are given in [8], where a method for calculating the power spectrum is given. For comparison of the spectral efficiency of the various schemes, figure 1 to 4 show some power spectra. In conclusion, GMSK and SRC schemes have lower sidelobes when the parameters are chosen to give a mainlobe of the same order as a RC scheme. Thus the spectral efficiency of GMSK and SRC schemes are better than for the RC schemes.

The received signal is $r(t)$

$$r(t) = s(t, \underline{\alpha}) + n(t) \quad (15)$$

where $n(t)$ is white Gaussian noise with one sided spectral density N_0 .

For $nT < t \leq (n+1)T$ the information carrying phase is given by

$$\varphi(t, \underline{\alpha}_n) = 2\pi h \sum_{i=-\infty}^n \alpha_i \cdot q(t-iT) \quad (16)$$

The frequency pulse $g(t)$ is time-limited [2]

$$g(t) \equiv 0 \quad t \leq 0, \quad t \geq LT \quad (17)$$

where L is the duration in symbol intervals of the frequency pulse, and

$$\int_0^{LT} g(\tau) d\tau = \frac{1}{2} \quad (18)$$

Thus

$$\begin{cases} q(t) \equiv 0 & t \leq 0 \\ q(t) \equiv \frac{1}{2} & t \geq LT \end{cases} \quad (19)$$

This gives

$$\begin{aligned} \varphi(t, \underline{\alpha}_n) &= 2\pi h \sum_{i=n-L+1}^n \alpha_i \cdot q(t-iT) + h\pi \sum_{i \leq n-L} \alpha_i = \\ &= \left[2\pi h \sum_{i=n-L+1}^n \alpha_i \cdot q(t-iT) - h\pi \sum_{i=n-L+1}^n \alpha_i \right] + h\pi \sum_{i \leq n} \alpha_i = \\ &= \left[2\pi h \sum_{i=n-L+1}^n \alpha_i \left[q(t-iT) - \frac{1}{2} \right] \right] + h\pi \sum_{i \leq n} \alpha_i \end{aligned} \quad (20)$$

The first term in (20) can be seen as an intersymbol interference (ISI) around the phase node. Define the phase node (phase state) Φ_n as

$$\vartheta_n = h\pi \sum_{i \leq n} \alpha_i \quad (21)$$

This implies that it is possible to detect the phase node ϑ_n , at time $t = (n+1)T$, and then by processing ϑ_n make an estimate on α_n . From (21) we have

$$\vartheta_{n-1} = h\pi \sum_{i \leq n-1} \alpha_i \quad (22)$$

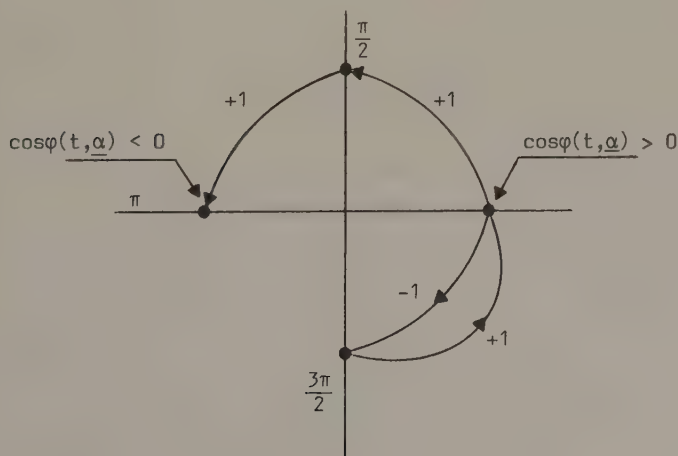
and

$$\vartheta_n = h\pi \sum_{i \leq n} \alpha_i \quad (23)$$

and therefore

$$\alpha_n = \frac{1}{h\pi} (\vartheta_n - \vartheta_{n-1}) \quad (24)$$

For the case of $h=1/2$ we have the 4 different phase states $0, \frac{\pi}{2}, +\pi, \frac{3\pi}{2}$. The $\cos\varphi(t, \underline{\alpha})$ eye pattern shows the open eye every second symbol, i.e. every interval $2T$. If $\cos\varphi(t, \underline{\alpha}) > 0$ corresponds to data symbol $+1$ or data symbol -1 depends on the prehistory however. The following phase diagram shows the problem.



The data sequence +1,+1 moves us from phase state 0 to π , thus changes the sign of $\cos\varphi(t, \underline{\alpha})$ in the eye pattern. The data sequence -1,+1 takes us from phase state 0 back to phase state 0. The sign of $\cos\varphi(t, \underline{\alpha})$ is now unchanged. Thus $\cos\varphi(t, \underline{\alpha}) > 0$ can correspond to different data symbols. The way to resolve this is shown in the appendix [14], [22], by means of differential encoding. With de Budas technique [22] the decisions in the quadrature channels can be made independently of each other. If we look at the function $\cos\varphi(t, \underline{\alpha})$ for modulation index $h=1/2$ this is the same as detecting whether $\cos\varphi(t, \underline{\alpha})$ is greater or less than zero. This is a typical binary hypothesis testing problem. The receiver is shown in figure 5.

Some examples of the signal set before the receiver filter in this receiver are given in figure 6 to 9. This is the signals for 2RC, 3RC, 4RC and TFM5. In the following we will denote these normalized quadrature signals $\tilde{s}(t, \underline{\alpha})$, i.e.

$$\tilde{s}(t, \underline{\alpha}) = \cos\varphi(t, \underline{\alpha}) \quad (25)$$

These signals give an open eye only every second symbol interval. However, the signal in the other quadrature arm is $\sin\varphi(t, \underline{\alpha})$ and these signals give an open eye for the time intervals inbetween. This means that the decisions can be made alternately from the two quadrature arms. From (20) it is seen that the ensemble of signals $\sin\varphi(t, \underline{\alpha})$ is just an one symbol interval replica of the ensemble of signals $\cos\varphi(t, \underline{\alpha})$. Thus only the decisions based on $\tilde{s}(t, \underline{\alpha})$ will be considered. From (20) it is also easily verified that the ensemble of these signals, are periodic with period $2T$. Thus without loss of generality the decision taken place at $t_0=0$ if L is odd and $t_0 = \frac{T}{2}$ if L is even can be considered. This difference of the time when the eye is opened is due to the definition of the CPM scheme. The ensemble of signals $\tilde{s}(t, \underline{\alpha})$ can be divided into two subsets. The subset having $\cos\varphi(t_0, \underline{\alpha}) > 0$, or equivalent $\tilde{s}(t_0, \underline{\alpha}) > 0$, will be denoted S^+ , while the subset having $\tilde{s}(t_0, \underline{\alpha}) < 0$ will be denoted S^- . The corresponding data sequences will in the same manner be denoted $\underline{\alpha}^+$ if the sequence gives rise to a signal in S^+ and $\underline{\alpha}^-$ if the corresponding signal belongs to S^- . In the vector signal space the signals will be denoted $\underline{s}_1^+$ or $\underline{s}_1^-$ where

$$\begin{aligned}\underline{s}_i^+ &\in S^+ & i = 1, \dots, m/2 \\ \underline{s}_i^- &\in S^- & i = 1, \dots, m/2\end{aligned}\quad (26)$$

where m is to total number of signals.

Due to the fact that $h=1/2$, the signals in S^- are equal to $-\underline{s}_i^+$, where $i = 1, \dots, m/2$.

The normalized received signal is given by

$$\tilde{r}(t) = \tilde{s}(t, \underline{\alpha}) + \sqrt{\frac{T}{E_b}} n_c(t) \quad (27)$$

where $n_c(t)$ is given by

$$n(t) = \sqrt{2} n_c(t) \cos \omega_0 t - \sqrt{2} n_s(t) \sin \omega_0 t \quad (28)$$

Thus the binary hypothesis testing problem is

$$\begin{aligned}H^+: \quad \underline{r} &= \underline{s}_i^+ + \underline{n} & \underline{s}_i^+ &\in S^+ \\ H^-: \quad \underline{r} &= \underline{s}_i^- + \underline{n} & \underline{s}_i^- &\in S^-\end{aligned}\quad (29)$$

$\underline{n}$ represents the additive noise and $\underline{r}$ the received signal in the vector signal space. Since the data symbols ± 1 occur with equal probability $1/2$, the probability of the signals $\underline{s}_i^+$ and $\underline{s}_i^-$ are equal to $1/m$. The linear receiver for this problem is given in figure 5. The threshold is equal to zero because of the symmetry of the signal sets S^+ and S^- . This means that there is $m/2$ signal points corresponding to H^+ and $m/2$ signal points corresponding to H^- in the signal space. This signal space has to be divided into two decision regions, and dependent on the received signal the decision can be made.

The optimum receiver filter is given by [9]

$$a(t) = \sum_{i=1}^{m/2} a_i \tilde{s}(t, \underline{\alpha}_i^+) \quad (30)$$

also because of the symmetry of the signal sets. In the vector space this is equal to

$$\underline{a} = \sum_{i=1}^{m/2} a_i \underline{s}_i^+ \quad (31)$$

The coefficients a_i , $i = 1, 2, \dots, m/2$, should be chosen to minimize the average error probability P . To do this the error probability needs to be known. The channel is assumed to be an additive white Gaussian noise channel. This means that the decision variable λ is a Gaussian variable and therefore its mean and variance determine the error probability. Let us assume signal $\underline{s}_j^+$ transmitted. Then the mean value M_j is

$$M_j = E\{\lambda | \underline{s}_j^+\} = \langle \underline{s}_j^+, \underline{a} \rangle \quad (32)$$

where $E\{\cdot\}$ denotes the expected value of $\cdot$, and $\langle \cdot, \cdot \rangle$ is the inner product in the Euclidean space. The variance $\sigma^2 = E\{(\lambda - M_j)^2\}$ is independent of any particular transmitted signal and is given by

$$\sigma^2 = \frac{1}{E_b} \cdot \frac{N_0}{2} \cdot |\underline{a}|^2 \quad (33)$$

since the variance of $\underline{n}$ is $1/E_b \cdot N_0/2$. N_0 is the one sided spectral density of the additive white noise.

The normalized squared distance is now defined through

$$d_j^2 = \frac{M_j^2}{\sigma^2} \cdot \frac{N_0}{E_b} \quad (34)$$

and thus

$$d_j^2 = \frac{2}{T} \cdot \frac{[\langle \underline{s}_j^+, \underline{a} \rangle]^2}{|\underline{a}|^2} \quad (35)$$

Note that this is not a distance in the signal space but acts as a Euclidean distance in the error probability calculation, compare part I. This distance can also be written

$$d_j^2 = \frac{2}{T} \cdot \frac{\left[\int_I a(-t) \tilde{s}(t, \underline{\alpha}_j^+) dt \right]^2}{\int_I a^2(t) dt} \quad (36)$$

where I denotes the N_T symbol interval long observation interval. For MSK

the squared normalized distance is 2, with matched filters of length $N_T=2$. The transmission is in this case free from intersymbol interference and is linear modulation in each quadrature arm [16], [22]. This is true for any full response $h=1/2$ scheme [1]. The asymptotic performance, i.e. for large signal to noise ratios, is given by the minimum of these distances, i.e.

$$d_{\min}^2 = \min_j \{d_j^2\} \quad (37)$$

The error probability in estimating the phase node ϕ_n on the Gaussian channel is then obtained, by averaging over all possible transmitted waveforms, as a function of the signal to noise ratio E_b/N_0 .

$$P = \frac{1}{m} \sum_{j=1}^m Q \left(\sqrt{d_j^2 \frac{E_b}{N_0}} \right) \quad (38)$$

where m is the number of transmitted waveforms. The function $Q(x)$ is the Gaussian error function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} dy \quad (39)$$

For QPSK and MSK we have

$$P = Q \left(\sqrt{\frac{2E_b}{N_0}} \right) \quad (40)$$

Methods to calculate $Q(x)$ are given in [26]. From (24) we can conclude, that since α_n depends on both ϕ_n and ϕ_{n-1} , i.e. a dependence on two symbol intervals, an error in estimating ϕ_n will affect both α_n and α_{n+1} . An error in estimating ϕ_n will therefore cause an error in both α_n and α_{n+1} alternatively, an error is made in α_n and α_{n+2} or α_{n+1} and α_{n+3} when the de Buda technique is used. Therefore the bit error probability P_e in estimating the data sequence α_n will be about a factor 2 larger than P . With differential encoding for resolving the relationship between phase nodes and data symbols [14], [22] and the length of the filter N_T not greater than 2 we have

$$P_e = 2P(1-P) \quad (41)$$

If $N_T > 2$ the decision variables for deciding about ϕ_n and ϕ_{n-2} are correlated and thus P_e has to be calculated from the joint distribution of these two variables.

Now consider the problem of calculating the average error probability for a fading channel [6]

$$P = \int_0^{\infty} f(\gamma) P(\gamma) d\gamma \quad (42)$$

where $P(\gamma)$ is the average error probability of the modulation scheme with ideal coherent MSK-type detection for Gaussian noise for the signal-to-noise ratio γ given by (38)

$$P(\gamma) = \frac{1}{m} \sum_{j=1}^m \cdot Q\left(\sqrt{d_j^2 \gamma}\right) \quad (43)$$

and where $f(\gamma)$ is the density for γ . Note that the receiver filter operates over N_T symbols typically smaller than 10. Slow fading is assumed and the averaging is performed over one "block", of length N_T intervals [6].

The instantaneous signal-to-noise ratio in diversity branch k is assumed to be γ_k with equal average value for all branches. For Rayleigh fading the γ_k has the probability density function

$$f(\gamma) = \frac{1}{\Gamma} e^{-\gamma/\Gamma} \quad (44)$$

where Γ is the average signal-to-noise ratio.

First assume ideal maximal ratio combining [4], [6]. This combiner must know each path magnitude and phase in order to perform the perfect combining and must have the property that the output signal-to-noise ratio is the sum of the instantaneous branch signal-to-noise ratios, i.e.

$$\gamma = \sum_{k=1}^M \gamma_k \quad (45)$$

where M is the number of branches. The random variable γ has the density function [4], [6]

$$f(\gamma) = \frac{1}{\Gamma} \left(\frac{\gamma}{\Gamma}\right)^{M-1} \frac{1}{(M-1)!} e^{-\gamma/\Gamma} \quad (46)$$

for M branch maximal ratio combining. We assume independent Rayleigh fading in each branch. The average receiver output signal-to-noise ratio is

$$\Gamma_R = E\{\gamma\} = M\Gamma \quad (47)$$

For the case of coherent MSK-type reception of partial response continuous phase modulated signals the error probability is given by (43). The result of averaging (42) with (43) and (46) yields [6]

$$P = \frac{1}{2^m} \sum_{j=1}^m \left[1 - \sqrt{\frac{\frac{d_j^2 \Gamma}{2}}{1 + \frac{d_j^2 \Gamma}{2}}} \cdot \left(1 + \frac{1}{1!2} \left(1 + \frac{d_j^2 \Gamma}{2} \right)^{-1} + \frac{1 \cdot 3}{2!2^2} \left(1 + \frac{d_j^2 \Gamma}{2} \right)^{-2} + \dots + \frac{1 \cdot 3 \cdot 5 \dots (2M-1)}{(M-1)!2^{(M-1)}} \left(1 + \frac{d_j^2 \Gamma}{2} \right)^{-(M-1)} \right) \right] \quad (48)$$

where d_j^2 , $j = 1, \dots, m$ depend on the modulation scheme and the receiver filter. Γ is the average per branch signal-to-noise ratio and M is the number of diversity branches.

An alternative to the optimum maximal ratio combining is the sub-optimum selection combining. The ideal selection diversity combiner is defined here as a device which selects that diversity branch with the largest signal-to-noise ratio for bit decisions. The same branch is used for all symbols over a time interval of the receiver filter under the assumption of slow fading.

Let γ_k be the instantaneous signal-to-noise ratio in branch k with average value Γ equal in all branches. From [4], [6] the probability density function for the ideal selection combiner output signal-to-noise ratio γ is

$$f(\gamma) = \frac{M}{\Gamma} e^{-\gamma/\Gamma} \left(1 - e^{-\gamma/\Gamma} \right)^{(M-1)} \quad (49)$$

where Γ is the average signal-to-noise ratio per branch and M is the number of branches. The average receiver output signal-to-noise ratio is in this case

$$\Gamma_R = E\{\gamma\} = \Gamma \cdot \sum_{k=1}^M \frac{1}{k} \quad (50)$$

which of course increases much slower with increasing M than the corresponding average for maximal ratio combining (47).

It is shown in [6] that the averaging of (42) with (43) and (49) yields the analytical error probability formula

$$P = \frac{1}{2^m} \sum_{j=1}^m \sum_{k=0}^M \frac{(-1)^k \binom{M}{k}}{\sqrt{1 + \frac{2k}{d_j^2 \Gamma}}} \quad (51)$$

where as before d_j^2 , $j = 1, \dots, m$ are defined by the modulation scheme and by the receiver.

III. THE OPTIMUM RECEIVER FILTER

In this section the methods for finding the optimum receiver filter for the parallel MSK-type receiver will be presented. The problem of finding the optimum receiver filter to a linear MSK-type receiver has been addressed in [9] - [12]. Most of the theoretical background is given in these papers. In [9] an algorithm giving the asymptotically optimum filter is presented. This algorithm however typically requires the solution of a quadratic program. Therefore we will present a simple alternative algorithm giving the asymptotically optimum filter. Numerical maximization methods giving the optimum receiver filter for a given signal-to-noise ratio both for the Gaussian channel and the Rayleigh fading channel will also be presented.

The optimum receiver filter is given in eq. (30). From eq. (35) it is seen that an increase in energy of the receiver filter will not change the performance. Thus the coefficients a_i have been normalized such that

$$\sum_{i=1}^{m/2} a_i = 1 \quad (52)$$

throughout this paper.

Low SNR's on the Gaussian channel

The average error probability for the additive Gaussian channel is given in eq (38). For very small SNR's, i.e. when $E_b/N_0 \rightarrow 0$, the error function Q is approximated by

$$Q(x) \approx \frac{1}{2} - \frac{1}{\sqrt{2\pi}} x \quad \text{small } x \quad (53)$$

This means that, when $E_b/N_0 \rightarrow 0$ ($|\cdot|$ denoted the absolute value of $\cdot$)

$$\begin{aligned} P &\approx \sum_{j=1}^m \frac{1}{m} \left[\frac{1}{2} - \frac{1}{\sqrt{2\pi}} |d_j| \sqrt{\frac{E_b}{N_0}} \right] = \\ &= \frac{1}{2} - \sqrt{\frac{E_b}{2\pi N_0}} \sum_{j=1}^{m/2} \frac{2}{m} d_j \end{aligned} \quad (54)$$

where the symmetry of the two sets have been used. This means that P is minimized if $\sum_{j=1}^{m/2} d_j$ is maximized. From eq (35) d_j is given by

$$d_j = \sqrt{\frac{2}{T}} \frac{\langle \underline{s}_j^+, \underline{a} \rangle}{|\underline{a}|} \quad (55)$$

where the inner product $\langle \underline{s}_j^+, \underline{a} \rangle$ is defined by

$$\langle \underline{s}_j^+, \underline{a} \rangle = \int_I \tilde{s}(t, \underline{\alpha}_j^+) a(-t) dt \quad (56)$$

and the absolute value $|\underline{a}|$ is defined by

$$|\underline{a}| = \sqrt{\int_I a^2(t) dt} \quad (57)$$

Thus

$$\frac{2}{m} \sum_{j=1}^{m/2} d_j = \sqrt{\frac{2}{T}} \frac{\sum_{j=1}^{m/2} \langle \underline{s}_j^+, \sum_{i=1}^{m/2} a_i \underline{s}_i^+ \rangle}{\left| \sum_{i=1}^{m/2} a_i \underline{s}_i^+ \right|} \quad (58)$$

where eq (55) has been used. Rewriting eq (58) and using Schwartz inequality gives

$$\begin{aligned} \frac{2}{m} \sum_{j=1}^{m/2} d_j &= \sqrt{\frac{2}{T}} \frac{\langle \sum_{j=1}^{m/2} \frac{2}{m} \underline{s}_j^+, \sum_{i=1}^{m/2} a_i \underline{s}_i^+ \rangle}{\left| \sum_{i=1}^{m/2} a_i \underline{s}_i^+ \right|} \leq \\ &\leq \sqrt{\frac{2}{T}} \left| \sum_{j=1}^{m/2} \frac{2}{m} \underline{s}_j^+ \right| \end{aligned} \quad (59)$$

with equality if and only if

$$\sum_{j=1}^{m/2} \frac{2}{m} \underline{s}_j^+ = \sum_{i=1}^{m/2} a_i \underline{s}_i^+ \quad (60)$$

Thus, by choosing

$$a_i = \frac{2}{m} \quad i = 1, \dots, m/2 \quad (61)$$

the average error probability will be minimized. This is the so-called Average Matched Filter (AMF) defined by

$$a(t) = \sum_{j=1}^{m/2} \frac{2}{m} \tilde{s}(t, \underline{a}_j^+) \quad (62)$$

and is seen to be the average of all the signals in S^+ .

Large SNR's on the Gaussian channel

In this subsection an algorithm giving the optimum receiver filter for large SNR's, i.e. $E_b/N_0 \rightarrow \infty$, is presented. The optimum filter in this case is derived in [9] but the algorithm presented requires solution of a quadratic program.

For large SNR's the error probability is determined by the squared minimum distance $d_{\min}^2$, since for large SNR's

$$Q(x) \approx e^{-x^2/2} \quad \text{large } x \quad (63)$$

Thus the error probability is minimized if the squared minimum distance $d_{\min}^2$ is maximized. Now $d_{\min}^2$ is given by

$$d_{\min}^2 = \min_j \frac{2}{T} \frac{\left[\left\langle \underline{s}_j^+, \sum_{i=1}^{m/2} a_i \underline{s}_i^+ \right\rangle \right]^2}{\left| \sum_{i=1}^{m/2} a_i \underline{s}_i^+ \right|^2} \quad (64)$$

From [9] it is known that since we are trying to maximize the minimum of a set of inner products, it follows that an optimum receiver vector $\underline{a}$ is characterized by the properties

$$\begin{cases} d_j^2 = d_{\min}^2 & \text{for } j \in J \\ d_j^2 > d_{\min}^2 & \text{for } j \notin J \end{cases} \quad (65)$$

for some $d_{\min}^2$ and set of indices $J \subset \{1, 2, \dots, m/2\}$, and that no other receiver filter $\underline{a}$ exists which satisfies (65) with a greater $d_{\min}^2$ and any set of indices J . In [9] the signal vectors $\underline{s}_j^+$ with $j \in J$ are referred to as the active vectors.

From (30) it is known that the receiver filter is a linear combination of all the signals. In [9] it is shown that, for $E_b/N_0 \rightarrow \infty$, in fact the receiver filter is a linear combination of the active signals only, i.e. the signals corresponding to the active vectors. Thus

$$\underline{a} = \sum_{j \in J} a_j \underline{s}_j^+ \quad (66)$$

It is also shown that $\underline{a}$ is unique, but $\underline{a}$ can be given by several different linear combinations, due to the linear dependency of the signals in S^+ .

In [9] it is also shown that it is possible to find at least one combination with nonnegative coefficients a_i , $i = 1, \dots, m/2$, i.e. the receiver filter is in the convex conical hull of the set S^+ .

Thus, the problem is to find the active signals and the corresponding coefficients. One problem is that the number of signals m is very large in most of the interesting cases. For a modulation scheme based on 3RC and an observation length of $N_T=4$, i.e. the length of the AMF-filter, the number of signals in S^+ is 32; i.e. m is 64. However, in some cases not all of these signals have to be considered, due to the linear dependency of the signals. In the above example only 8 signals have to be considered. Only a smaller part of the signals are symmetric. Since the ensemble of signals in S^+ form a symmetric eye pattern, and the signals have equal a priori probability it is clear that the receiver filter impulse response must be symmetric. It can easily be shown that the non-symmetric signals occur in pair if $g(t)$ is symmetric, where the two signals are related through

$$\tilde{s}(t-t_0, \underline{\alpha}_j^+) = \tilde{s}(-(t-t_0), \underline{\alpha}_i^+) \quad i \neq j \quad (67)$$

and therefore their sum is symmetric. t_0 is the time when the quadrature eye is opened, i.e. $t_0=0$ if L is odd and $t_0 = \frac{T}{2}$ if L is even. This means that the coefficients for these two signals must be equal in order to give a symmetric filter. Thus instead of considering all the nonsymmetric signals in S^+ these signals can be combined according to eq (67) and only the average signals of these combinations have to be considered. Furthermore we will show that if $L \leq 3$ then only the symmetric signals in S^+ have

to be considered because the above linear combinations of the nonsymmetric signals are linearly dependent of the symmetric signal set. To show this the state description of the CPM signal will be introduced [2].

The phase function $\varphi(t, \underline{\alpha})$ is given in (2). Now consider the interval $nT \leq t \leq (n+1)T$. Let $\underline{\alpha}_n$ denote the sequence $\underline{\alpha}_n = \dots, \alpha_{n-1}, \alpha_n$. Then

$$\varphi(t, \underline{\alpha}_n) = \pi \sum_{i=n-(L-1)}^n \alpha_i q(t-iT) + \frac{\pi}{2} \sum_{i \leq n-L} \alpha_i \quad (68)$$

where the properties of $q(t)$ and $h=1/2$ have been used. Now define

$$\theta(t, \underline{\alpha}_n) = \pi \sum_{i=n-(L-1)}^n \alpha_i q(t-iT) \quad (69)$$

and

$$\theta_n = \frac{\pi}{2} \sum_{i \leq n-L} \alpha_i \mod. 2\pi \quad (70)$$

where mod. 2π means modulo 2π . Thus it is seen that the shape of the phase in $nT \leq t \leq (n+1)T$ depends only on the last L data symbols and the prior symbols just correspond to a constant phase shift. Now define the state vector as

$$\underline{\sigma}_n = (\theta_n, \alpha_{n-1}, \alpha_{n-2}, \dots, \alpha_{n-L+1}) \quad (71)$$

and it is seen [2] that $\underline{\sigma}_n$ and α_n exactly determine the phase in $nT \leq t \leq (n+1)T$. The vector $(\alpha_{n-1}, \dots, \alpha_{n-L+1})$ is referred to as the correlative state vector.

Figure 6 to 8 show the quadrature eye of 2RC, 3RC and 4RC respectively with the states assigned to the branches. Note that the phases corresponding to different states sometimes coincides. Note also that the definition of phase state is not unique, compare eq (21) and part I [2].

Let us start with the case $L=2$. Figure 6 shows this case. Consider two nonsymmetric signals $\tilde{s}(t, \underline{\alpha}_i^+)$ and $\tilde{s}(t, \underline{\alpha}_j^+)$ where $i \neq j$, belonging to the set S^+ where $\underline{\alpha}_i^+$ and $\underline{\alpha}_j^+$ are chosen so that the signals satisfy eq (67). From figure 6 it is seen that the state at time $t=0$ is $(\frac{3\pi}{2}, -1)$ or $(\frac{\pi}{2}, +1)$. This means that $\alpha_{i0}^+ = \alpha_{j0}^+$ if both signals are in the same state at $t=0$ or $\alpha_{i0}^+ = -\alpha_{j0}^+$ if they are in different states at $t=0$. In both cases they

give rise to the same branch in the eye-pattern in the interval $0 \leq t \leq T$, since being in state $(\frac{\pi}{2}, +1)$ with input -1 gives exactly the same signal as being in state $(\frac{3\pi}{2}, -1)$ with input $+1$, if the frequency response is symmetric. When both signals are in the same state at $t=0$ and the inputs are the same they trivially give the same signals.

Now consider two symmetric signals $\tilde{s}(t, \underline{\beta}_i^+)$ and $\tilde{s}(t, \underline{\beta}_j^+)$ where $i \neq j$, belonging to S^+ . Let $\beta_{ik}^+ = \alpha_{ik}^+$ for $k \leq 0$ and $\beta_{jk}^+ = \alpha_{jk}^+$ for $k \leq 0$. This means that

$$\begin{aligned} \tilde{s}(t, \underline{\alpha}_i^+) &= \tilde{s}(t, \underline{\beta}_i^+) \\ \tilde{s}(t, \underline{\alpha}_j^+) &= \tilde{s}(t, \underline{\beta}_j^+) \end{aligned} \quad t \leq T \quad (72)$$

Then choose $\underline{\beta}_i^+$ and $\underline{\beta}_j^+$ such that $\tilde{s}(t, \underline{\beta}_i^+)$ and $\tilde{s}(t, \underline{\beta}_j^+)$ are symmetric around $t = \frac{T}{2}$. Then it is seen from figure 6 that

$$\begin{aligned} \tilde{s}(t, \underline{\alpha}_i^+) &= \tilde{s}(t, \underline{\beta}_j^+) \\ \tilde{s}(t, \underline{\alpha}_j^+) &= \tilde{s}(t, \underline{\beta}_i^+) \end{aligned} \quad t \geq 0 \quad (73)$$

and therefore

$$\tilde{s}(t, \underline{\alpha}_i^+) + \tilde{s}(t, \underline{\alpha}_j^+) = \tilde{s}(t, \underline{\beta}_i^+) + \tilde{s}(t, \underline{\beta}_j^+) \quad (74)$$

In this way the sum of all pairs of nonsymmetric signals satisfying eq (67) can be shown to be equal to the sum of two symmetric signals chosen as described above. Note that even if figure 6 shows the signals for 2RC, the above holds for every possible scheme based on a symmetric pulse $g(t)$, since the states are independent of the pulse shape. Therefore, when $L=2$, only the symmetric signals in S^+ have to be considered, when the asymptotically optimum filter has to be found.

Figure 7 shows the signals when $L=3$. Now the eye is opened at time $t_0=0$. Consider two nonsymmetric signals $\tilde{s}(t, \underline{\alpha}_i^+)$ and $\tilde{s}(t, \underline{\alpha}_j^+)$ where $i \neq j$, belonging to the set S^+ , where $\underline{\alpha}_i^+$ and $\underline{\alpha}_j^+$ are chosen so that the signals satisfy eq (67). The possible states at $t=0$ are $(3\pi/2, +1, -1)$, $(\pi/2, -1, +1)$, $(3\pi/2, -1, -1)$ and $(\pi/2, +1, +1)$. Since the nonsymmetric signals satisfy eq (67) this means that the signals must be either in some of the states $(3\pi/2, +1, -1)$ and $(\pi/2, -1, +1)$ or in some of the other two states.

Now assume the signals are in some of the states $(3\pi/2, +1, -1)$ and $(\pi/2, -1, +1)$ at $t=0$. The possible transitions into state $(3\pi/2, +1, -1)$ are

$$\begin{aligned} (\pi, -1, +1) &\xrightarrow{+1} (3\pi/2, +1, -1) \\ (0, -1, -1) &\xrightarrow{+1} (3\pi/2, +1, -1) \end{aligned} \quad (75)$$

and into state $(\pi/2, -1, +1)$ are

$$\begin{aligned} (\pi, +1, -1) &\xrightarrow{-1} (\pi/2, -1, +1) \\ (0, +1, +1) &\xrightarrow{-1} (\pi/2, -1, +1) \end{aligned} \quad (76)$$

It is easily checked that the first transitions in (75) and (76) respectively give the same signal in $-\Gamma \leq t \leq 0$ and also the second transitions in (75) and (76) give the same signal. The possible transitions out of state $(3\pi/2, +1, -1)$ are

$$\begin{aligned} (3\pi/2, +1, -1) &\xrightarrow{+1} (\pi, +1, +1) \\ (3\pi/2, +1, -1) &\xrightarrow{-1} (\pi, -1, +1) \end{aligned} \quad (77)$$

and out of $(\pi/2, -1, +1)$ are

$$\begin{aligned} (\pi/2, -1, +1) &\xrightarrow{-1} (\pi, -1, -1) \\ (\pi/2, -1, +1) &\xrightarrow{+1} (\pi, +1, -1) \end{aligned} \quad (78)$$

Also here it is easily checked that the first transitions in (77) and (78) respectively give the same signal in $0 \leq t \leq \Gamma$ and also the second transitions in (77) and (78) give the same signals.

Now it is easily checked that for every pair of nonsymmetric signals satisfying (67) it is possible to find two symmetric signals $\tilde{s}(t, \beta_1^+)$ and $\tilde{s}(t, \beta_j^+)$ such that

$$\begin{aligned} \tilde{s}(t, \beta_1^+) &= \tilde{s}(t, \alpha_1^+) \\ \tilde{s}(t, \beta_j^+) &= \tilde{s}(t, \alpha_j^+) \end{aligned} \quad t \leq 0 \quad (79)$$

and

$$\begin{aligned}\tilde{s}(t, \underline{\beta}_{-1}^+) &= \tilde{s}(t, \underline{\alpha}_j^+) \\ t &\geq 0 \\ \tilde{s}(t, \underline{\beta}_j^+) &= \tilde{s}(t, \underline{\alpha}_{-1}^+)\end{aligned}\tag{80}$$

and therefore

$$\tilde{s}(t, \underline{\beta}_{-1}^+) + \tilde{s}(t, \underline{\beta}_j^+) = \tilde{s}(t, \underline{\alpha}_{-1}^+) + \tilde{s}(t, \underline{\alpha}_j^+)\tag{81}$$

The above arguments can in the same way be verified for the states $(3\pi/2, -1, -1)$ and $(\pi/2, +1, +1)$ and therefore the sum of every pair of non-symmetric signals satisfying (67) can be shown to be equal to a sum of symmetric signals. Thus also when $L=3$ it is enough to consider the symmetric signal set.

Unfortunately this is not so when $L \geq 4$. We will give a counter example when $L=4$. The signals for this case are shown in figure 8 and some of the states are also given. Consider the nonsymmetric signals making the transitions

$$\begin{aligned}(\pi/2, +1, +1, -1) &\xrightarrow{+1} (0, +1, +1, +1) \\ (3\pi/2, +1, +1, +1) &\xrightarrow{-1} (0, -1, +1, +1)\end{aligned}\tag{82}$$

These two signals satisfy eq (67) in $0 \leq t \leq T$ and assume they are chosen to satisfy eq (67) also outside this interval. The main point for $L \leq 3$ is that it is possible to find two symmetric signals with the same sum as the sum of the two nonsymmetric signals. These symmetric signals must be chosen to follow the transitions

$$\begin{aligned}(\pi/2, +1, +1, -1) &\xrightarrow{-1} (0, -1, +1, +1) \\ (3\pi/2, +1, +1, +1) &\xrightarrow{+1} (0, +1, +1, +1)\end{aligned}\tag{83}$$

otherwise they will not be symmetric in $0 \leq t \leq T$. Now it is easily checked that the two sums are not equal in $0 \leq t \leq T$ and thus not equal. The reason why this does not hold for $L \geq 4$ is the correlative states. When $L=4$ the correlative state consists of the prior 3 symbols. But the eye-opening is only 2 symbols wide and then also the nonsymmetric signals are needed to represent the signals.

Numerical, however it has been concluded that the sum of two nonsymmetrical signals satisfying eq (67) are very close to a properly chosen linear combination of symmetric signals.

We are now presenting an algorithm for calculating the asymptotically optimum filter. The main idea behind the algorithm is the observation that often only a small number of signals contribute to the filter. The algorithm first checks if one signal is enough to give the optimum filter. If not, the algorithm considers all pairs of symmetric signals and if the optimum filter is not found the algorithm considers all 3-tuples of symmetric signals, and so on until the filter is found. It is known from (65) that all the active vectors are giving the minimum distance for the optimum filter. Therefore, for every n-tuple the coefficients are chosen so that the signals contributing to the filter give the same distance. To find these coefficients n-1 linear equations must be solved. The last coefficient is one minus the other n-1 coefficients. Therefore only n-1 equations are needed. Since for a given filter

$$\underline{a} = \sum_{i \in \text{the } n\text{-tuple}} a_i \underline{s}_i^+,$$

$|\underline{a}|$ is constant independent of $\underline{s}_j^+$, from (64) these equations are

$$\langle \underline{s}_j^+, \sum_{i \in \text{the } n\text{-tuple}} a_i \underline{s}_i^+ \rangle = \langle \underline{s}_k^+, \sum_{i \in \text{the } n\text{-tuple}} a_i \underline{s}_i^+ \rangle \quad (84)$$

for all
j ∈ the n-tuple
but j ≠ k

where k is one signal in the n-tuple and

$$a_k = 1 - \sum_{\substack{i \in \text{the } n\text{-tuple} \\ i \neq k}} a_i \quad (85)$$

Rewriting (84) gives

$$\sum_{i \in \text{the } n\text{-tuple}} a_i s_{ij} = \sum_{i \in \text{the } n\text{-tuple}} a_i s_{ik} \quad (86)$$

for all
j ∈ the n-tuple
but j ≠ k

where

$$s_{ij} \doteq \langle \underline{s}_i^+, \underline{s}_j^+ \rangle \quad (87)$$

is the correlation between signal $\underline{s}_i^+$ and $\underline{s}_j^+$. It is known that at least one of the combinations giving the optimum filter is a combination having non-negative coefficient, therefore if at least one a_i is negative the next n -tuple is considered, otherwise this n -tuple is continued. Then the minimum distance for all the symmetric signals are calculated. The n -tuple giving the largest minimum distance is found and if this distance is given by the signals contributing to the filter we know from (65) that the optimum filter is found otherwise the filter can be improved by letting at least one more signal contribute to the filter. In this way the algorithm proceeds until the optimum filter is found. If however $L \geq 4$ this filter is optimum only for the symmetric signal set as shown before. Therefore if $L \geq 4$ the distances for the whole set S^+ are calculated and if the minimum distance for this set is the same as the minimum distance of the symmetric set the filter is optimum also in the set S^+ of signals. Otherwise some nonsymmetric signals have to be used to improve the filter. Then the algorithm just gives a suboptimum filter. As seen in the next section, this does not happen very often.

Finally the average error probability is calculated for the optimum filter and therefore all the distances for the signals in S^+ are calculated also when $L \leq 3$. A block diagram of the algorithm is given in figure 10. All the results presented in the next section are calculated with the above algorithm. For all of the cases at most 4 symmetric signals have contributed to the filter and therefore the algorithm works very fast. If many signals had contributed to the filter this algorithm should be impractical. Schemes with as many as 2048 signals, where 64 are symmetric, have been solved with the algorithm and the optimum filter was found. In this case 4 signals contributed to the filter. In this case there are 2016 pairs, 41664 3-tuples and 635376 4-tuples. This is of course quite many but the computation goes fast.

We have included a possibility to work with an even smaller signal set in the algorithm. It was noted that the signals contributing to the filter for an observation length of N_T symbol intervals, mostly were the same as the signals giving the filter at observation length N_T-1 , but extended to length N_T . Therefore we can choose to work with the whole symmetric set or just use the extensions of the signals giving the optimum filter of length N_T-1 . This set is mostly very small, i.e. the number of

signals is normally between 4 and 16. The problem however is to find these extensions. Of course when this reduced set is used the minimum distance for the symmetric set or if L is greater than three, the minimum distance for the whole set S^+ have to be calculated and compared with the minimum distance in the reduced set, to check if the filter is optimum also for the set S^+ .

Intermediate SNR's on the Gaussian channel

For intermediate values of SNR (E_b/N_0) no analytic solution to the optimum filter has been found. This is a difficult minimization problem, where a sum of Q-functions, eq (38), has to be minimized. The arguments of the Q-function depend on the coefficients a_i , which have to be chosen to minimize P . Of course it is possible to find the derivatives of P on a_i , and set these equations equal to zero. This will however result in a quite large number of nonlinear equations, and seems hard to solve analytically. Thus the minimizations are done numerically with a steepest descent algorithm [25]. As shown before only the symmetric signals contribute to the filter if L is less than four and this is used to reduce the dimensionality of the minimization. Minimizations have also been done for some schemes with L equal to four and in this case the filter obtained is optimum in the symmetric set only. As mentioned before however, it has been noted that the nonsymmetric pairs of signals can be approximated with a very small error, by linear combinations of symmetric signals. Therefore this suboptimum filter seems very close to the optimum filter. To find the overall optimum filter seems very hard since the number of signals is large.

Optimum filters on fading channels

Now the optimum filter for the MSK-type receiver on the fading channel will be considered. The average error probability for this channel is given in eq (48) for maximal ratio combining and in eq (51) for selection combining. The error probability without diversity is obtained by letting $M=1$ in (48) or (51). In the low SNR case, i.e. $\Gamma \rightarrow 0$ the average error probability is approximated by

$$P \approx \sum_{j=1}^m \frac{1}{m} \cdot \frac{1}{2} \left(1 - \sqrt{\frac{d_j^2 \Gamma}{2}} \cdot K \right) \quad \Gamma \rightarrow 0 \quad (88)$$

where

$$K = K_{MR} = 1 + \frac{1}{1! \cdot 2} + \frac{1 \cdot 3}{2! 2^2} + \dots + \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2M-1)}{(M-1)! 2^{(M-1)}} \quad (89)$$

if maximal ratio combining is used or

$$K = K_S = \sum_{k=1}^M \frac{(-1)^k \binom{M}{k}}{\sqrt{k}} \quad (90)$$

if selection combining is used, i.e. K is independent of a_i , $i = 1, \dots, m/2$.

Now (88) has to be minimized. Rewriting (88) gives

$$P = \frac{1}{2} - K \sqrt{\frac{\Gamma}{2}} \sum_{j=1}^{m/2} \frac{1}{m} d_j \quad \Gamma \rightarrow 0 \quad (91)$$

where the symmetry of the signals in S^+ and S^- has been used. From (91) it is seen that minimizing P is equivalent to maximizing $\sum_{j=1}^{m/2} d_j$ and from (55) to (62) it is known that the solution to this is

$$a_i = \frac{2}{m} \quad i = 1, \dots, m/2 \quad (92)$$

i.e. the AMF-filter. Therefore the same filter as on the Gaussian channel is optimum for low SNR's.

For high SNR's on the other hand, the following approximation can be used

$$P \approx \sum_{j=1}^{m/2} \frac{1}{m} \cdot \frac{1}{d_j^2 \Gamma} \quad \Gamma \rightarrow \infty \quad (93)$$

From (93) it is seen that

$$\sum_{j=1}^{m/2} \frac{1}{m} \frac{1}{d_j^2} \quad (94)$$

has to be minimized. But this sum is given by

$$\sum_{j=1}^{m/2} \frac{1}{m} \frac{1}{d_j^2} = \sum_{j=1}^{m/2} \frac{1}{m} \cdot \frac{\left| \sum_{i=1}^{m/2} a_i s_i^+ \right|^2}{\left[\sum_{i=1}^{m/2} a_i s_{ij} \right]^2} \cdot \frac{\Gamma}{2} \quad (95)$$

where s_{ij} is the correlation given in (87). The derivatives on a_i , $i = 1, \dots, m/2$ of this sum can of course be calculated and set to zero.

A nonlinear system of equations will result and these equations seem hard to solve analytically. Also for intermediate Γ an analytical solution to the optimum filter problem is hard to find. Thus we are referred to numerical minimizations, when Γ is not approaching zero on the fading channel.

IV. NUMERICAL RESULTS

In this section a variety of numerical results are given. More results are given in [7]. The impulse response of the optimum filter is given in figures, as well as the error probability for the different receivers. Different transmitted schemes are compared according to both the detection and the spectral efficiency. The receivers are also compared with the optimum Viterbi receivers [2].

The nonfading Gaussian channel

Here the results for the nonfading Gaussian channel are given. The schemes are divided into classes according to the main-lobe width of the power spectrum.

2REC

Some of these results for 2REC are also given in [9]-[12]. In this paper they are reproduced with another algorithm however, and will be used as comparison.

Figure 11 gives the impulse response of the filters optimum for $N_T=6$. Figure 12 shows the corresponding error probabilities. Note that the error probability for the filter optimum at an intermediate SNR value is given only for that specific SNR value (marked with x). The minimum distance for the asymptotically optimum receiver and the Viterbi receiver are summarized in table 1. For $N_T=3$ to $N_T=5$ two signals contribute to the asymptotically optimum filter, for $N_T=6$ and $N_T=7$ three signals contribute, while when $N_T=8$ four signals contribute to the asymptotically optimum filter. From figure 12 it is seen that the asymptotically optimum filter is close to optimum for most SNR values. The loss at large SNR's compared to the Viterbi receiver is as small as 0.27dB.

	Asymptotic optimum							Viterbi
	$N_T=2$	$N_T=3$	$N_T=4$	$N_T=5$	$N_T=6$	$N_T=7$	$N_T=8$	
2REC	1.3634	1.5331	1.5889	1.6029	1.6126	1.6203	1.6211	1.7268

Table 1. Minimum distance for various receivers to 2REC.

2RC-type schemes

No error probabilities or filters will be given for these schemes, since the error probabilities are very close to MSK [7]. However the minimum distances are given in table 2.

	Asymptotic optimum							Viterbi
	$N_T=2$	$N_T=3$	$N_T=4$	$N_T=5$	$N_T=6$	$N_T=7$	$N_T=8$	
2RC	1.9320	1.9631	1.9650	1.9653	1.9654	1.9654	1.9654	1.9666
GMSK2 $B_b T=0.7$	1.9541	1.9757	1.9766	1.9767	1.9767	1.9768	1.9768	1.9773
GMSK3 $B_b T=0.5$	1.8784	1.9304	1.9357	1.9364	1.9367	1.9368	1.9368	1.9405

Table 2. Minimum distance for various receivers to schemes of 2RC-type.

From figure 1 it is known that GMSK2 with $B_b T=0.7$ has a main-lobe of about the same order as 2RC, while the side-lobes are slightly larger. GMSK3 with $B_b T=0.5$ on the other hand has slightly smaller side-lobes. For all these schemes the asymptotic loss compared to the Viterbi receiver and also compared to MSK is small. The asymptotic loss compared to MSK is less than 0.05dB, and even smaller for intermediate SNR's.

For the above schemes with $N_T=8$ only four out of 32 symmetric signals contribute to the asymptotically optimum filter.

3RC-type schemes

Now schemes with a power spectral density main-lobe of the same order as 3RC are considered. Figure 13 shows the optimum filters for $N_T=6$ and figure 14 shows the asymptotically optimum filter for $N_T=9$ for 3RC. Figure 17 shows the corresponding error probabilities. From figure 17 it is seen that also for 3RC, the asymptotically optimum filter is close to optimum also for intermediate SNR's. The minimum distances are given in table 3. The asymptotic loss for $N_T=9$ compared to the Viterbi receiver is about

0.16dB while the loss compared to MSK is 0.70dB. The loss at 10^{-4} compared to MSK is however only about 0.3dB for the asymptotically optimum receiver. The number of signals contributing to the asymptotically optimum filter with $N_T=9$ is 4 out of 64 symmetric signals.

From figure 2 it is known that GMSK4 with $B_b T=0.25$ has lower side-lobes than 3RC, and a main-lobe of the same order as 3RC. Figure 15 gives the asymptotically optimum filter with $N_T=9$ for this scheme. The corresponding error probability is given in figure 17. The asymptotic loss compared to the Viterbi receiver is about 0.26dB while the loss compared to MSK is about 1.0dB. At 10^{-4} the loss compared to MSK is about 0.5dB. The minimum distances are given in table 3. Thus the detection properties of GMSK4 are slightly worse than for 3RC, but the spectral properties are better.

	Asymptotic optimum								Viterbi
	$N_T=2$	$N_T=3$	$N_T=4$	$N_T=5$	$N_T=6$	$N_T=7$	$N_T=8$	$N_T=9$	
3RC	1.4886	1.6328	1.6841	1.6928	1.6967	1.7001	1.7004	1.7005	1.7647
GMSK4 $B_b T=0.25$	1.3185	1.4811	1.5653	1.5805	1.5854	1.5896	1.5899	1.5899	1.6890
3SRC6 $\beta=0.75$	1.6757	1.6989	1.7102	1.7119	1.7131	1.7135	----	----	1.8959

Table 3. Minimum distance for various receivers to 3RC-type schemes.

Another scheme of 3RC-type is 3SRC6 with various values of the roll-off factor β . From [7] it is known that $\beta=0.75$ gives the best asymptotic performance. Figure 16 shows the optimum filter for $N_T=7$ when $\beta=0.75$. The corresponding error probability is shown in figure 17. The minimum distances are given in table 3. With this scheme the asymptotic loss compared to MSK is about 0.67dB, i.e. a small gain compared to 3RC. Note also the similarity of the asymptotically optimum filters for 3SRC6 with the asymptotically optimum filter for 3RC. The loss compared to MSK at 10^{-4} is about 0.20dB, which is also a gain compared to 3RC. Thus this scheme performs better than 3RC both spectrally and according to detection performance.

TFM-type schemes

TFM5 and GMSK5 with $B_b T = 0.20$ both have spectra with about the same main-lobe, while GMSK5 has lower side-lobes. The optimum filters for TFM5 have not been found, but the filters optimum in the symmetric signal set are found and they are very close to optimum also for the whole set of signals. For $N_T = 6$ the minimum distance in the symmetric set, for the filter optimum in this set, is 1.3131 while the minimum distance in the whole set for the same filter is 1.3079. The difference is of the same order for $N_T = 7$ and $N_T = 8$. Therefore the loss compared to the overall optimum filter is small. Figure 18 shows the filter optimum in the symmetric set and figure 20 shows the corresponding error probability when $N_T = 8$. The minimum distances are given in table 4. The asymptotic loss for TFM5 compared to the optimum Viterbi receiver is about 0.82dB, while the asymptotic loss compared to MSK is about 1.82dB. The loss at 10^{-4} compared to MSK is however only 1.1dB. Figure 20 gives the error probability for TFM5 with a SPAM filter of length $N_T = 6$ as comparison. The SPAM filter is based on a PAM approximation of the eye pattern [3].

	Asymptotic optimum							Viterbi
	$N_T = 2$	$N_T = 3$	$N_T = 4$	$N_T = 5$	$N_T = 6$	$N_T = 7$	$N_T = 8$	
TFM5	1.0239	1.1564	1.2751	1.3016	1.3079	1.3126	1.3144	1.5865
GMSK5 $B_b T = 0.20$	0.9353	1.1153	1.2541	1.2905	1.2994	1.3089	1.3113	1.5331

Table 4. Minimum distance for various receivers to TFM-type schemes.

The filters for TFM5 are optimum in the symmetric set only.

The results for GMSK5 with $B_b T = 0.20$ are given in figure 19 and 20. The filter given in figure 19 is optimum in the whole signal set. The minimum distances are given in table 4. In figure 20 the error probability for an AMF receiver with $N_T = 5$ to GMSK5 with $B_b T = 0.20$ is given as comparison. The asymptotic loss compared to a Viterbi receiver is about 0.68dB, and the asymptotic loss compared to MSK is about 1.83dB. The loss at 10^{-4} compared to MSK is about 1.1dB. Thus the above two schemes have about the same error probability performance. For $N_T = 8$ only 3 signals out of 64 symmetric signals contributed to the filters for both schemes.

4RC-type schemes

From figure 4 it is known that 4RC, 4SRC8 with β -values close to 1.0 and GMSK5 with $B_b T = 0.18$ have spectra of the same order. These schemes will now be considered.

Figure 21 shows the optimum filters for 4RC when $N_T = 7$ and figure 22 shows the asymptotically optimum filter when $N_T = 9$. Note that the filters optimum at intermediate values of SNR are optimum only in the symmetric signal set. The corresponding error probability is given in figure 24. It is seen that the asymptotically optimum filter gives an error probability very close to the filters optimum at an intermediate value of SNR, thus this filter is close to optimum for all practical SNR values. The minimum distances are given in table 5. The asymptotic loss compared to a Viterbi receiver is about 0.91dB while the asymptotic loss compared to MSK is about 2.12dB. However the loss at 10^{-4} compared to MSK is about 1.4dB.

	Asymptotic optimum								Viterbi
	$N_T = 2$	$N_T = 3$	$N_T = 4$	$N_T = 5$	$N_T = 6$	$N_T = 7$	$N_T = 8$	$N_T = 9$	
4RC	0.8779	1.0277	1.1724	1.2055	1.2140	1.2239	1.2269	1.2272	1.5135
GMSK5 $B_b T = 0.18$	0.7342	0.9084	1.0746	1.1140	1.1251	1.1379	1.1427	----	1.4523

Table 5. The minimum distance for various receivers to 4RC-type schemes.

The asymptotically optimum filter for GMSK5 with $B_b T = 0.18$ is given in figure 23, and the corresponding error probability is given in figure 24 when $N_T = 8$. As comparison the error probability for an AMF receiver with $N_T = 5$ are shown. The minimum distances are given in table 5. The asymptotic loss compared to the Viterbi receiver is about 1.04dB, while the asymptotic loss compared to MSK is about 2.43dB. The loss compared to MSK at 10^{-4} is however only about 1.7dB.

The fading channel

Some results are given for the Rayleigh fading channel. Numerical optimizations are done for the average SNR equal to 0, 5, 10 and 15dB. Figure 25 shows the filters for 3RC with observation length $N_T=6$ intervals. Figure 28 gives the corresponding error probabilities and it is seen that a small gain is obtained compared to AMF.

The optimum filters for 3RC with a two branch maximal ratio combiner are shown in figure 26, and with a two branch selection combiner in figure 27. In both cases the observation length is $N_T=5$. The corresponding error probabilities are also shown in figure 28. The gain compared to the AMF receiver is small but the performance is close to the MSK performance.

Figure 29 gives the filters for 4RC with $N_T=5$. Note that, since this scheme has $L=4$, these filters are optimum only in the symmetric signal set. A large improvement by using also nonsymmetric signals are however not expected. Figure 32 show the corresponding error probabilities. A gain of about 0.5dB relative to AMF is obtained at 10^{-2} . The loss at 10^{-2} compared to MSK is of about the same order. Figure 32 also shows the error probability when the asymptotically optimum filter for the nonfading channel is used on the fading channel. It can be concluded that this filter is very close to the optimum filters. Thus for practical reasons this filter can be used for most SNR's also on the fading channel.

Figure 30 and 31 show the optimum filters for 4RC with $N_T=5$ for maximal ratio combining and selection combining respectively. The corresponding error probabilities are given in figure 32. These filters are also optimum only in the symmetric signal set. In figure 32 the error probability obtained with the asymptotically optimum filter on the nonfading channel used on the fading channel with two branch diversity is also shown as comparison. It is seen that this filter is very close to optimum also on the fading channel, at least for average signal-to-noise ratios larger than about 5dB. Thus in practice the asymptotically optimum filter on the nonfading channel can be used also for lower signal-to-noise ratios.

V. DISCUSSION AND CONCLUSIONS

A parallel MSK-type receiver for CPM signals is considered in this part. This is a receiver for binary modulation schemes with modulation index $h=1/2$. A distance measure is derived and used in analysing the performance of the schemes. Coherent transmission on the Gaussian channel and the Rayleigh fading channel is considered. The minimum distance gives the asymptotic error probability behaviour on the Gaussian channel. An exact expression for the error probability in detecting the phase node is given for both the Gaussian and the fading channel.

The optimum receiver filters are calculated for a variety of schemes both on the nonfading and fading channel. On the fading channel also diversity schemes are considered, both with a maximal ratio and a selection combiner. The optimum filter depends on the signal-to-noise (SNR) ratio on the channel. The optimum filter for very small SNR's is derived for both type of channels. An algorithm giving the asymptotically (high SNR's) optimum filter on the Gaussian channel is presented as well as a numerical minimization procedure for intermediate SNR's. This numerical minimization however only gives an optimum filter in the symmetric signal set if the pulse length L is greater than 3. Comparisons are given between different transmitted schemes, i.e. REC, RC, SRC, TFM and GMSK.

It is shown that the loss for large SNR's compared to the optimum Viterbi receiver is small for schemes based on a frequency pulse with a mainlobe of length less than or equal to about 4 symbol intervals. Figure 33 summarizes the asymptotic behaviour for the different receivers for the RC schemes. As comparison the behaviour for the AMF receiver and the SPAM receiver is shown. The SPAM receiver is based on a PAM approximation of the decision eye [3]. Also shown is the performance for 3SRC6 with $\beta=0.75$ and TFM5, both with an optimum Viterbi receiver and with an optimum MSK-type receiver. All the receiver filters for the MSK-type receiver are time limited. In figure 33 the asymptotic performance for an optimum Viterbi receiver to 3SRC6 and 4SRC8 both with $\beta=0.0$ are also shown [7]. These are two very good schemes both spectrally and detectionably, but they are too complicated to be detected in an MSK-type receiver.

These optimum MSK-type receivers have a performance slightly better than the WAMF receiver considered in [24]. The WAMF filter is based on a minimum squared error approach. The filter consists of a weighted AMF filter, where the trajectories giving the inner boundary of the eye have weight $1+\lambda$ and all the others have weight 1. The coefficient λ is then chosen to

minimize the error probability. It is surprising that the WAMF receiver performs as close to optimum. This receiver has however only been applied to TFM5 but for this scheme the loss compared to the optimum for most SNR's is only a fraction of a dB. However it is noted from the results in this paper, that as long as the filter do not differ too much from the optimum filter the performance is very close to optimum. For the schemes considered in this paper for example, the asymptotically optimum filter on the Gaussian channel can be used for both types of channels and all SNR's greater than around 5dB with a performance close to optimum. Thus for practical reasons the same filter can be used independent of SNR.

Another MSK-type receiver for TFM is given in [14]. In this paper the TFM eye pattern is approximated with a 4RC PAM eye and the optimum filter for this PAM system is used in the receiver. The results in [14] show that the loss for this receiver is about 1dB. This type of filter has also been investigated in [3]. In this paper it is shown that the minimum distance is 1.02 and thus the asymptotic loss is about 2.9dB. This type of filter is quite good for error probabilities greater than about 10^{-4} , but is not as good asymptotically.

In [19] an MSK-type receiver to a GMSK scheme is considered. This receiver uses a predetection Gaussian bandpass filter with a normalized 3dB-down bandwidth of 0.63. It was found through practical experiments that this bandwidth was close to optimum. From practical experiments the loss compared to ideal MSK is found to be around 1.7dB at 10^{-3} for $B_bT=0.25$ and around 3.0dB for $B_bT=0.20$. The paper also contains some measurement on fast Rayleigh fading for various maximum Doppler frequencies.

The MSK-type receiver has also been studied in [9]-[12]. In these papers another algorithm giving the asymptotically optimum filter is given. The algorithm has however only been applied to schemes with piecewise constant frequency responses. These schemes are not very attractive according to spectral efficiency.

A modulation scheme called Frequency Shift Offset Quadrature modulation (FSOQ) is introduced in [15]. This is a modulation scheme of OQPSK type, but the pulse shape of the two modulation bit streams is varied, depen-

dent on the data bits. Since it is an OQPSK type of modulation, an MSK-type receiver can be used. It is claimed that the average bit error probability is minimized by optimizing the filter for the pulse shape of least energy and this filter is used in [15]. This idea has also been investigated in [3]. However, in this paper it is shown that this filter is not optimum. For most schemes this filter is optimum for observation length $N_T=2$, but a longer observation length improves the performance. Error probability results are presented in the paper and the loss compared to MSK at 10^{-3} is around 0.8dB, but the asymptotic degradation is around 1.7dB. The spectral properties are however better than for MSK. The performance of the FSOQ scheme has also been simulated for a nonlinear satellite channel and the results are given in [15].

In this part correct carrier recovery and timing are assumed. This is however a difficult task to obtain. The next problem to be solved for the above MSK-type receiver, is the synchronization procedure. The performance will naturally depend highly on the synchronization properties.

The general problem of jointly optimizing the choice of transmitter pulses and receiver filters for the MSK-type of receiver remains unsolved. If the joint optimization of transmitter pulse shape and receiver filter is to be solved the problem also must be formulated in terms of bandwidth.

APPENDIX

The use of differential encoding

In this appendix the use of differential encoding of the data sequence α_n , is applied to the problem of estimating the data symbol α_n directly, instead of the phase node ϕ_n [22].

First we consider the modulation scheme using a 1REC, $h=1/2$ transmitter and show that in this case a differential encoder solves the problem.

By considering the intersymbol interference in the partial response modulation scheme, using a Raised Cosine pulse (RC), we show that a differential encoder solves the problem also here.

The receiver uses $\tilde{s}(t, \underline{\alpha}) = \cos\varphi(t, \underline{\alpha})$ to make a decision of every second symbol. The other symbols are detected from $\sin\varphi(t, \underline{\alpha})$, but this is just a shifted version of $\cos\varphi(t, \underline{\alpha})$. Therefore only $\cos\varphi(t, \underline{\alpha})$ is considered. For the 1REC, $h=1/2$ modulation scheme, which is also known as a Minimum Shift Keying system (MSK), $q(t)$ is

$$q(t) = \begin{cases} 0 & ; \quad t < 0 \\ \frac{t}{2T} & ; \quad 0 \leq t \leq T \\ \frac{1}{2} & ; \quad t \geq T \end{cases} \quad (A1)$$

Therefore the phase at time $t = (n+1)T$ is ($h=1/2$)

$$\varphi((n+1)T, \underline{\alpha}_n) = \pi \sum_{i=-\infty}^n \alpha_i \cdot q((n+1-i)T) = \frac{\pi}{2} \sum_{i=-\infty}^n \alpha_i \quad (A2)$$

Using the fact that $\alpha_i = \pm 1$ and taking the phase response modulo 2π gives

$$\varphi((n+1)T, \underline{\alpha}_n) = \begin{cases} 0 \text{ modulo } \pi & ; \quad n \text{ odd} \\ \frac{\pi}{2} \text{ modulo } \pi & ; \quad n \text{ even} \end{cases} \quad (A3)$$

Thus

$$\tilde{s}((n+1)T, \underline{\alpha}_n) = \cos\varphi((n+1)T, \underline{\alpha}_n) = \begin{cases} \pm 1 & ; \quad n \text{ odd} \\ 0 & ; \quad n \text{ even} \end{cases} \quad (A4)$$

Since the phase response $\varphi(t, \alpha)$ typically varies smoothly between the phase points at the $t=nT$, n integer values, then it follows that, in any of the MSK-type schemes, $\cos\varphi(t, \alpha)$ will be very similar to a Pulse Amplitude Modulated (PAM) signal with eye diagram with no Intersymbol Interference (ISI) at the sampling points.

To recover the data symbols α_n , let us use a differential encoder as shown in figure A1.

Now

$$\varphi((n+1)T, \alpha_n) = \frac{\pi}{2} \sum_{i=-\infty}^n \alpha'_i \quad (A5)$$

where α'_i is the encoded data symbols. Consider the phase difference

$$\Delta\varphi = \varphi((n+1)T, \alpha_n) - \varphi((n-1)T, \alpha_{n-2}) = \frac{\pi}{2} (\alpha'_n + \alpha'_{n-1}) \quad (A6)$$

Let

$$\alpha'_n = -\alpha_n \cdot \alpha'_{n-1} = \alpha_n \oplus \alpha'_{n-1} \quad (A7)$$

where $\oplus$ means modulo 2 addition. This gives

$$\Delta\varphi = \frac{\pi}{2} (-\alpha_n \alpha'_{n-1} + \alpha'_{n-1}) = \frac{\pi}{2} (1 - \alpha_n) \cdot \alpha'_{n-1} = \begin{cases} 0 & \text{if } \alpha_n = 1 \\ \pm\pi & \text{if } \alpha_n = -1 \end{cases} \quad (A8)$$

Thus the data symbol, α_n , can be recovered from the polarity changes of $\cos\varphi(t, \alpha_n)$. The differential coding circuit using (A7) is shown in figure A2.

For the system to be free from intersymbol interference at the sampling point, the frequency pulse must be such that

$$\int_{\left(\frac{L+1}{2} + n\right)T}^{\left(\frac{L+1}{2} + n-1\right)T} g(\tau) d\tau = \begin{cases} 1/2 & ; \quad n = 0 \\ 0 & ; \quad n \neq 0 \end{cases} \quad (A9)$$

This is not so for the LRC systems, with L greater than 1.

First consider the 2RC scheme, where the frequency pulse $g(t)$ is non-zero for $0 \leq t \leq 2T$. Since the frequency pulse is symmetric around $t=T$, while for the MSK system it is symmetric around $t = \frac{T}{2}$, we will have a delay of $\delta \cdot T$ where $\delta = \frac{1}{2}$, relative to the MSK system. Thus $t = (n+1+\delta)T$ must be considered instead of $t = (n+1)T$.

$$\begin{aligned} \varphi\left((n+1+\delta)T, \alpha'_{-n+[L/2]}\right) &= \frac{\pi}{2} \sum_{i=-\infty}^{n+[L/2]} \alpha'_i \cdot q((n+1+\delta-i)T) = \\ &= \frac{\pi}{2} \sum_{i=-\infty}^n \alpha'_i + \pi \cdot (c_{-1} \alpha'_{n+1} - c_1 \alpha'_n) \end{aligned} \quad (A10)$$

where

$$c_{-1} = \int_0^{1/2} g(\tau) d\tau \quad (A11)$$

and

$$c_1 = \int_{3/2}^2 g(\tau) d\tau \quad (A12)$$

and $[L/2]$ means the integer part of $L/2$, i.e. 1 in this case.

The first term is just the phase of the MSK system at time $t = (n+1)T$ and therefore

$$\varphi\left((n+1+\delta)T, \alpha'_{-n+[L/2]}\right) = \varphi_{\text{MSK}}\left((n+1)T, \alpha'_n\right) + \pi(c_{-1} \alpha'_{n+1} - c_1 \alpha'_n) \quad (A13)$$

The second term in (A13) can be interpreted as an intersymbol interference term. Quite clearly then, providing the intersymbol interference term is less than $\pi/2$, the polarity of $\cos\varphi(t, \alpha'_n)$ is maintained and the same differential encoding rule can be applied for 2RC as for MSK. This is the case however, if and only if

$$\max_{\alpha'_n, \alpha'_{n+1}} |c_{-1} \alpha'_{n+1} - c_1 \alpha'_n| < \frac{1}{2} \quad (A14)$$

This is equivalent to

$$|c_{-1}| + |c_1| < \frac{1}{2} \quad (A15)$$

and, subject to this constraint, we find that the eye diagram for $\cos\varphi(t, \underline{\alpha})$ has an open eye.

In the 2RC case

$$c_1 = c_{-1} \approx 0.0454 \quad (\text{A16})$$

and therefore

$$|c_{-1}| + |c_1| \approx 0.09 < 0.5 \quad (\text{A17})$$

Therefore the same differential encoder can be used to 2RC.

The analysis for 2RC holds also for 3RC, with the only difference that the delay is T , i.e. $\delta=1$, and

$$c_1 = \int_0^1 g(\tau) d\tau \quad (\text{A18})$$

and

$$c_{-1} = \int_2^3 g(\tau) d\tau \quad (\text{A19})$$

For 3RC, $c_{-1} = c_1 \approx 0.0978$ and therefore

$$|c_{-1}| + |c_1| < 0.20 < 0.5 \quad (\text{A20})$$

Thus we can use the encoder also for 3RC.

The general case with an arbitrary L yields the delay $\delta = \frac{L-1}{2}$ and the coefficients c_i

$$c_i = \begin{pmatrix} \frac{L+1}{2} + i \\ \frac{L+1}{2} + i-1 \end{pmatrix} T \int g(\tau) d\tau \quad i = 0, \pm 1, \pm 2, \pm [L/2] \quad (\text{A21})$$

where $[L/2]$ means the integer part of $L/2$.

Considering the phase response at time $t = (n+1+\delta)T$ gives

$$\begin{aligned} \varphi\left((n+1+\delta)T, \alpha_{n+[L/2]}\right) = \\ = \varphi_{\text{MSK}}\left((n+1)T, \alpha_n\right) + \pi \sum_{i=1}^{[L/2]} \sum_{j=i}^{[L/2]} (c_{-j} \alpha'_{n+i} - c_j \alpha'_{n+1-i}) \end{aligned} \quad (\text{A22})$$

To make sure that the interference term is less than $\pi/2$ we must require that

$$\sum_{i=1}^{[L/2]} \left(\left| \sum_{j=i}^{[L/2]} c_{-j} \right| + \left| \sum_{j=i}^{[L/2]} c_j \right| \right) < \frac{1}{2} \quad (\text{A23})$$

and for the systems yielding this we can use the differential encoder of figure A2 and the decision rule for MSK.

For the case with 4RC we have

$$\begin{aligned} c_1 = c_{-1} &\approx 0.1250 \\ c_2 = c_{-2} &\approx 0.0062 \end{aligned} \quad (\text{A24})$$

which gives

$$|c_{-1}+c_{-2}| + |c_1+c_2| + |c_{-2}| + |c_2| \approx 0.27 < 0.5 \quad (\text{A25})$$

The above analysis is done for RC schemes. It can however be applied to any type of pulses. Therefore the above differential encoding rule can be applied to all the schemes considered in this paper.

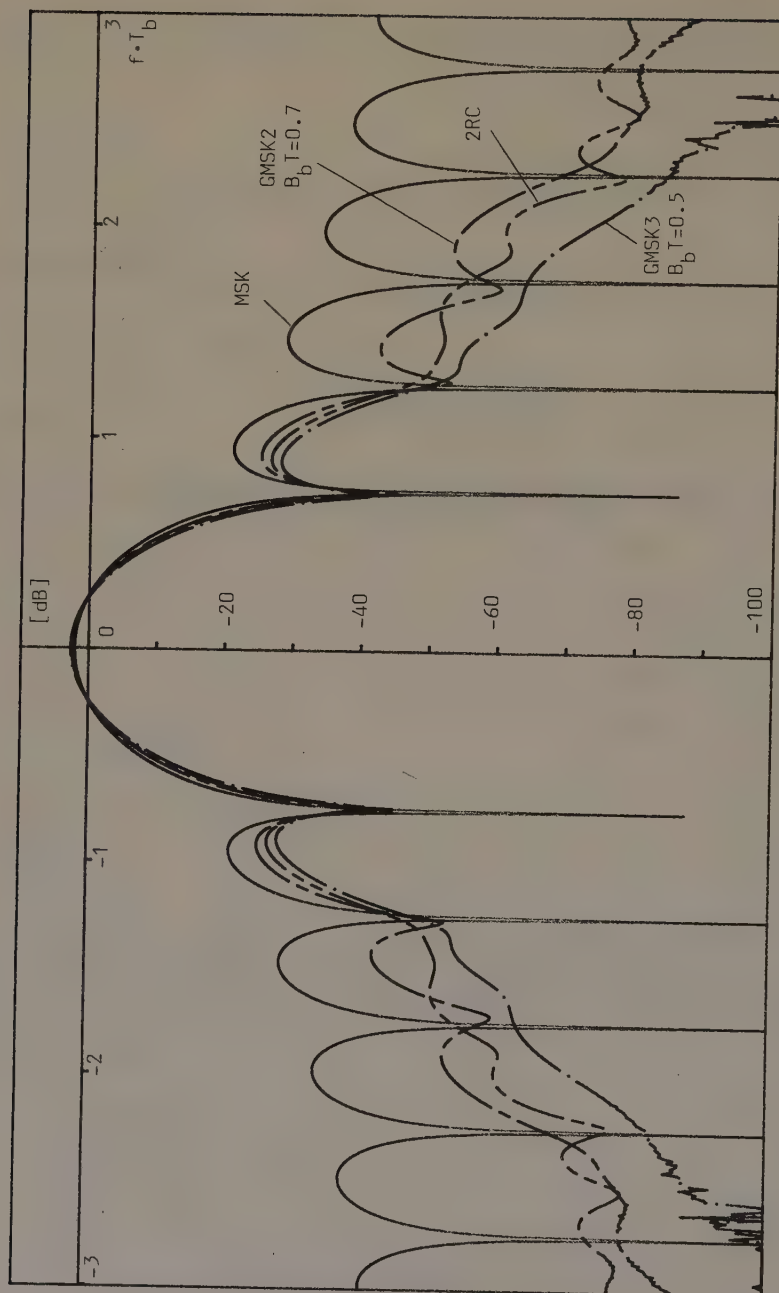


Figure 1. Power spectrum for CPM schemes with $h=1/2$.

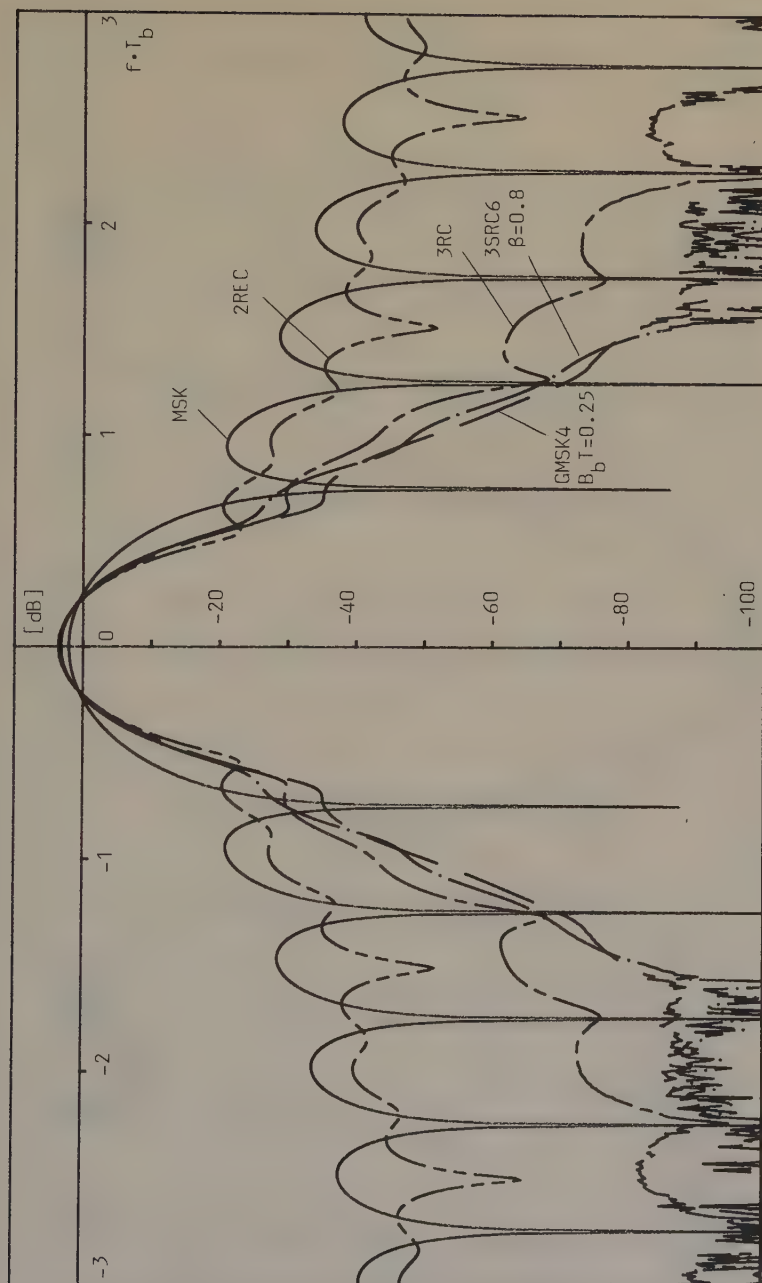


Figure 2. Power spectrum for CPM schemes with $h=1/2$.

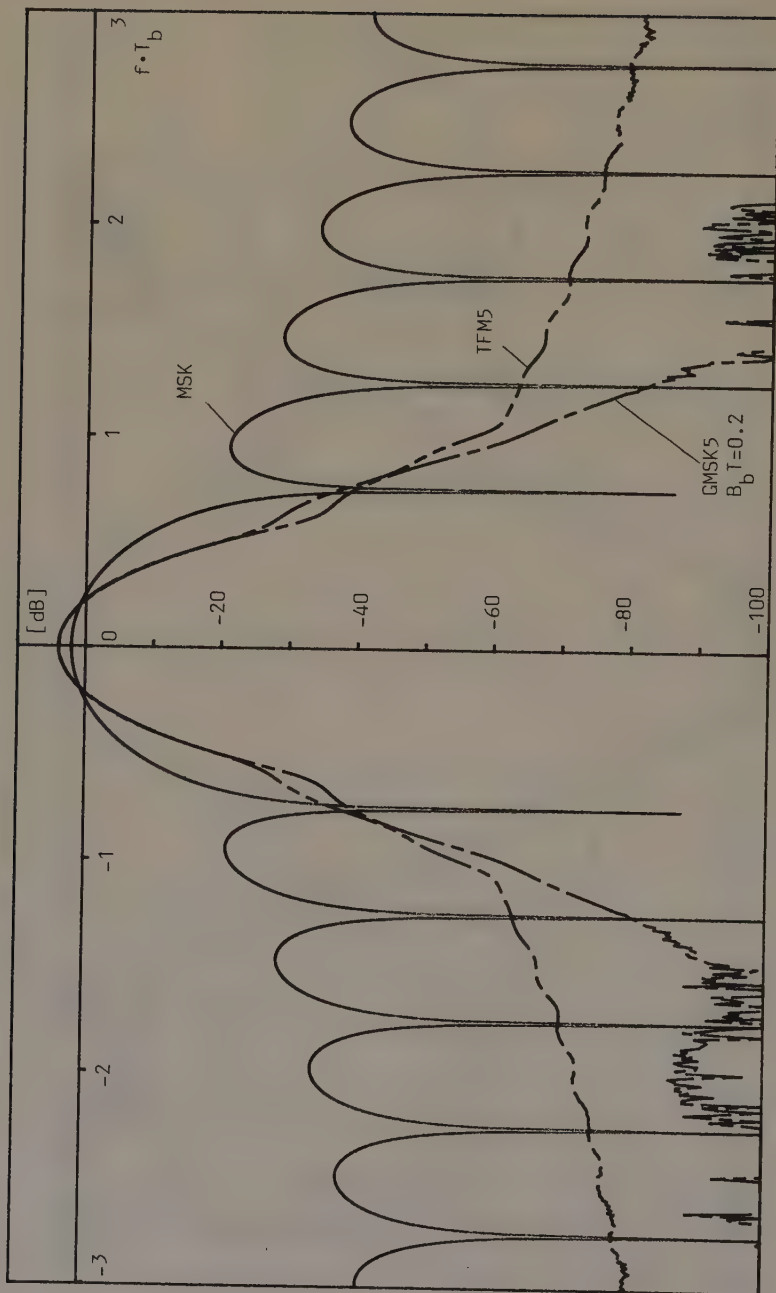


Figure 3. Power spectrum for CPM schemes with $h=1/2$.

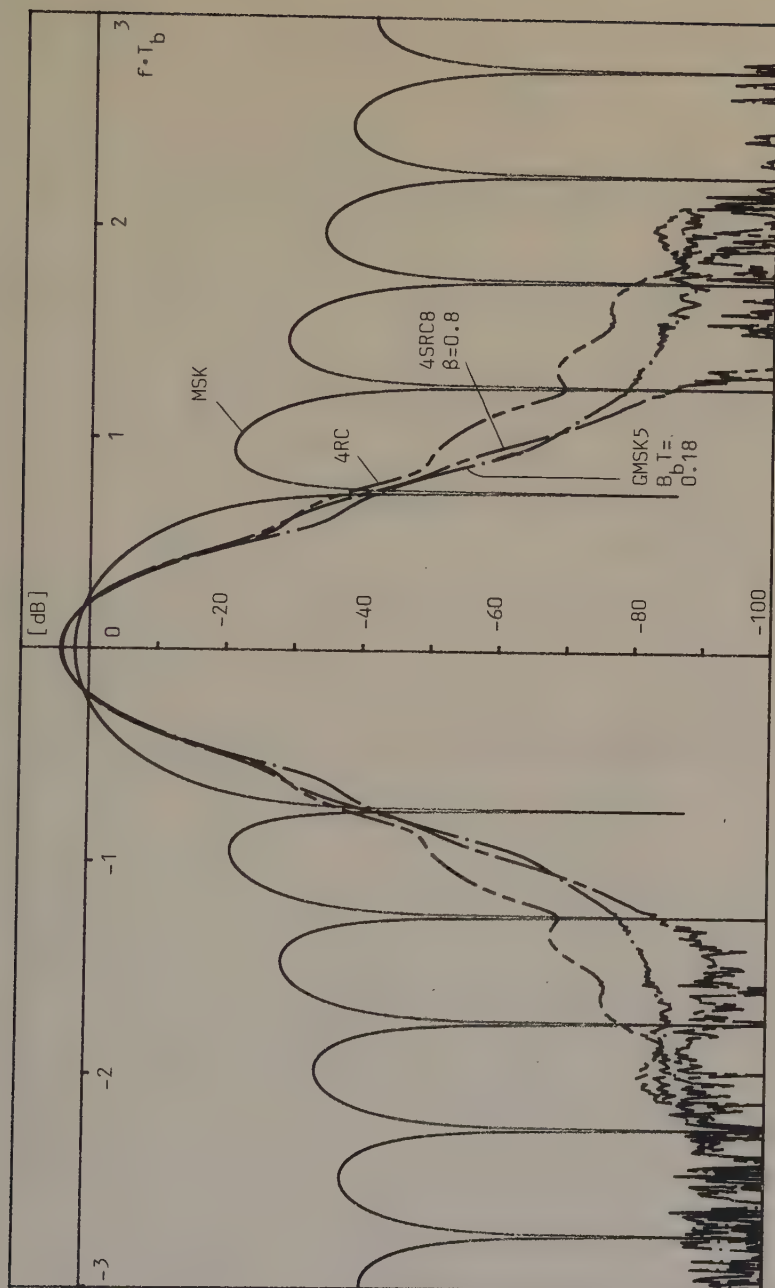


Figure 4. Power spectrum for CPM schemes with $h=1/2$.

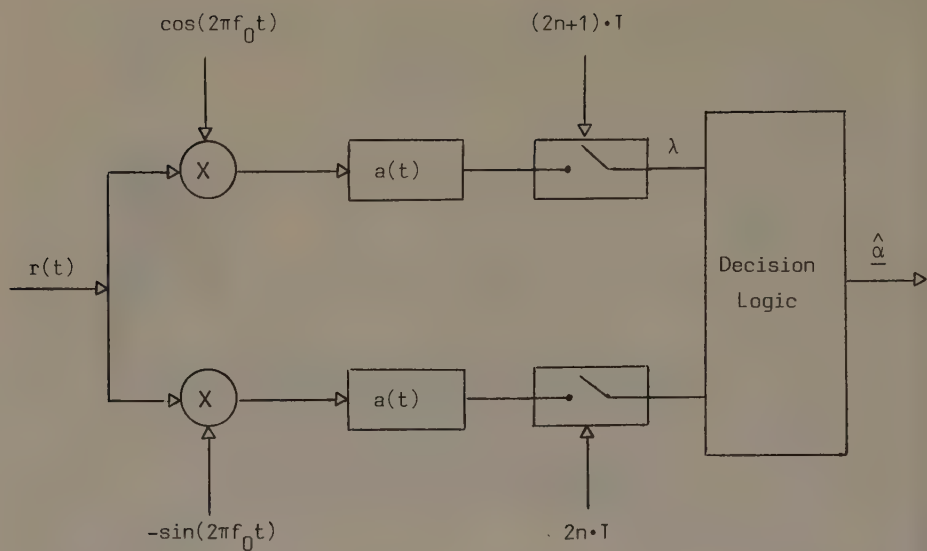


Figure 5. Receiver structure for the parallel MSK-type receiver for partial response CPM. $a(t)$ is the receiver filter.

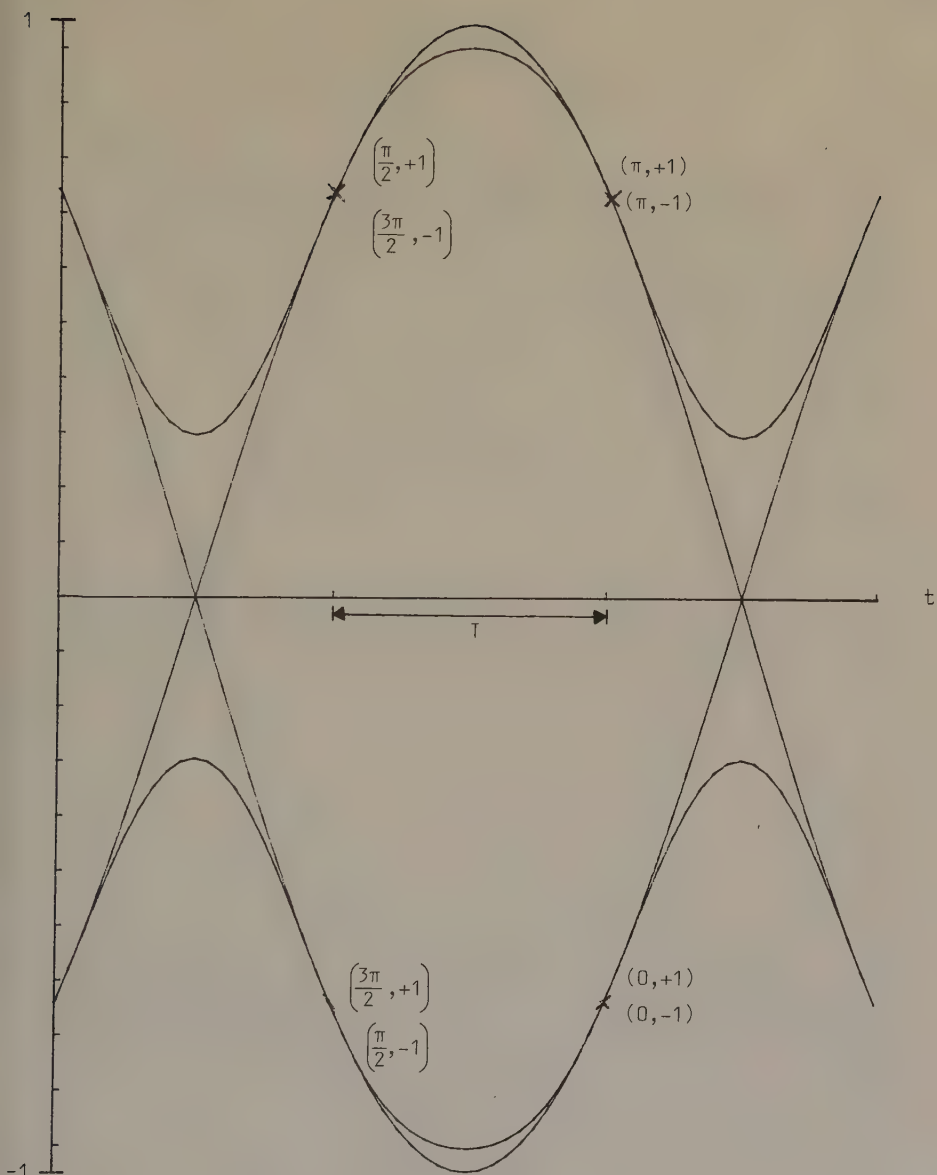


Figure 6. $\cos\phi(t, \underline{\alpha})$ eye pattern of a binary 2RC, $h=1/2$, scheme.
The states are assigned to the nodes.

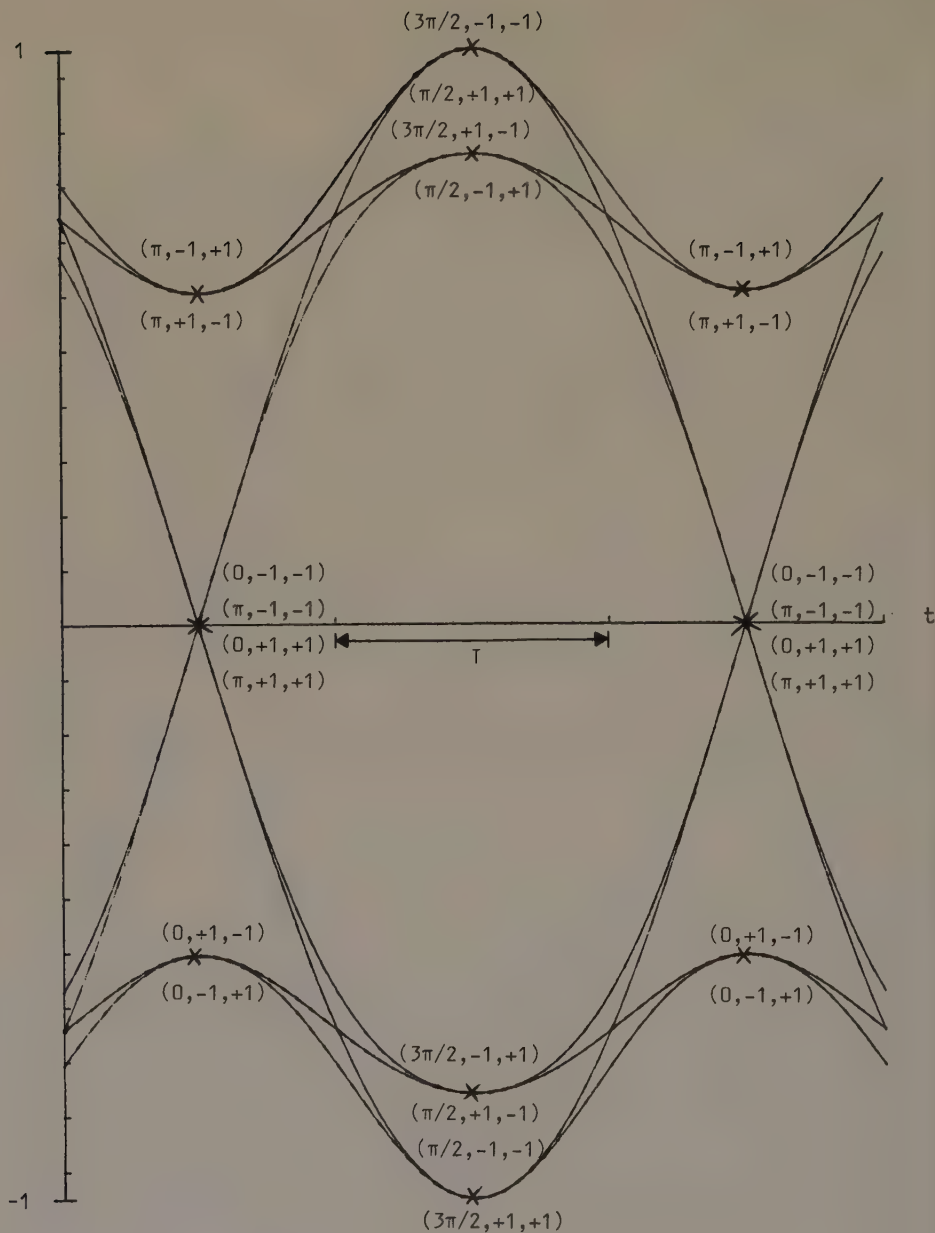


Figure 7. $\cos\varphi(t, \alpha)$ eye pattern of a binary 3RC, $h=1/2$, scheme.
The states are assigned to the nodes.

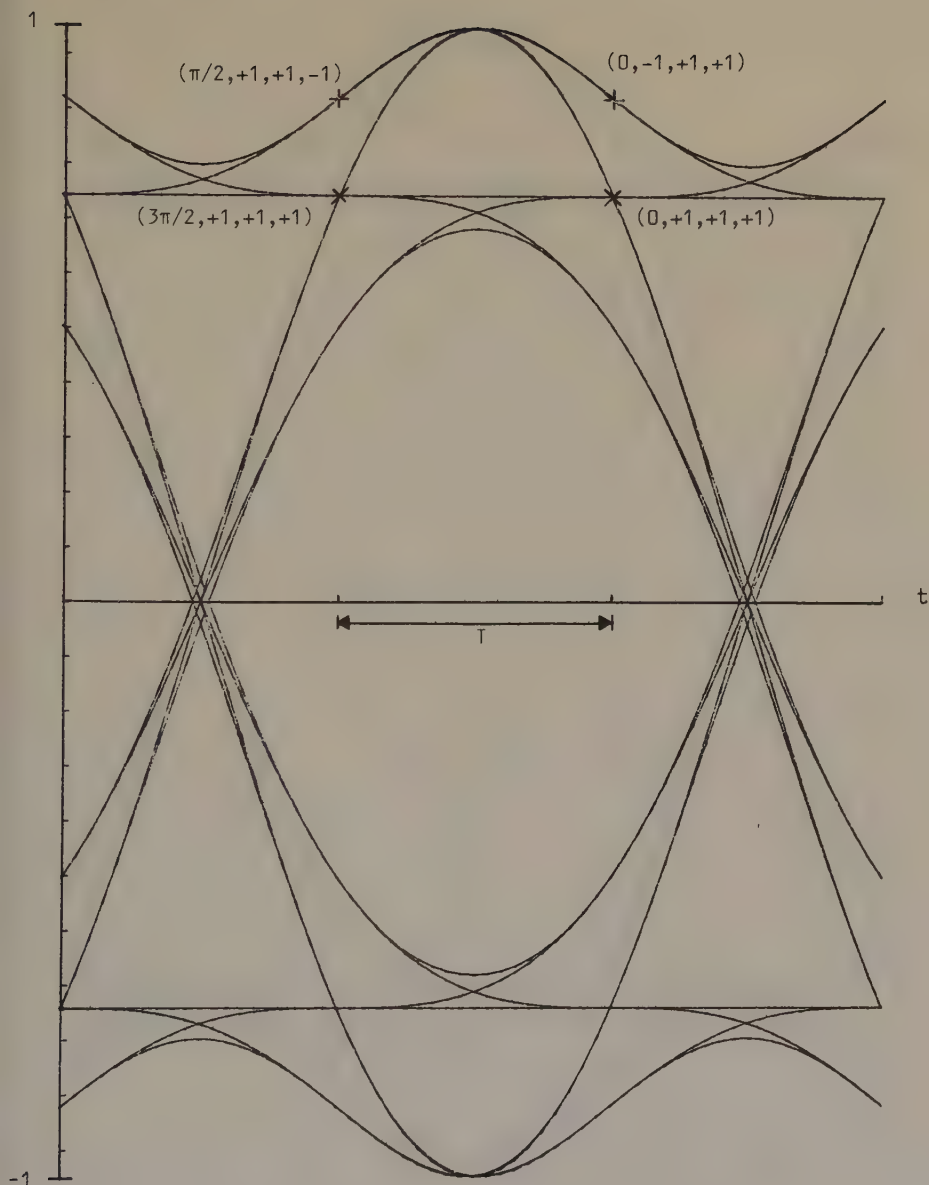


Figure 8. $\cos\varphi(t, \alpha)$ eye pattern of a binary 4RC, $h=1/2$, scheme.
Some of the states are assigned to the nodes.

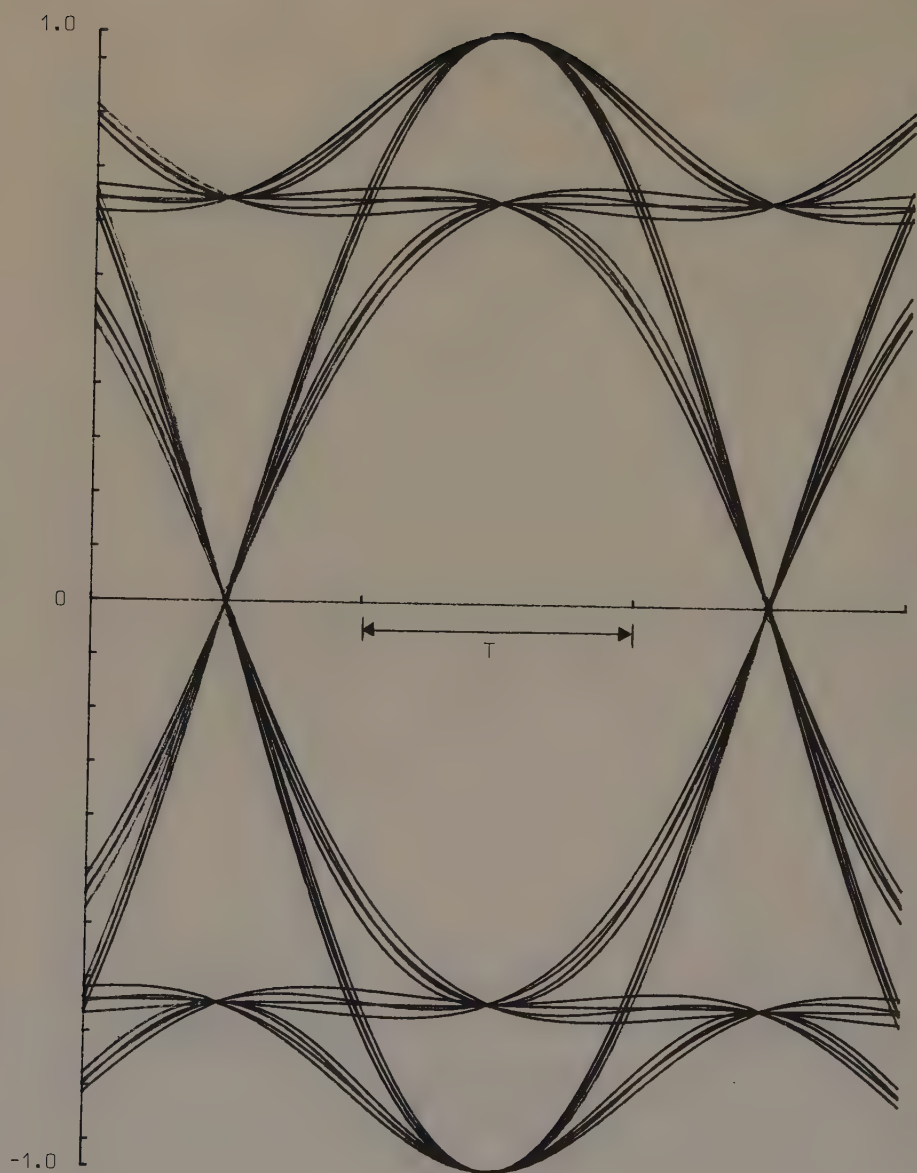


Figure 9. Quadrature eye-pattern before the receiver filter of a binary TFM5, $h=1/2$, scheme.

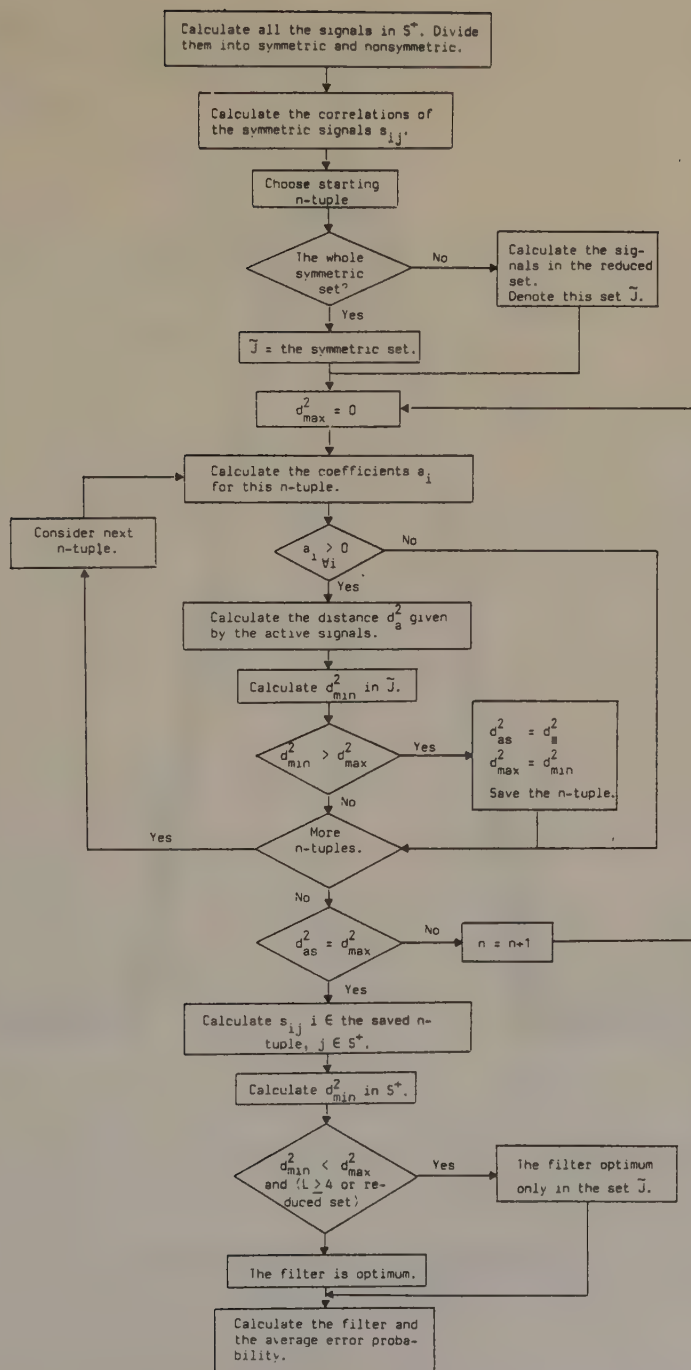


Figure 10. Flowchart for the algorithm giving the asymptotically optimum filter on the Gaussian nonfading channel.

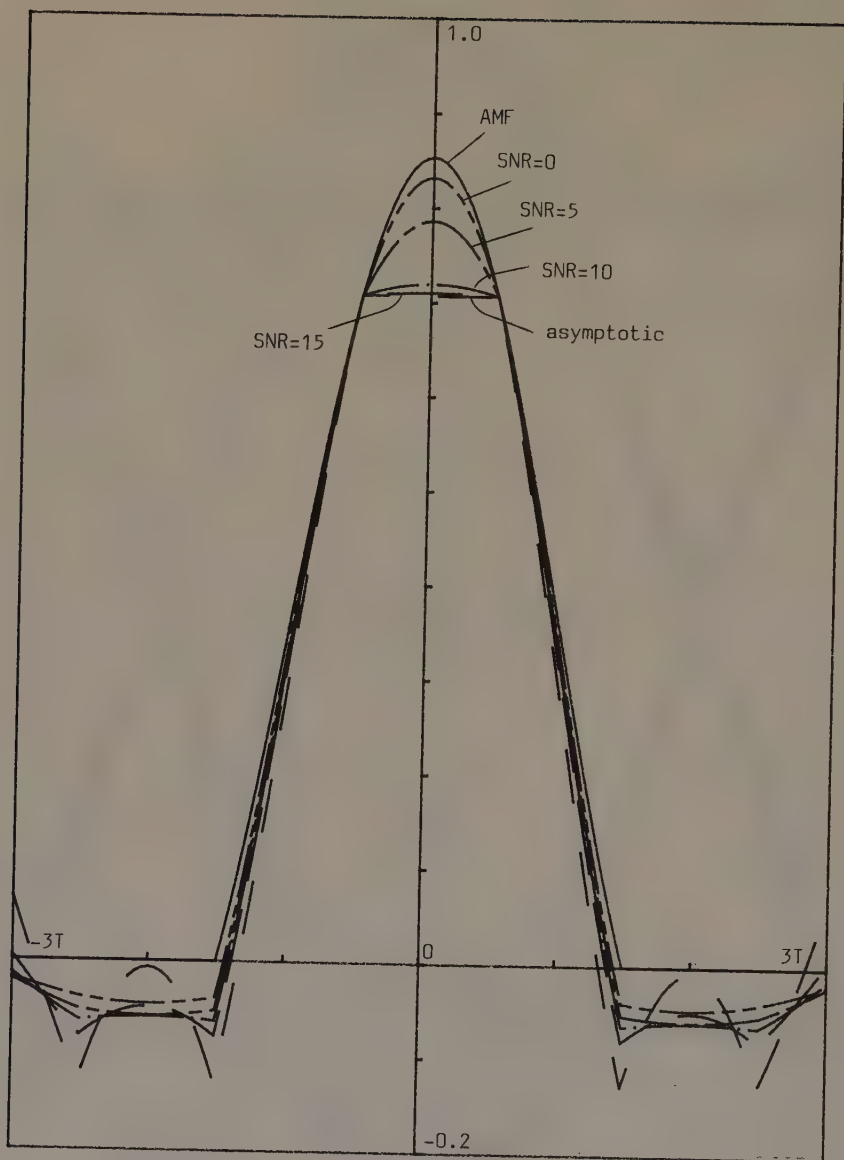


Figure 11. The optimum filters for 2REC with $N_T=6$.

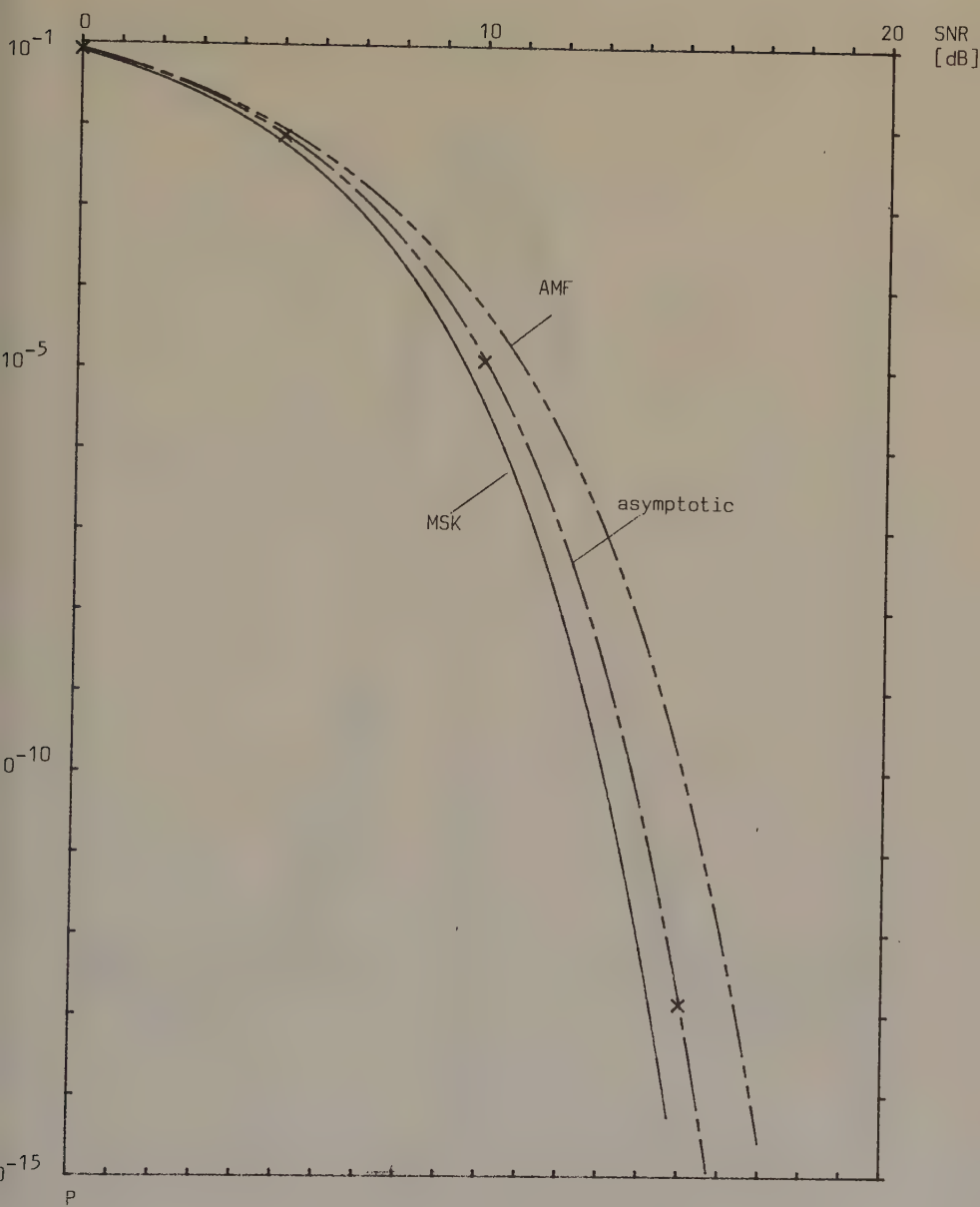


Figure 12. The error probability for the optimum MSK-type receiver to 2REC with $N_T=6$.

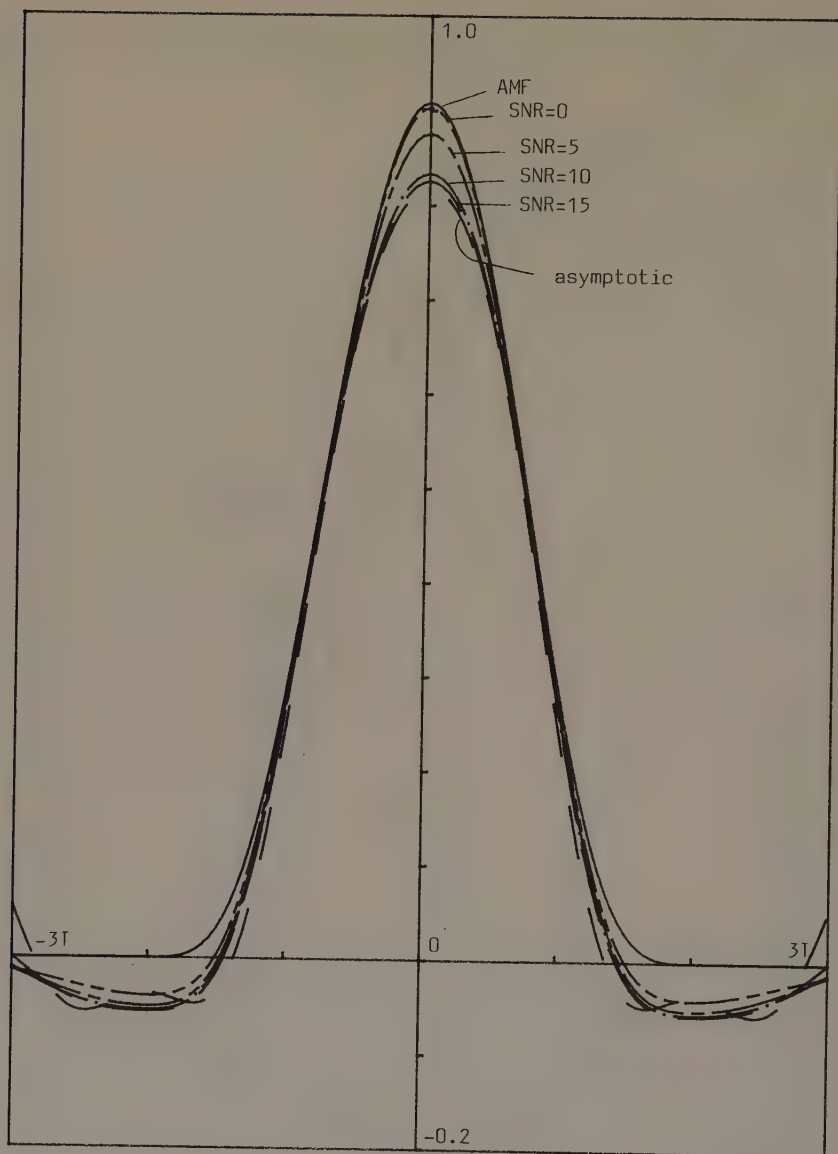


Figure 13. The optimum filters for 3RC with $N_T = 6$.

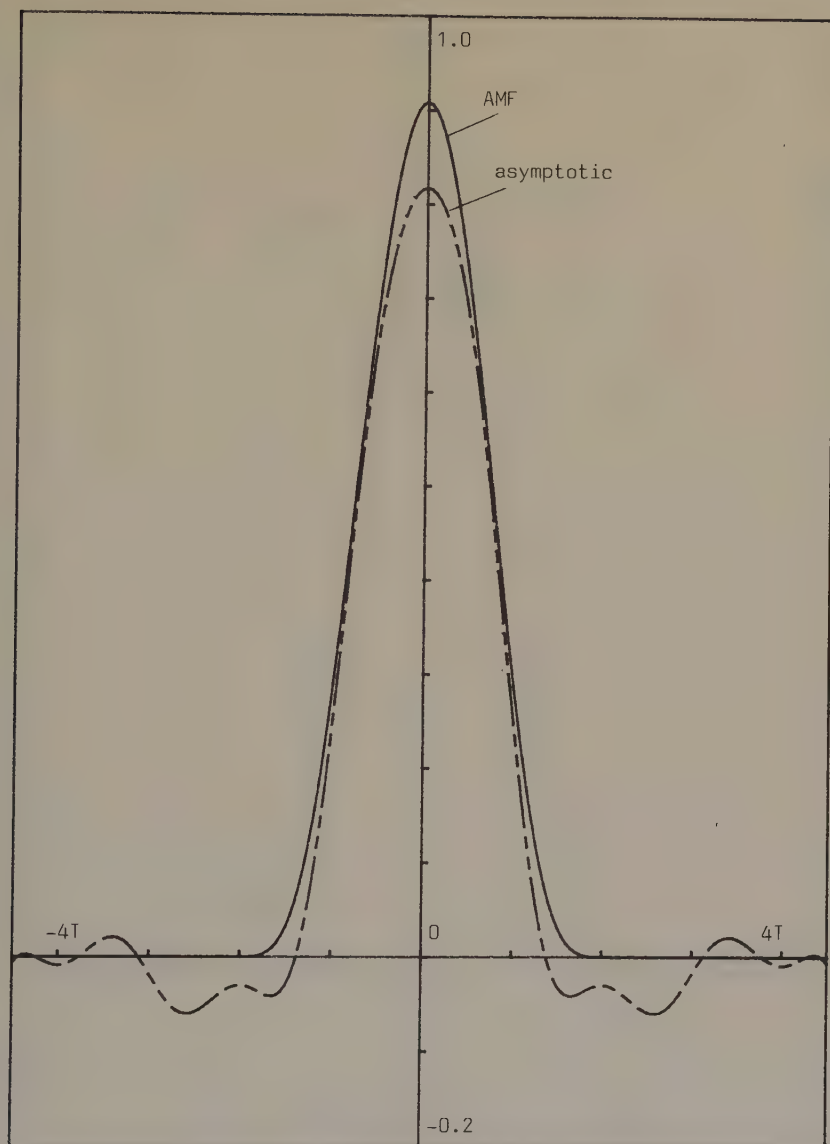


Figure 14. The asymptotically optimum filter for 3RC with $N_T=9$.

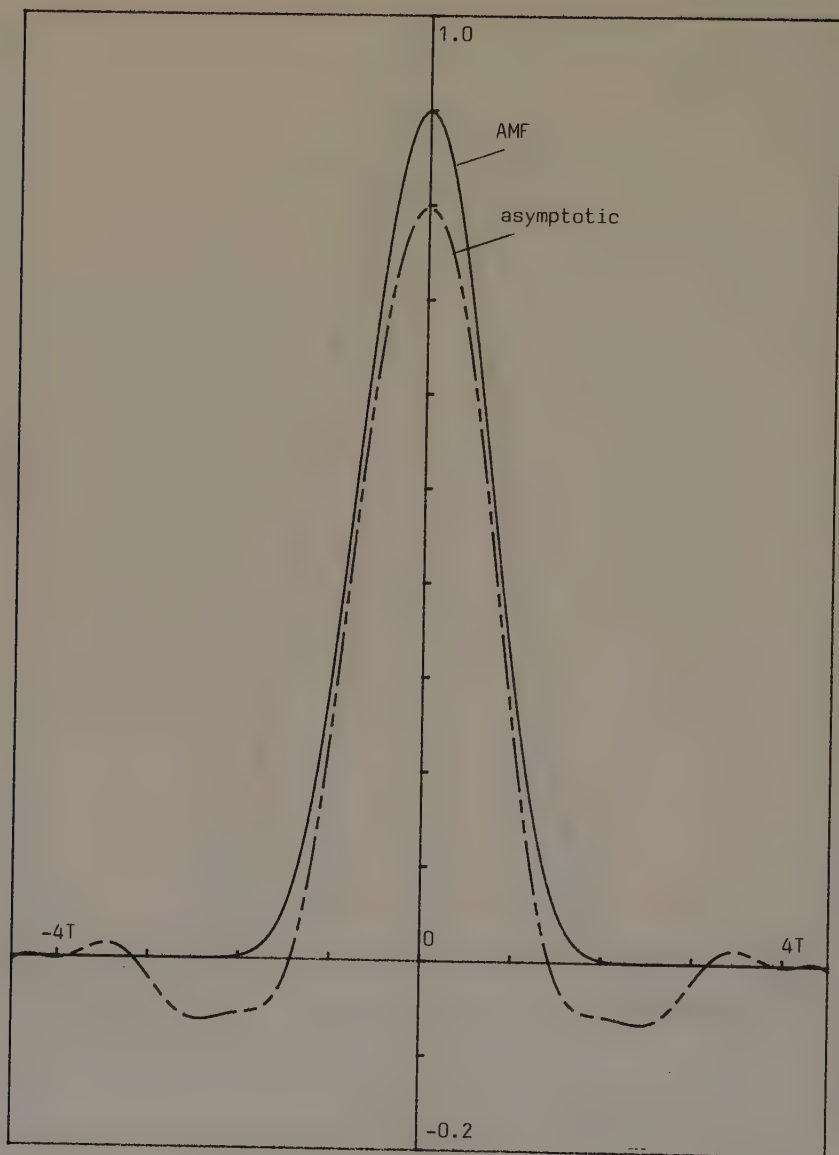


Figure 15. The asymptotically optimum filter for GMSK4 with $B_b T = 0.25$ and $N_T = 9$.

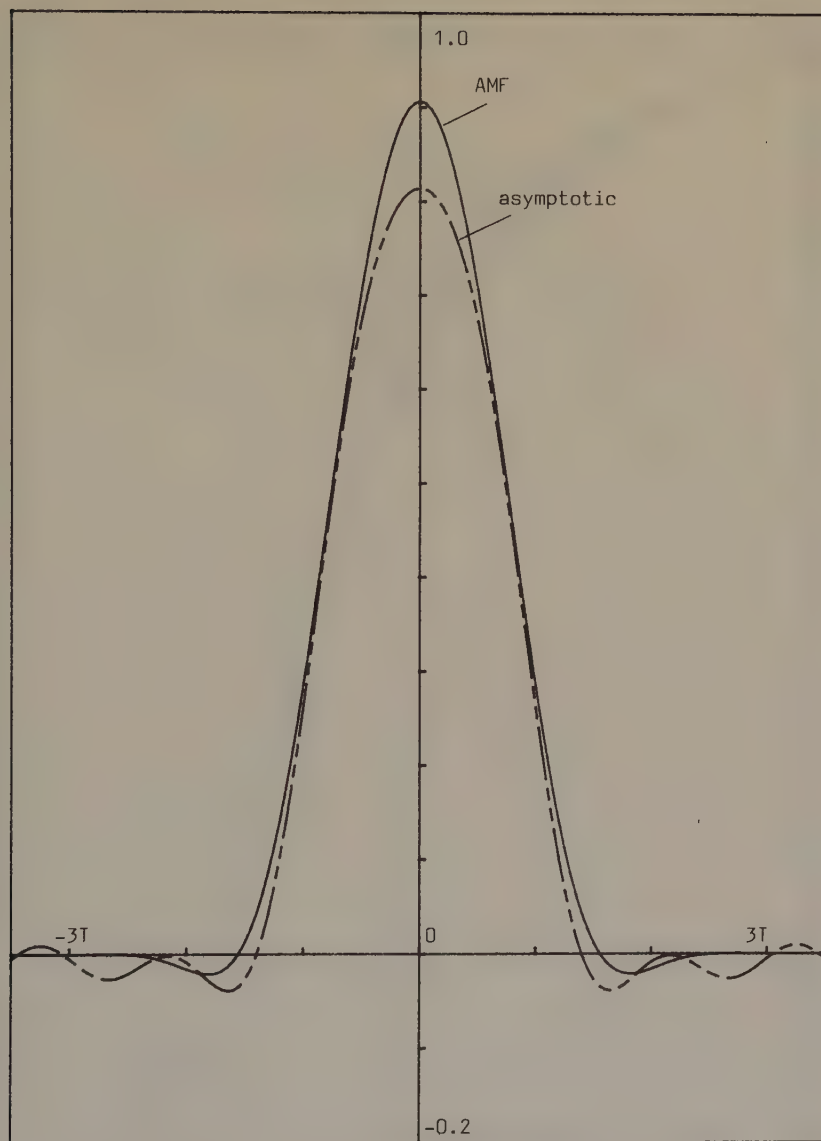


Figure 16. The asymptotically optimum filter for 3SRC6 with $\beta=0.75$ and $N_T=7$.

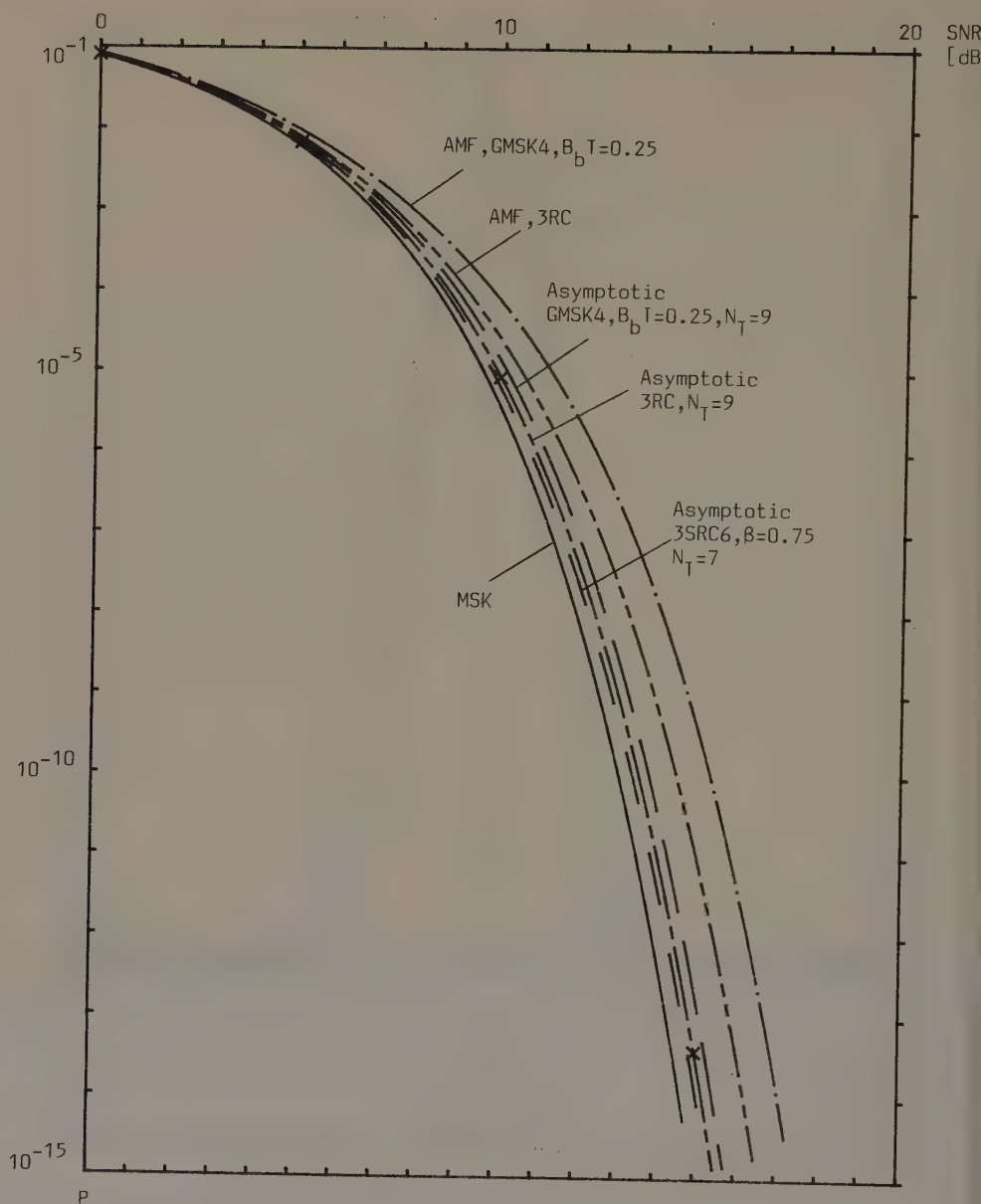


Figure 17. The error probability for 3RC with optimum filters of length $N_T=6$ (x) together with the error probability for the asymptotically optimum filters for 3RC with $N_T=9$, GMSK4 with $B_d T=0.25$ and $N_T=9$ and 3SRC6 with $\beta=0.75$ and $N_T=7$. As comparison the error probability for AMF filters to 3RC and GMSK4 are shown.

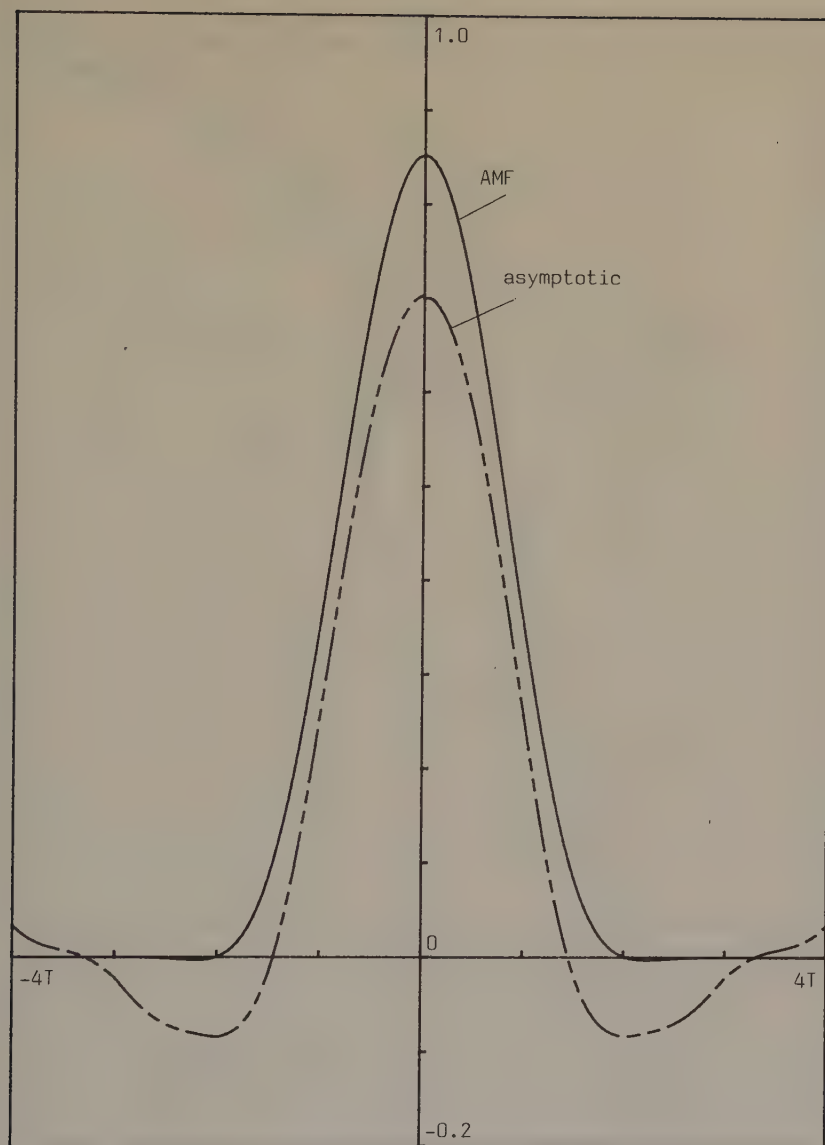


Figure 18. The asymptotically optimum filter in the symmetric signal set for TFM5 with $N_T=8$.

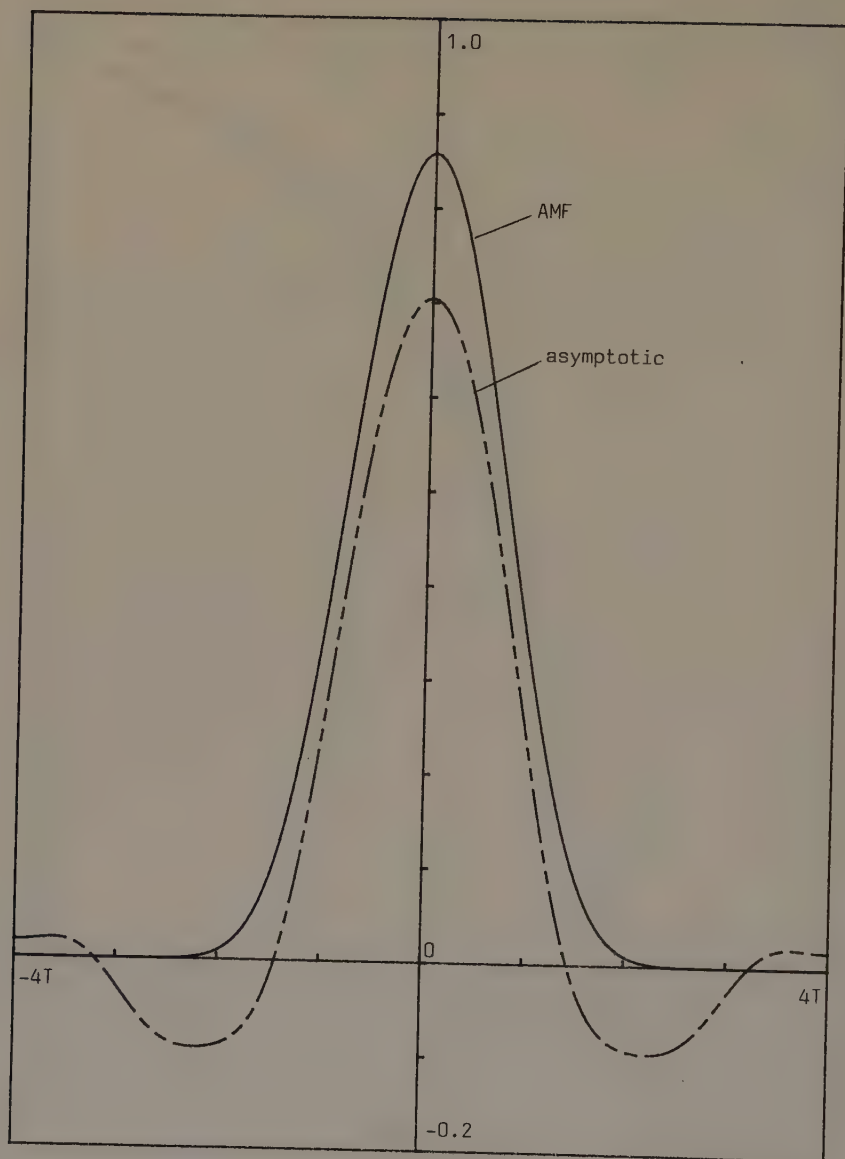


Figure 19. The asymptotically optimum filter for GMSK5 with $B_b T = 0.20$ and $N_T = 8$.

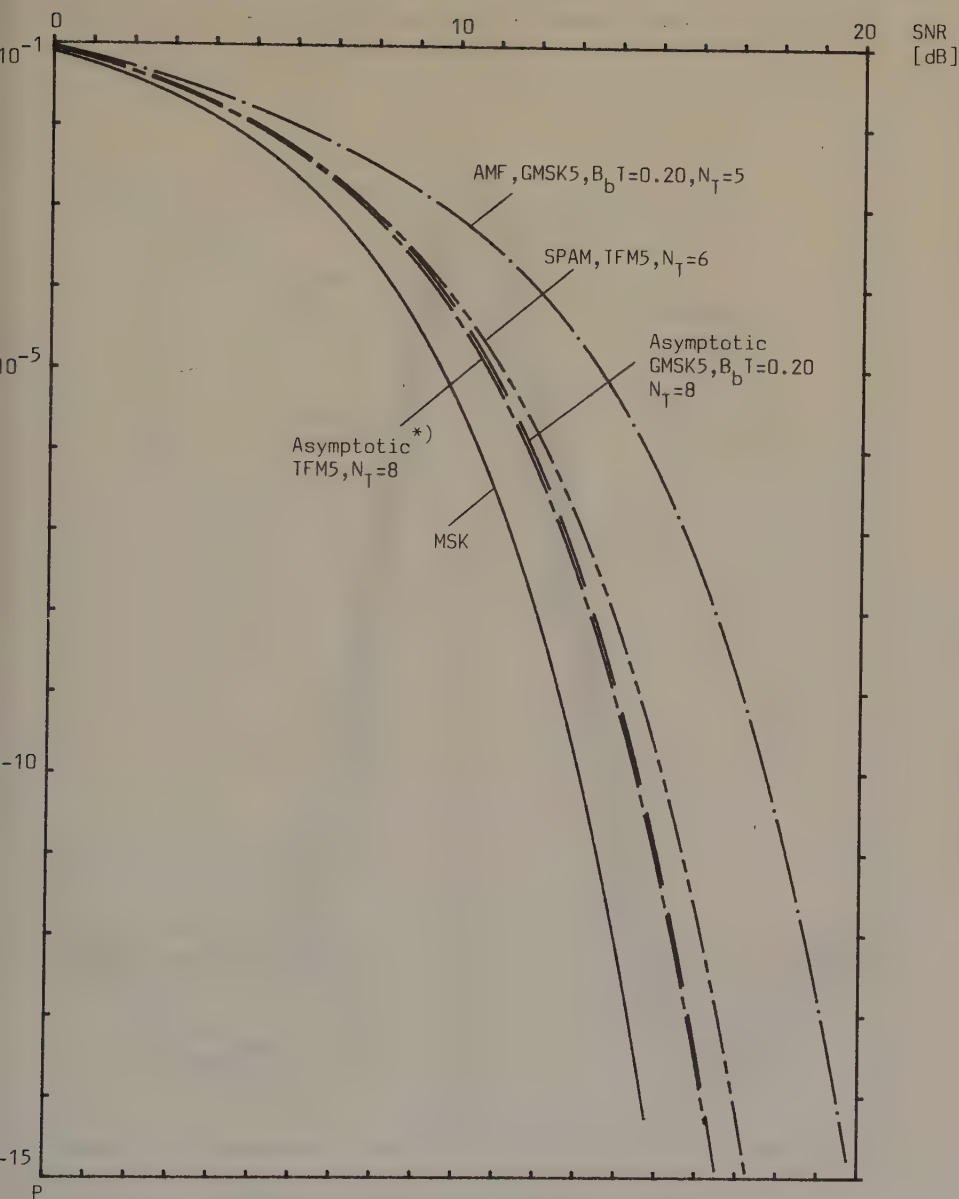


Figure 20. The error probability for the asymptotically optimum receivers to TFMS^{*)} with $N_T = 8$ and GMSK5 with $B_b T = 0.20$ and $N_T = 8$. As comparison the error probability for TFMS with a SPAM filter [3] with $N_T = 6$ and for GMSK5 with an AMF filter of length $N_T = 5$ are shown.

*) This filter is only optimum in the symmetric set.

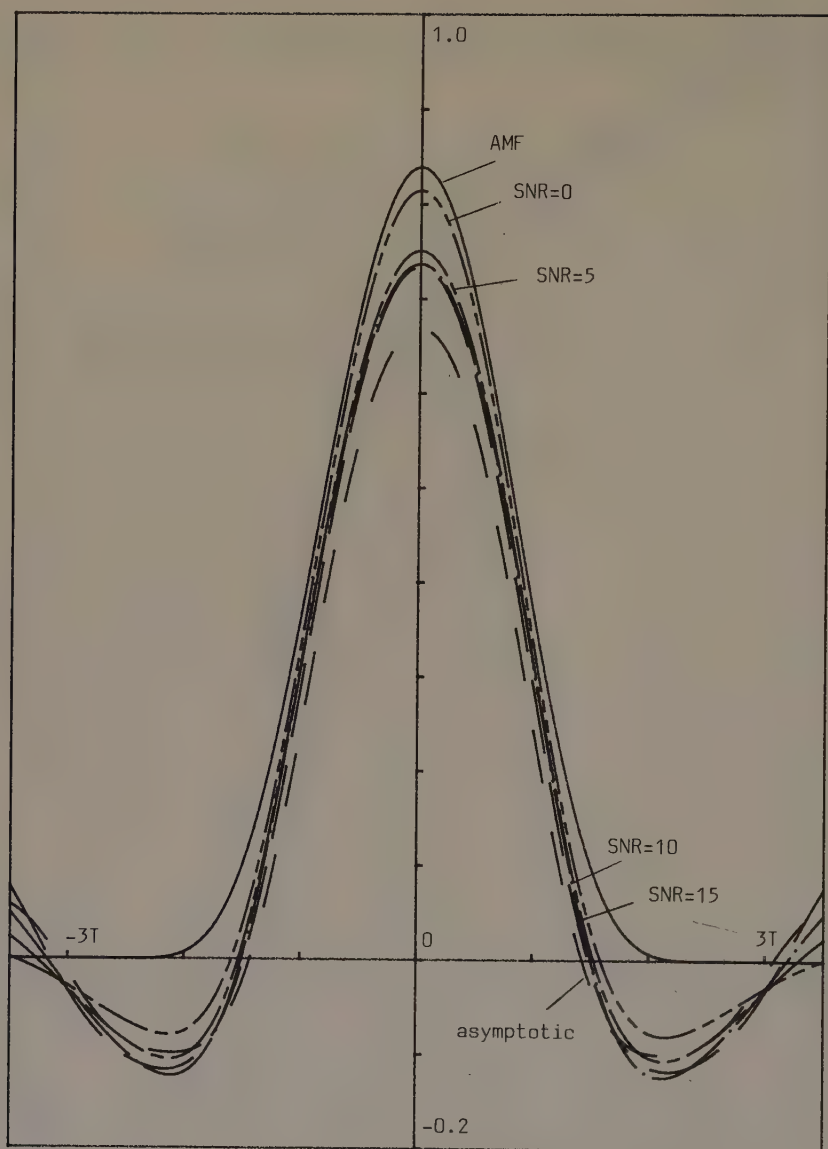


Figure 21. The optimum filters for 4RC with $N_T=7$. The filters optimum at intermediate values of SNR are optimum only in the symmetric signal set.

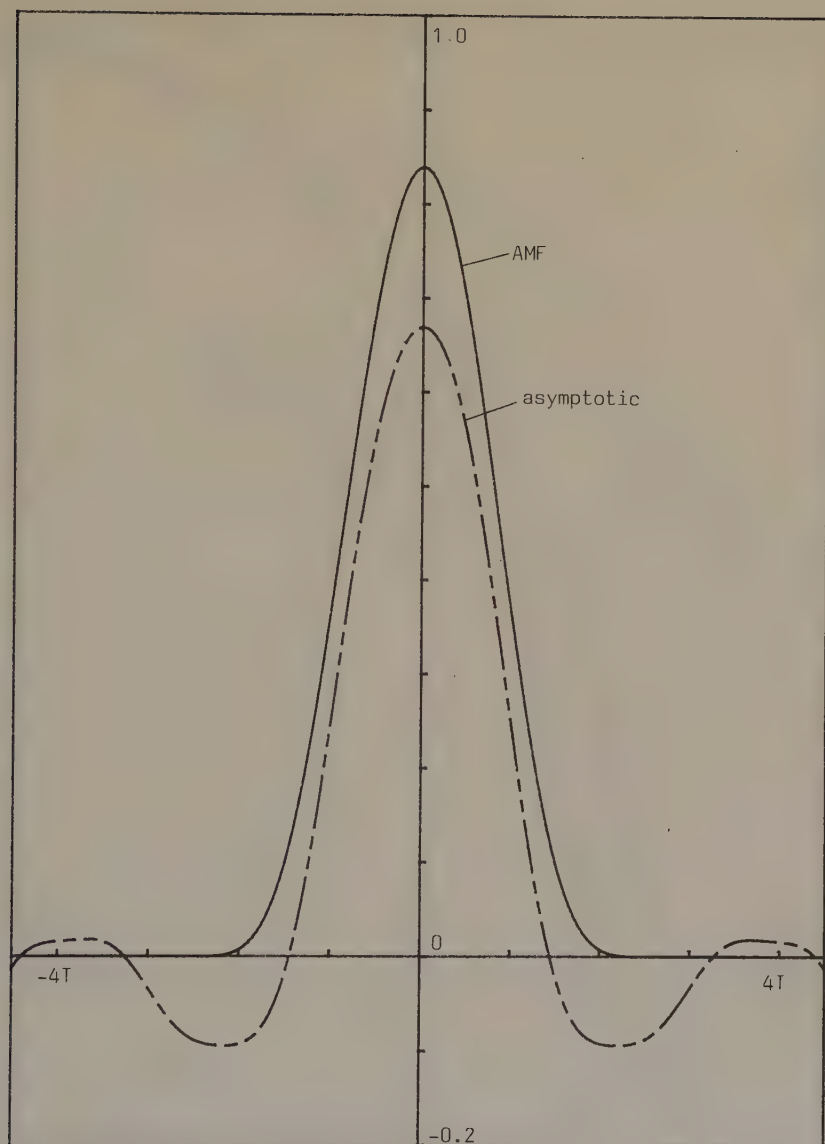


Figure 22. The asymptotically optimum filter for 4RC with $N_T=9$.

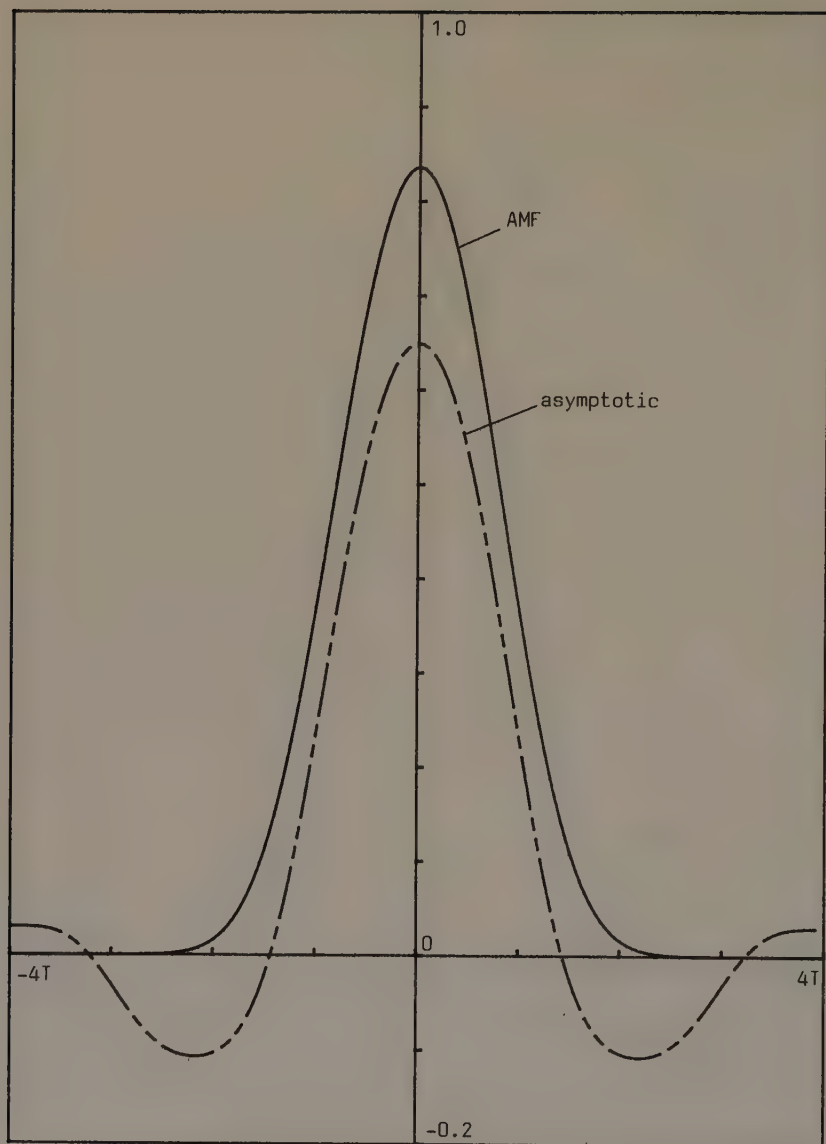


Figure 23. The asymptotically optimum filter for GMSK5 with $B_b T = 0.18$ and $N_T = 8$.

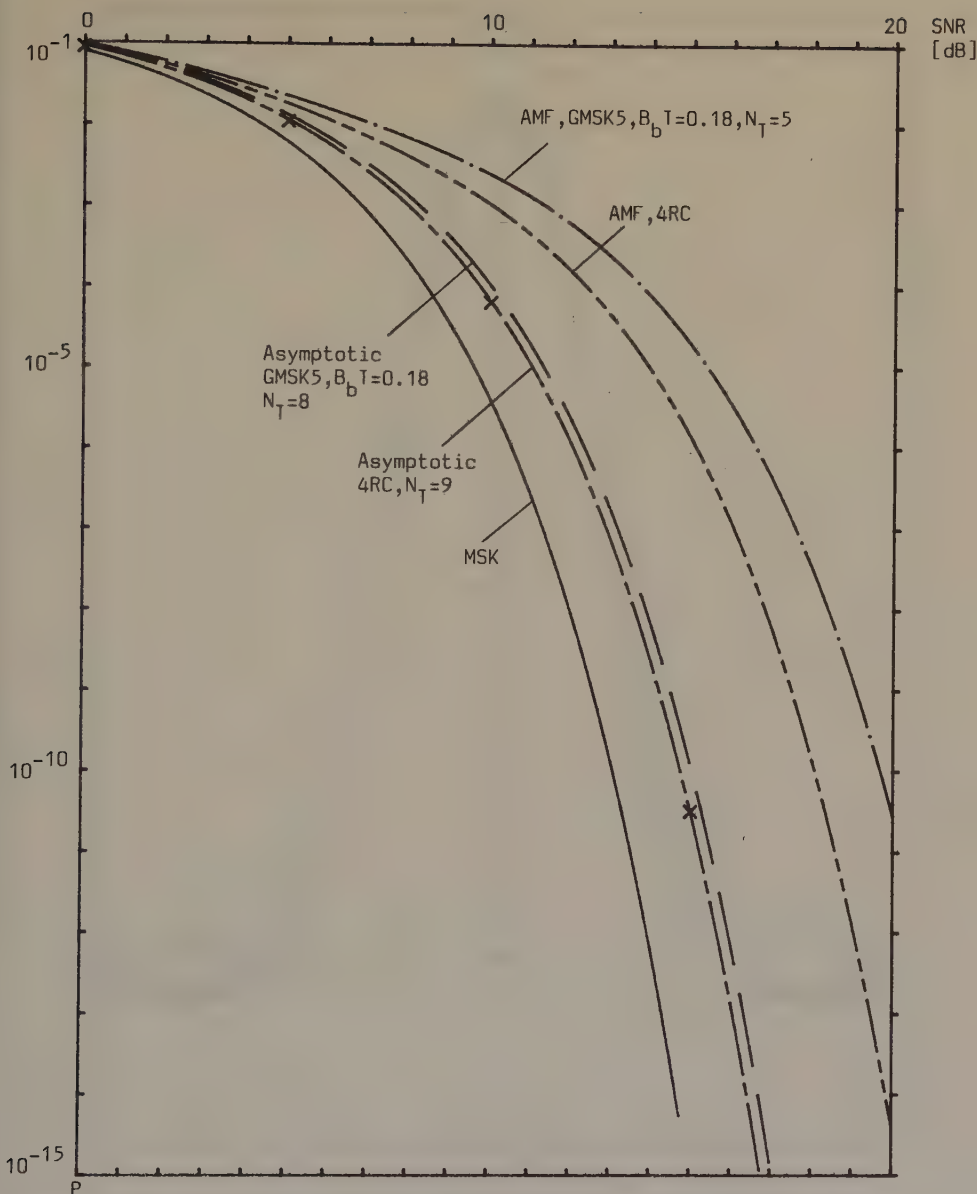


Figure 24. The error probability for 4RC with optimum filters of length $N_T=7$ (x) *) together with the error probability for the asymptotically optimum filters for 4RC with $N_T=9$ and GMSK5 with $B_D T=0.18$ and $N_T=8$. As comparison the error probability for AMF filters to 4RC and GMSK5 with $B_D T=0.18$ and $N_T=5$ are shown.

*) These are optimum only in the symmetric signal set.

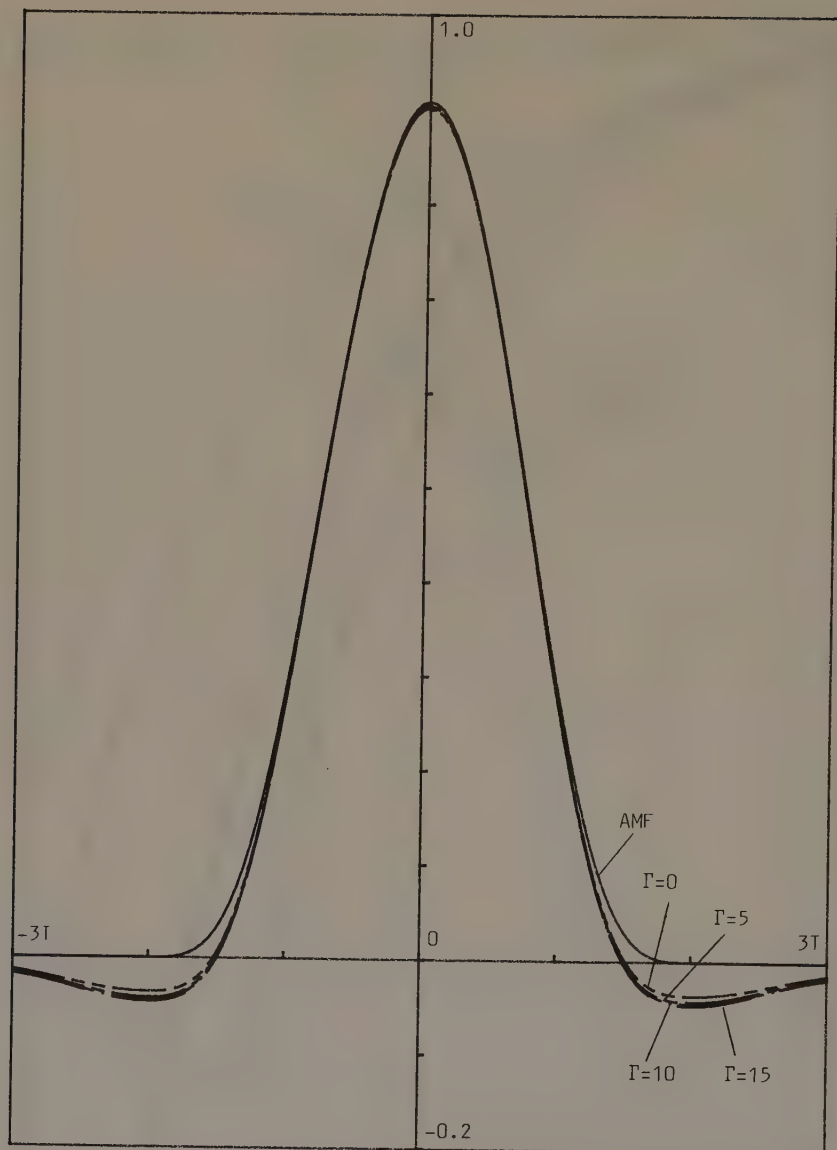


Figure 25. The optimum filters on a Rayleigh fading channel for 3RC with $N_T=6$.

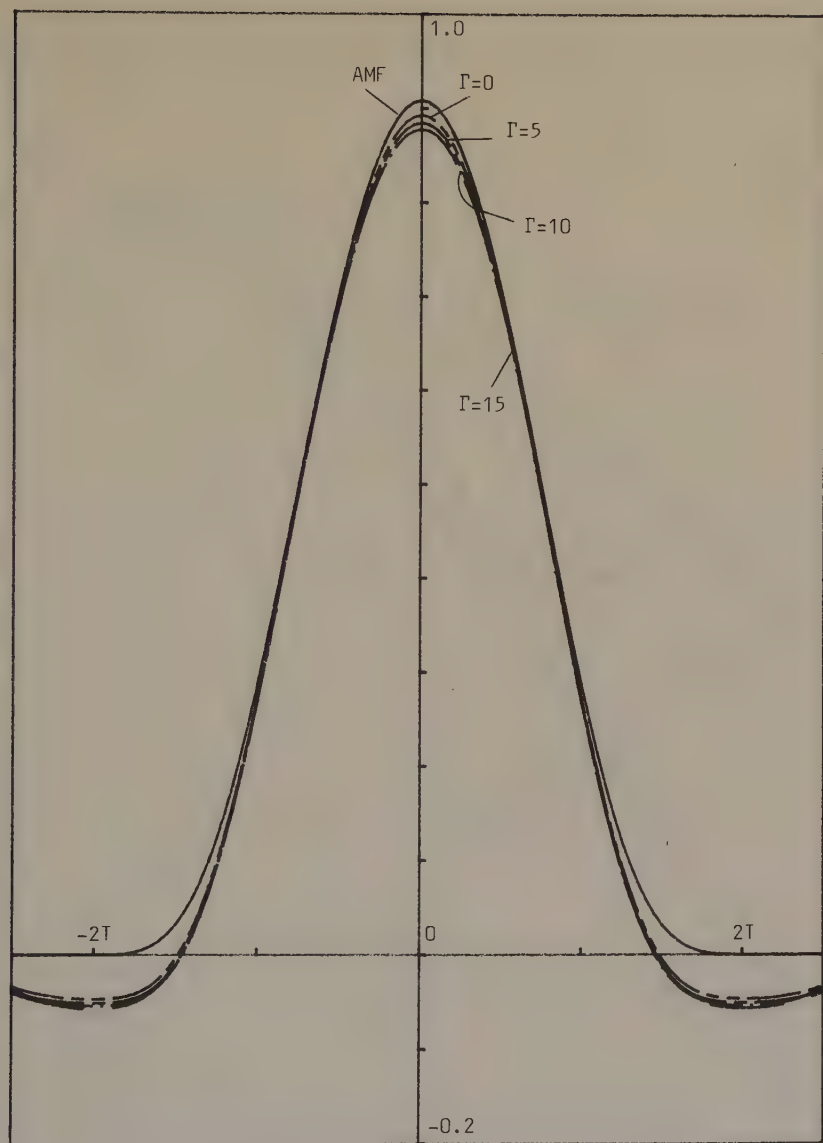


Figure 26. The optimum filters on a Rayleigh fading channel with a two branch maximal ratio combiner for 3RC with $N_T=5$.

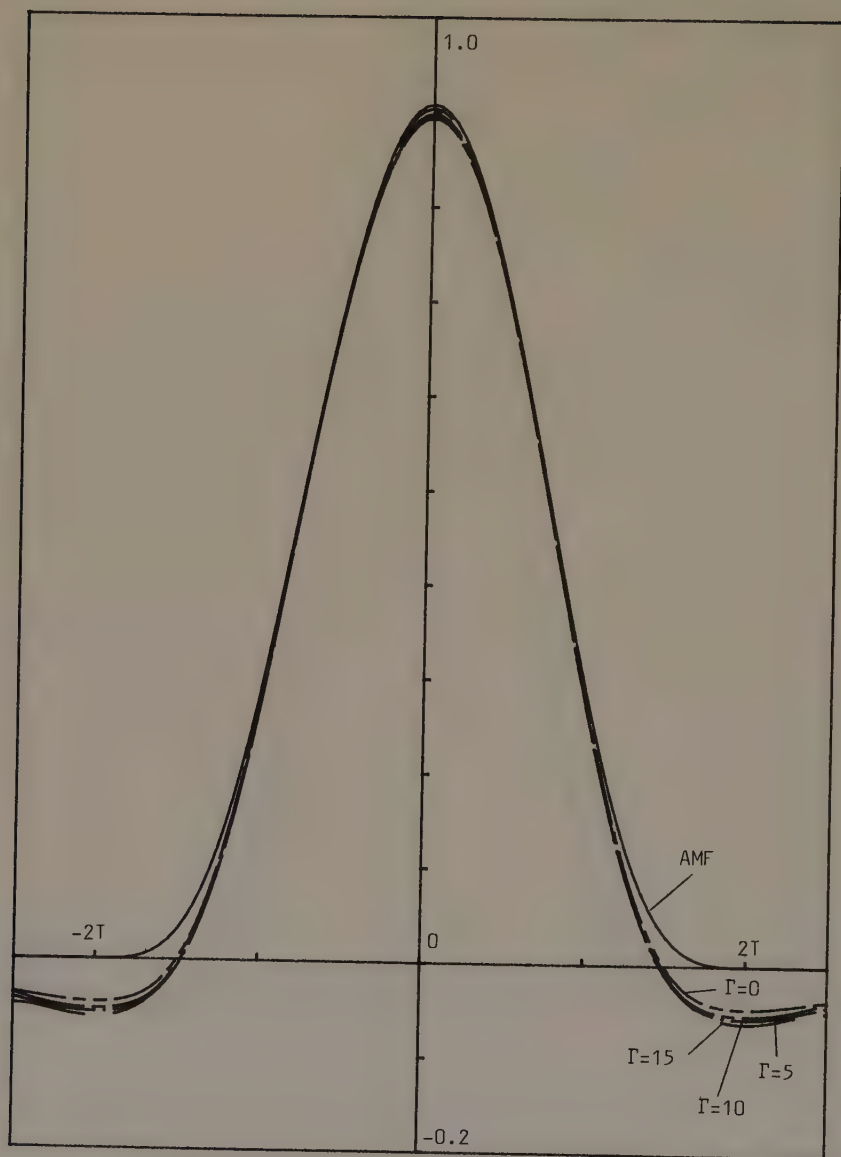


Figure 27. The optimum filters on a Rayleigh fading channel with a two branch selection combiner for 3RC with $N_T=5$.

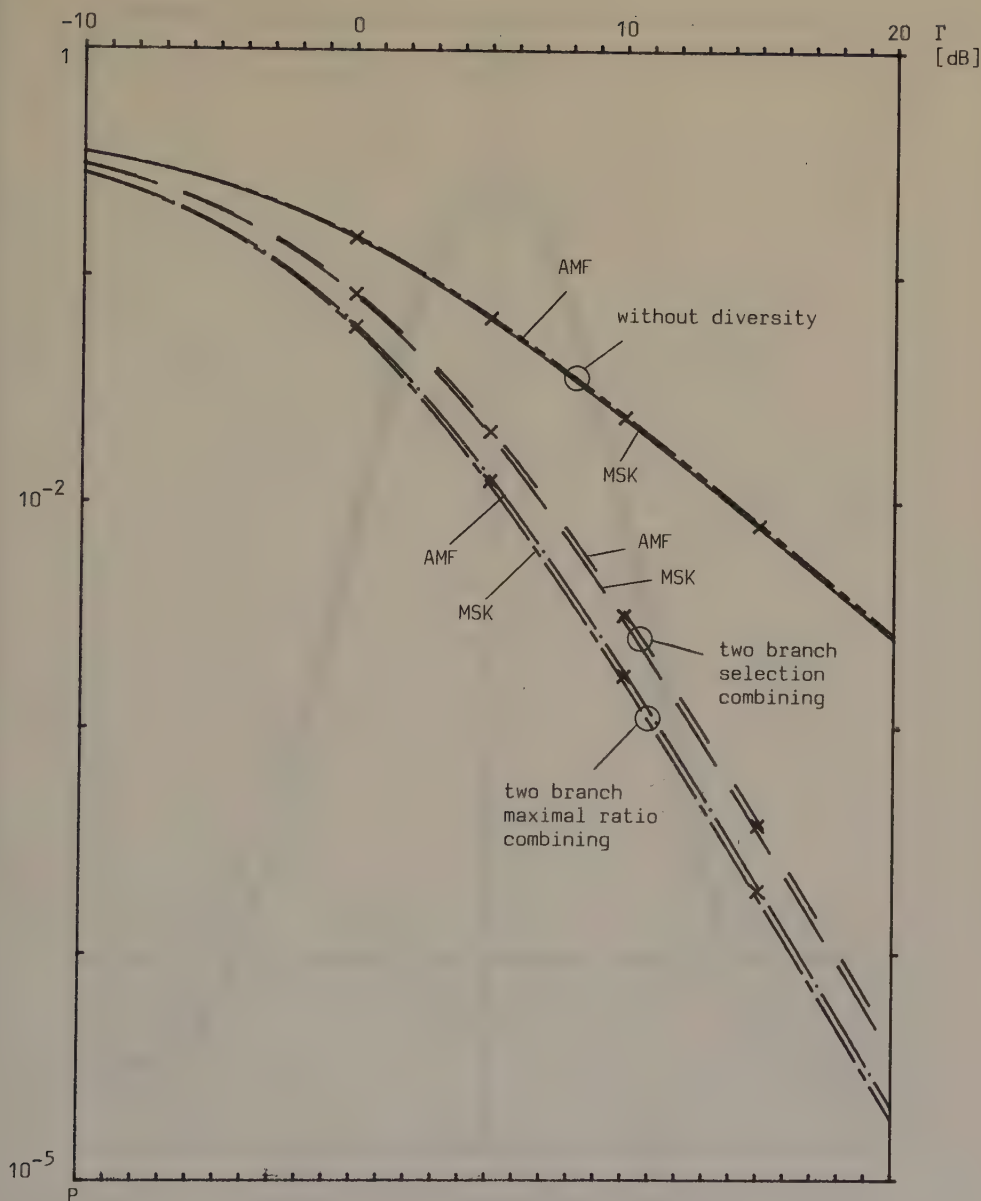


Figure 28. The error probability (x) for the optimum receiver on a Rayleigh fading channel with and without diversity for 3RC. The error probability for an AMF receiver is shown as comparison.

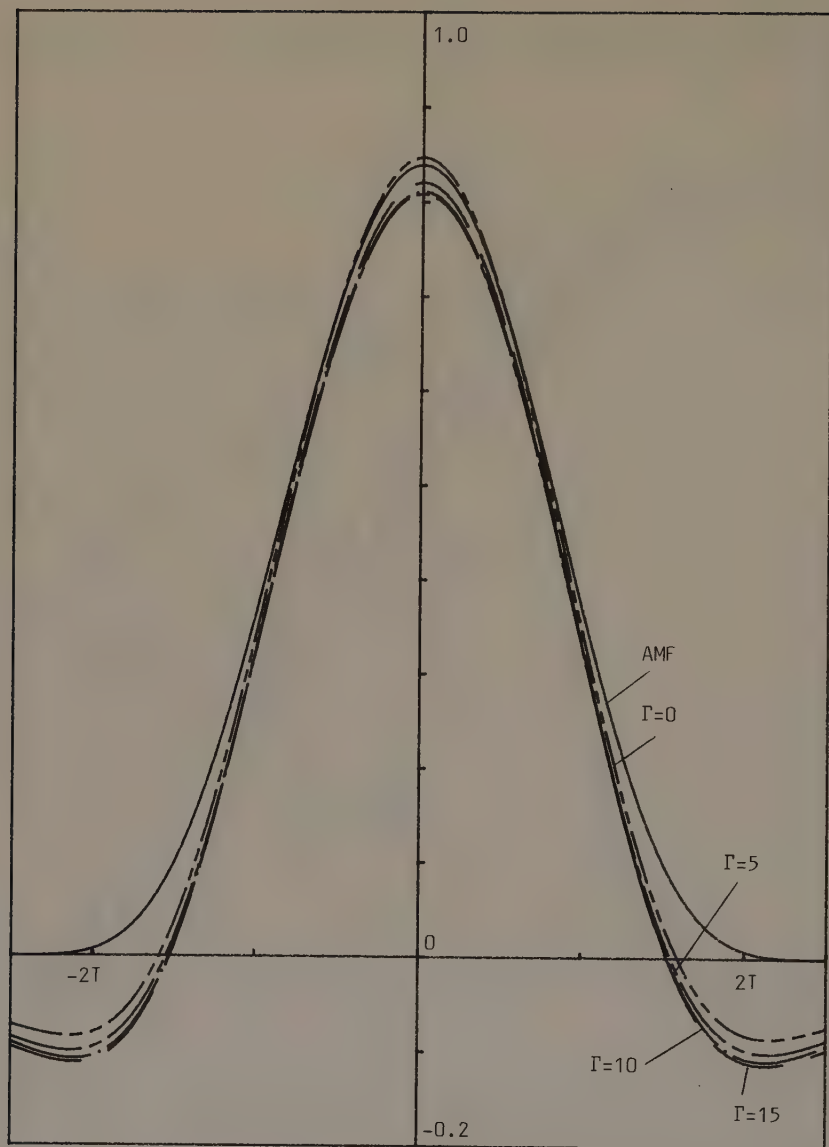


Figure 29. The optimum filters in the symmetric signal set, on a Rayleigh fading channel for 4RC with $N_T=5$.

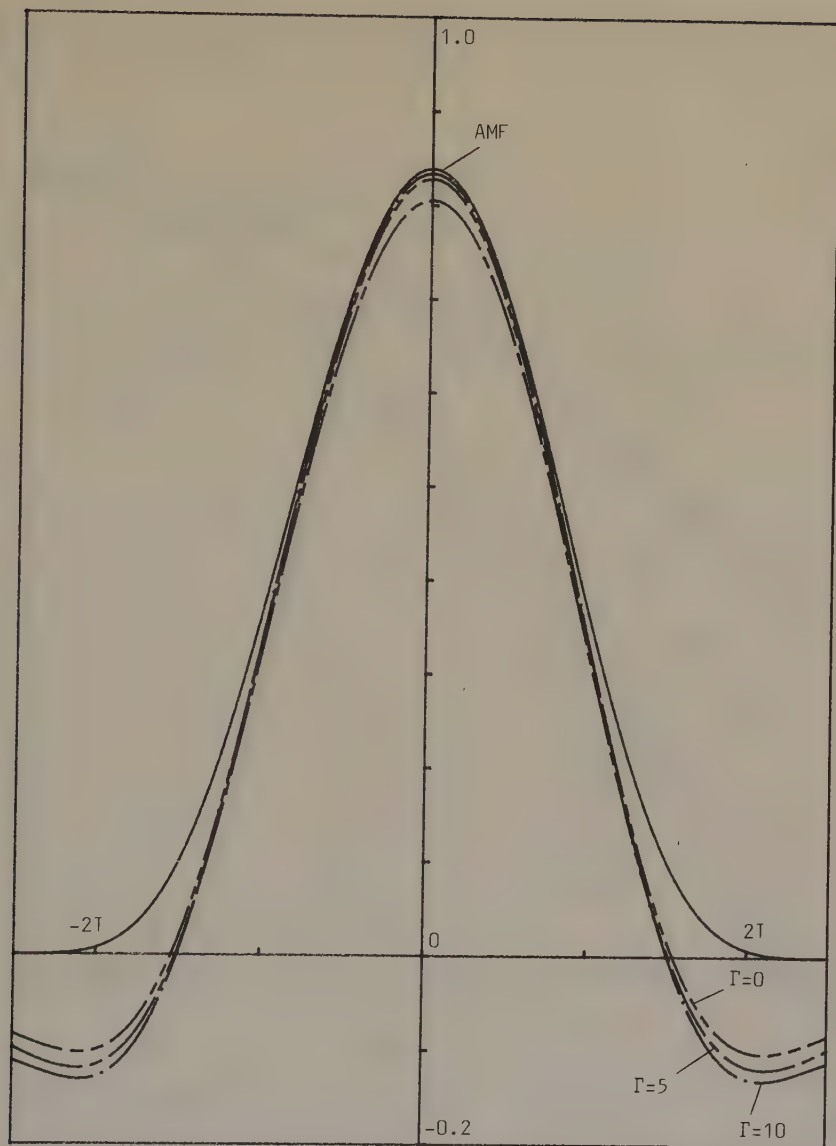


Figure 30. The optimal filters in the symmetric signal set on a Rayleigh fading channel with a two branch maximal ratio combiner for 4RC with $N_T=5$.

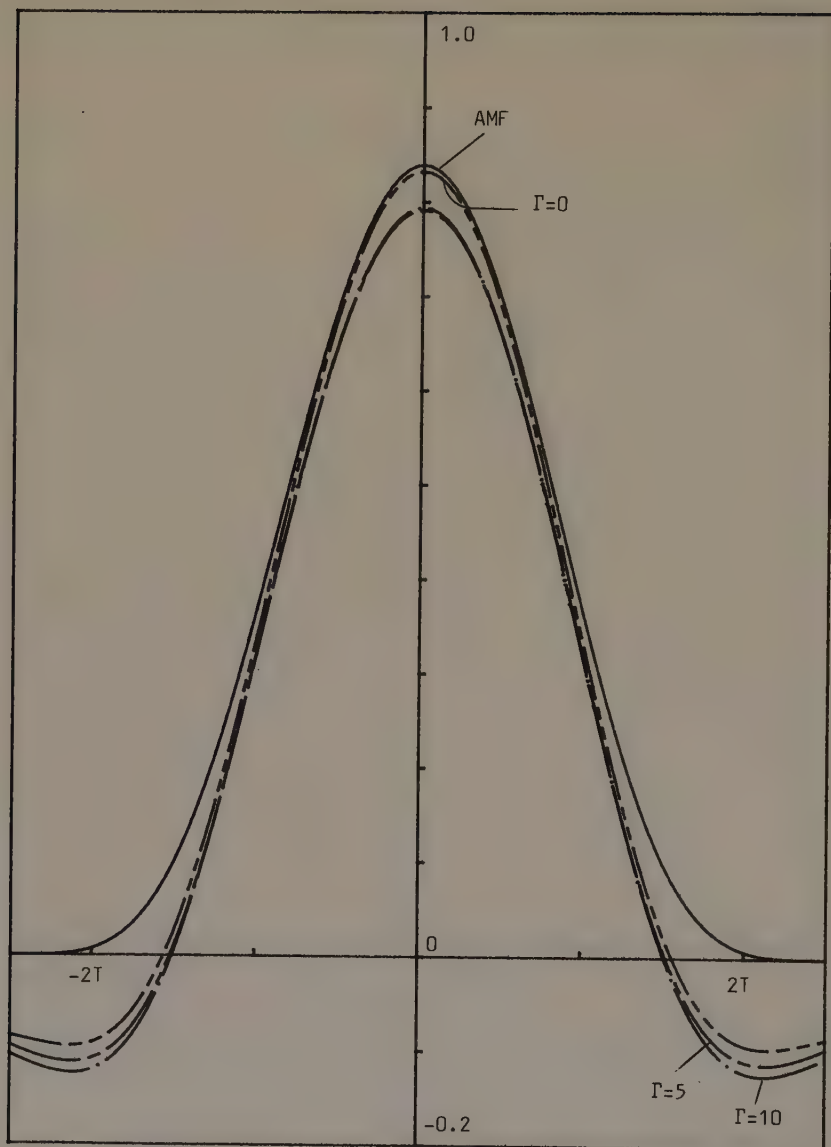


Figure 31. The optimum filters in the symmetric signal set on a Rayleigh fading channel with a two branch selection combiner for 4RC with $N_T=5$.

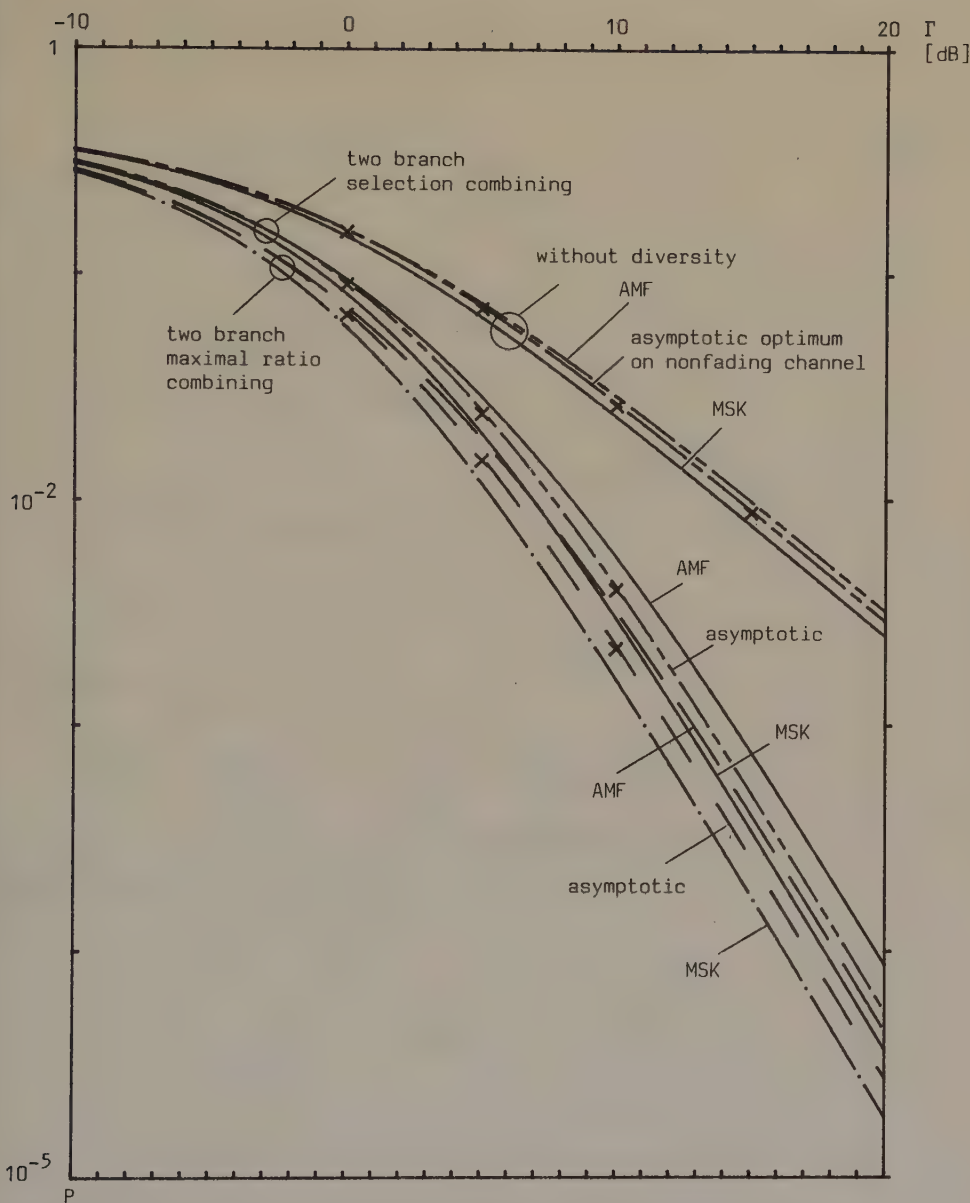


Figure 32. The error probability (x) for the receiver optimum in the symmetric signal set on a Rayleigh fading channel with and without diversity for 4RC. The error probability for an AMF receiver is shown as comparison. Also shown is the error probability for the asymptotically optimum receiver on the nonfading channel used on the fading channel.

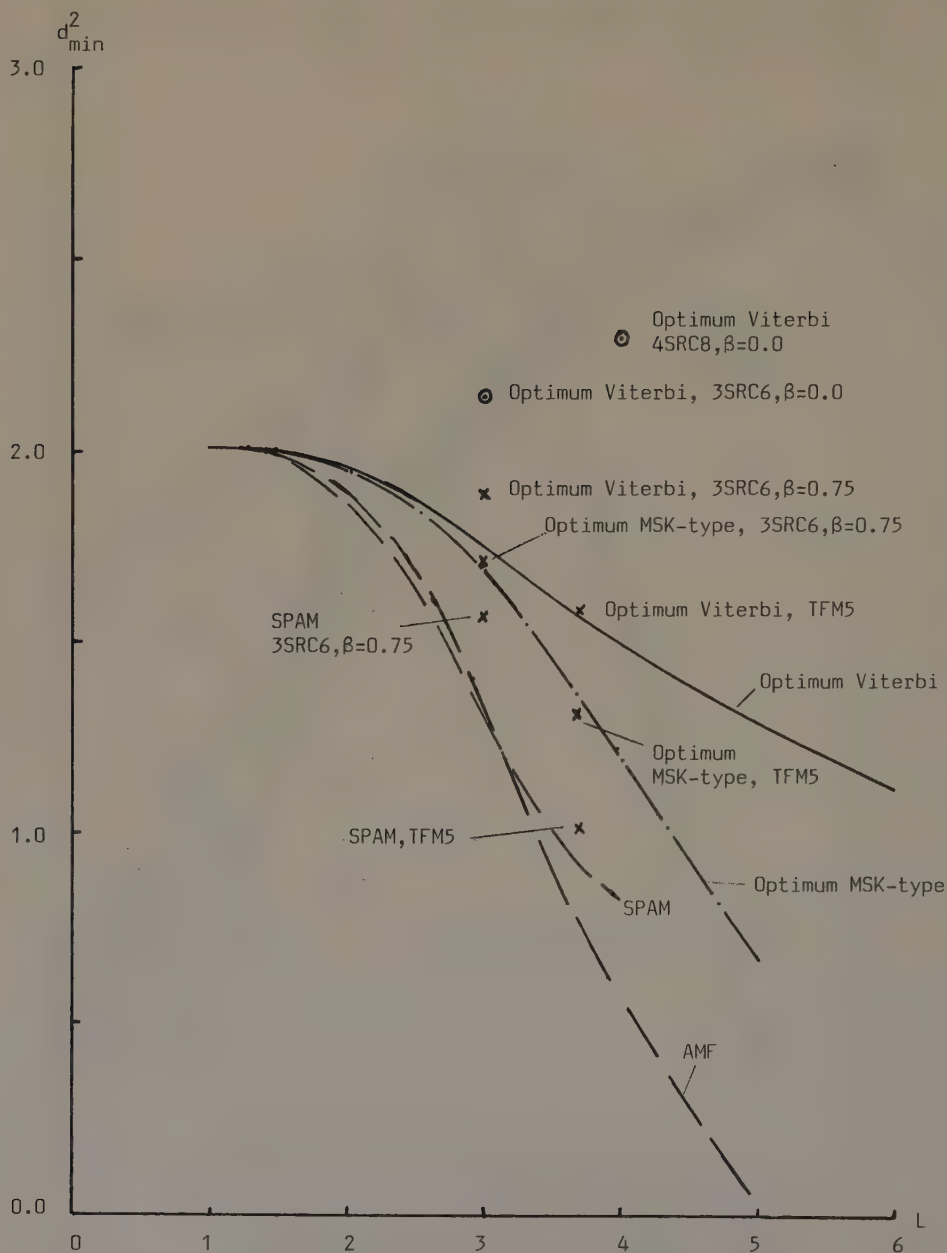


Figure 33. Asymptotic performance for various receivers on the nonfading channel to transmitted RC schemes of length LT . The performance is also shown for 3SRC6, TFM5 and 4SRC8.

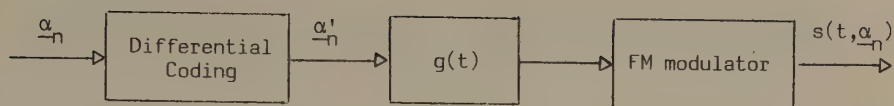


Figure A1. The use of a differential encoder in the transmitter of a CPM signaling scheme.

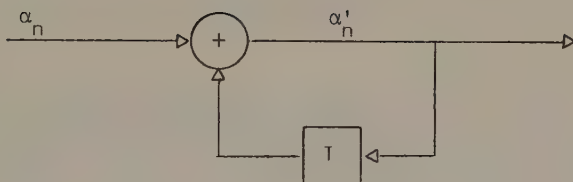


Figure A2. The differential encoder used in the CPM modulation scheme.

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RECEIVERS FOR CPM

PART III

SERIAL MSK-TYPE DETECTION

ABSTRACT

In this part another MSK-type receiver is studied. The MSK-type receiver can also be implemented as a serial receiver, compared to the parallel receiver considered in part II. This serial receiver is useful for the same schemes as the parallel, i.e. binary modulation schemes with modulation index $h=1/2$. Only coherent detection of signals transmitted over an additive white Gaussian noise channel is considered.

The serial receiver can be implemented with only two filters and simple decision logic. The decisions are made serially in one decision eye pattern. Three types of receiver filters are considered. Error probability results are presented for the receiver, both with and without a phase error and a timing error present in the receiver. It is shown that assuming perfect phase and time synchronization, the optimum serial receiver performs equal to the previously considered optimum parallel receiver. The optimum filters for the serial receiver are easily calculated from the optimum filter for the parallel receiver. The advantages with the serial receiver over the parallel receiver are the same for partial response continuous phase modulation as for classical Minimum Shift Keying, MSK, i.e. the reduced sensitivity to phase errors.

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I. INTRODUCTION

From parts I and II of this thesis, it is known that schemes with combined good spectral and detection properties can be obtained in the CPM-family of schemes. It is also known that the optimum Viterbi receiver, which in many cases is complex, does not have to be used. In many cases the reduced-complexity Viterbi receiver has a performance very close to the optimum receiver. For schemes with modulation index $h=1/2$, it is known from part II that a parallel MSK-type receiver can be used. This receiver works well for schemes based on smooth frequency pulses of length up to about four symbol intervals.

The parallel MSK-type receiver has, however, at least one disadvantage. The receiver is of parallel type and the decisions have to be made alternately in the quadrature arms. Recently another receiver of serial type has been investigated [6]–[12]. This receiver has not the above disadvantage of the parallel receiver. It is also concluded that the serial receiver is easier to implement and is less sensitive to phase errors [6], [7]. This serial MSK-type receiver has been studied for MSK [6]–[12] and the optimum receiver filters have been found. The performance of this receiver has been analysed and various results for phase errors have been presented.

In this part the serial receiver is generalized to include also partial response schemes based on smooth frequency pulses. A distance measure giving the asymptotic error probability performance is derived for this serial receiver and an error probability formula is presented. It is shown that the optimum serial receiver and the optimum parallel receiver performs equal, under the assumption of no phase and timing errors. The performance for various schemes are calculated and given as error probability curves. The performance degradation due to phase errors and timing errors is investigated and presented both for the serial and the parallel receiver. Comparisons with BPSK and QPSK are given.

Throughout part III we will assume coherent detection on the additive white Gaussian noise channel.

This part is organized as follows. In section II the modulation scheme and the serial type of receiver are presented. In section III the error

probability is derived when no phase errors or timing errors are present. Comparisons will be made with the parallel receiver. The sensitivity to phase errors is derived in section IV both for the serial and parallel receiver. A variety of results are presented. Then section V presents the results when both a phase error and also a timing error are present. Also here results are shown. Finally section VI is a discussion and conclusion section.

II. THE MODULATION SCHEME

This part deals with a problem related to the one considered in [3], [4] and part II of this thesis, i.e. one approach to the problem of coherent detection of CPM-signals, with simple receivers. In this part a serial MSK-type detector is investigated [6]-[12]. Binary signaling with modulation index $h=1/2$ is assumed. In this section and also the next section perfect timing and synchronization is assumed.

First a brief introduction to the CPM signaling scheme. Let $\underline{\alpha} = \dots, \alpha_{n-2}, \alpha_{n-1}, \alpha_n, \alpha_{n+1}, \alpha_{n+2}, \dots$ be an infinitely long sequence of statistically independent binary data symbols, taking the values ± 1 with equal probability $1/2$. The transmitted modulated constant envelope signal is given by

$$s(t, \underline{\alpha}) = \sqrt{\frac{2E}{T}} \cos \left(2\pi f_0 t + \varphi(t, \underline{\alpha}) + \varphi_0 \right) \quad (1)$$

where the information carrying phase is

$$\varphi(t, \underline{\alpha}) = 2\pi h \sum_{i=-\infty}^{\infty} \alpha_i \cdot q(t-iT) \quad (2)$$

The symbol energy is denoted E , the symbol time T and the carrier frequency f_0 . h is the modulation index. φ_0 is an arbitrary constant phase shift, and can be set to zero without loss of generality. The phase response $q(t)$ is defined by

$$q(t) = \int_{-\infty}^t g(\tau) d\tau \quad (3)$$

where $g(t)$ is the frequency response. For details see [1], [2].

Schemes based on various frequency pulses are compared. The Raised Cosine pulse of length L symbol intervals, denoted LRC, is given by

$$g(t) = \begin{cases} \frac{1}{2LT} \left[1 - \cos \left(\frac{2\pi t}{LT} \right) \right] & ; \quad 0 \leq t \leq LT \\ 0 & ; \quad \text{otherwise} \end{cases} \quad (4)$$

The Tamed Frequency Modulation scheme (TFM) is given by

$$g(t) = \frac{1}{8} [g_0(t-T) + 2g_0(t) + g_0(t+T)] \quad (5)$$

where

$$g_0(t) \approx \frac{1}{T} \left[\frac{\sin\left(\frac{\pi t}{T}\right)}{\frac{\pi t}{T}} - \frac{\pi^2}{24} \cdot \frac{2\sin\left(\frac{\pi t}{T}\right) - \frac{2\pi t}{T} \cos\left(\frac{\pi t}{T}\right) - \left(\frac{\pi t}{T}\right)^2 \sin\left(\frac{\pi t}{T}\right)}{\left(\frac{\pi t}{T}\right)^3} \right] \quad (6)$$

Next, the pulse Spectral Raised Cosine with roll-off factor β and main-lobe width L symbol intervals (LSRC) is given by

$$g(t) = \frac{1}{LT} \frac{\sin\left(\frac{2\pi t}{LT}\right)}{\frac{2\pi t}{LT}} \cdot \frac{\cos\left(\beta \cdot \frac{2\pi t}{LT}\right)}{1 - \left(\frac{4\beta}{LT} \cdot t\right)^2} ; \quad 0 \leq \beta \leq 1 \quad (7)$$

In the frequency domain this pulse is defined by

$$G(\omega) = \begin{cases} 1 & ; \quad |\omega| < \frac{2\pi}{LT} (1-\beta) \\ \frac{1}{2} \left(1 - \sin \left(\frac{LT}{4\beta} \left(\omega - \frac{2\pi}{LT} \right) \right) \right) & ; \quad \frac{2\pi}{LT} (1-\beta) \leq |\omega| < \frac{2\pi}{LT} (1+\beta) \\ 0 & ; \quad \text{otherwise} \end{cases} \quad (8)$$

Both the TFM pulse and the SRC pulse are not time limited, therefore these have been truncated symmetrically around $t=0$, to the total length of L_T symbol intervals. The truncated pulse is normalized to have

$$\int_{-\infty}^{\infty} g(\tau) d\tau = \frac{1}{2} \quad (9)$$

by definition of the CPM scheme [1], [2]. These truncated schemes are denoted $TFML_T$ and $LSRCL_T$ respectively.

Finally the Rectangular pulse of length L symbol intervals is given by

$$g(t) = \begin{cases} \frac{1}{2LT} & ; \quad 0 < t < LT \\ 0 & ; \quad \text{otherwise} \end{cases} \quad (10)$$

In particular the full response scheme 1REC ($L=1$) is identical to Minimum Shift Keying, MSK, when $h=1/2$.

The received signal is $r(t)$

$$r(t) = s(t, \underline{\alpha}) + n(t) \quad (11)$$

where $n(t)$ is white Gaussian noise with one-sided spectral density N_0 .

In [3], [4] the parallel MSK-type of receiver with a receiver filter in each of the quadrature arms, is considered. This is a parallel type of receiver where the decisions are made parallel in the quadrature arms. The receiver structure is shown in figure 1a. The optimum filter for this parallel receiver is derived and given in part II. The performance for these optimum receivers are also given. In [3] some other type of ad-hoc filters are considered, which are not optimum. However for error probabilities greater than around 10^{-4} the loss is small for these ad-hoc filters. Some of these filters will be considered in this part. When the degradation due to phase and timing errors are compared one can as well study these filters, which make the computational efforts smaller. However, the asymptotically optimum filters from part II will also be considered.

The MSK-filter is given by the impulse-response

$$a(t) = \begin{cases} \cos\left(\frac{\pi t}{2T}\right) & ; \quad |t| \leq T \\ 0 & ; \quad \text{otherwise} \end{cases} \quad (12)$$

The SPAM filter is based on an approximation of the unfiltered quadrature eye. The performance of the receiver depends on the opening in this eye. This means that the inner part of the eye is the limiting part. This inner part of the eye, can be considered as an eye of a PAM system. Since the decision is made alternately from the quadrature arms, the approximating PAM system has symbol time $T_0 = 2T$. The transmitter pulse $g_{TPAM}(t)$ corresponding to the PAM approximation can easily be derived. From Lucky, Salz and Weldon [14] it is known that the optimum receiver filter to a PAM system, with the transmitter filter $g_{TPAM}(t)$ is

$$g_{RPAM}(t) = F^{-1} \left\{ G_{RPAM}(\omega) \right\} \quad (13)$$

where $F^{-1}\{\cdot\}$ denotes the inverse Fourier transform. $G_{\text{RPAM}}(\omega)$ is defined by

$$G_{\text{RPAM}}(\omega) = \frac{G_{\text{TPAM}}^*(\omega)}{\sum_k \left| G_{\text{TPAM}}\left(\omega + \frac{2\pi k}{T_0}\right) \right|} \quad (14)$$

where $G_{\text{TPAM}}(\omega)$ is the Fourier transform of $g_{\text{TPAM}}(t)$ and $G_{\text{TPAM}}^*(\omega)$ is the complex conjugate of the Fourier transform. This receiver filter is used as a receiver filter to the CPM scheme.

This means that

$$a(t) = g_{\text{RPAM}}(t) \quad (15)$$

An alternative serial type of receiver has recently been considered for MSK [6]-[12]. It is claimed [6]-[9], [11] that this receiver has advantages in implementation and is less sensitive to phase errors. In [8] the optimum serial MSK receiver is derived for MSK signals. This receiver is shown in figure 1b, where the optimum filters for an MSK signal are given by

$$\begin{cases} h_1(t) = \begin{cases} \cos^2 \frac{\pi t}{2T} & ; \quad |t| \leq T \\ 0 & ; \quad \text{otherwise} \end{cases} \\ h_2(t) = \begin{cases} -\frac{1}{2} \sin \frac{\pi t}{T} & ; \quad |t| \leq T \\ 0 & ; \quad \text{otherwise} \end{cases} \end{cases} \quad (16)$$

These impulse responses are shown in figure 2. Perfect synchronization in figure 1b corresponds to $\Phi=0$. The difference from the parallel receiver is the demodulation frequency (IF). In the serial receiver the signal is demodulated with the frequency $f_1 = f_0 - \frac{1}{4T}$. This frequency corresponds to the transmitted frequency, for the transmitted data sequence $\dots, -1, -1, -1, \dots$, i.e. the all -1 sequence. Thus the signals in the quadrature arms of the serial receiver are $\cos\varphi_1(t, \underline{\alpha})$ and $\sin\varphi_1(t, \underline{\alpha})$ where

$$\varphi_1(t, \underline{\alpha}) = \varphi(t, \underline{\alpha}) + \frac{\pi t}{2T} \quad (17)$$

This is equivalent to the unfiltered quadrature signals for a parallel receiver, when the transmitted signal has the phase function $\varphi_1(t, \underline{\alpha})$. Figure 3 shows the phase tree for this scheme, i.e. the ensemble of $\varphi_1(t, \underline{\alpha})$. Note that a transmitted -1 now corresponds to a constant phase trajectory. The quadrature signals are then filtered and added. The detection eye for MSK is shown in figure 4. This eye pattern should be compared with the corresponding eye pattern for a parallel MSK receiver given in figure 5. The advantage for the serial eye is the opened eye at every symbol interval. This means that all the symbols can be detected from this eye. The eye opening is also of the same size, and therefore the performance for these two receivers are equal. Note however that the serial eye is more involved than the parallel eye.

It is easily seen that the filters $h_1(t)$ and $h_2(t)$ in eq. (16) can be derived from the MSK-filter for the parallel receiver in eq. (12). The relation is given by

$$\begin{cases} h_1(t) = a(t) \cdot \cos \frac{\pi t}{2T} \\ h_2(t) = -a(t) \cdot \sin \frac{\pi t}{2T} \end{cases} \quad (18)$$

where $a(t)$ is the filter used in the parallel receiver. Therefore we suggest to use this serial receiver also for partial response schemes, where the filters are chosen according to eq. (18) and where $a(t)$ is the corresponding filter for the parallel receiver given in eq. (12) and eq. (15). As said before, the quadrature signals are equivalent to the quadrature signals of the parallel receiver for a signal with phase $\varphi_1(t, \underline{\alpha})$ according to eq. (17). The corresponding phase tree for $\varphi_1(t, \underline{\alpha})$ is given in figure 6 for 3RC.

Finally two detection eyes are given in figure 7 and 8. Figure 7 shows the eye pattern for the serial receiver to 2RC, based on the MSK filter. This receiver will in the following be referred to as SMSK, where the first S is for serial. In the same manner the parallel receiver will be referred to as PMSK. Correspondingly the SPAM filters will be called PSPAM and SSPAM respectively. The type of filters will as before be called MSK and SPAM. Figure 8 shows the eye pattern for 3RC with a SMSK

receiver. Note that these eyes are more involved than the corresponding parallel eyes, but they are opened every T seconds. Thus decisions of all the transmitted data symbols can be made from this eye, compared to the parallel receiver where the decisions are made alternately from two different eyes.

III. PERFORMANCE ANALYSIS

In this section, the minimum Euclidean distance and an exact expression of the error probability in estimating the phase node ϑ_n for the coherent serial MSK-type receiver, described in section II, are derived. Note that the phase node ϑ_n for the serial receiver is defined in the phase tree given by the ensemble of phases $\varphi_1(t, \underline{\alpha})$. Thus ϑ_n takes only two values, i.e. 0 and π .

We assume zero-mean additive white Gaussian noise and perfect synchronization and timing in the receiver. The receiver observes the signal over N_T symbol intervals and makes an optimum decision on one symbol. Thus it is easily seen that the decision variable λ is a Gaussian random variable. Therefore its mean and variance determine the error probability.

Assume the sequence $\underline{\alpha}_j$ is transmitted, where index j is an enumeration of the sequences. From eq. (1), the transmitted signal can be written

$$s(t, \underline{\alpha}_j) = \sqrt{\frac{2E_b}{T}} \cos \left[\left(\omega_0 - \frac{\pi}{2T} \right) t + \varphi(t, \underline{\alpha}_j) + \frac{\pi t}{2T} \right] \quad (19)$$

where $\omega_0 = 2\pi f_0$.

Using trigonometric formulas this can be rewritten

$$\begin{aligned} s(t, \underline{\alpha}_j) = \sqrt{\frac{2E_b}{T}} & \left[\cos \left[\varphi(t, \underline{\alpha}_j) + \frac{\pi t}{2T} \right] \cos \left(\omega_0 - \frac{\pi}{2T} \right) t - \right. \\ & \left. - \sin \left[\varphi(t, \underline{\alpha}_j) + \frac{\pi t}{2T} \right] \sin \left(\omega_0 - \frac{\pi}{2T} \right) t \right] \end{aligned} \quad (20)$$

Using common techniques [13], the noise can be written

$$\begin{aligned} n(t) &= \sqrt{2} \, n_c(t) \cos \omega_0 t - \sqrt{2} \, n_s(t) \sin \omega_0 t = \\ &= \sqrt{2} \left[n_c(t) \cos \frac{\pi t}{2T} - n_s(t) \sin \frac{\pi t}{2T} \right] \cos \left(\omega_0 - \frac{\pi}{2T} \right) t - \\ &- \sqrt{2} \left[n_c(t) \sin \frac{\pi t}{2T} + n_s(t) \cos \frac{\pi t}{2T} \right] \sin \left(\omega_0 - \frac{\pi}{2T} \right) t \end{aligned} \quad (21)$$

where

$$E\{n_c(t)\} = E\{n_s(t)\} = 0 \quad (22)$$

$$E\{n_c(\tau_1)n_c(\tau_2)\} = E\{n_s(\tau_1)n_s(\tau_2)\} = \frac{N_0}{2} \delta(\tau_1 - \tau_2) \quad (23)$$

and

$$E\{n_c(\tau_1)n_s(\tau_2)\} = E\{n_s(\tau_1)n_c(\tau_2)\} = 0 \quad (24)$$

$E\{\cdot\}$ denotes expectation and $\delta(t)$ is the Dirac's delta function.

Assuming $t=0$, and using eq. (18) for $h_1(t)$ and $h_2(t)$ gives

$$\begin{aligned} E\{\lambda_j\} &= \sqrt{\frac{E_b}{2T}} \int_I \left[\cos\left[\varphi(t, \underline{\alpha}_j) + \frac{\pi t}{2T}\right] \cos \frac{\pi t}{2T} a(-t) + \right. \\ &\quad \left. + \sin\left[\varphi(t, \underline{\alpha}_j) + \frac{\pi t}{2T}\right] \sin \frac{\pi t}{2T} a(-t) \right] dt = \\ &= \sqrt{\frac{E_b}{2T}} \int_I \cos\varphi(t, \underline{\alpha}_j) a(-t) dt \end{aligned} \quad (25)$$

where I denotes the observation interval. This is equal to the expected value of the decision variable for the parallel receiver. This means that the eye opening of the two different receivers are equal. Compare with the eye diagrams in figure 7 and 8.

The variance $\text{Var}\{\lambda_j\} = E\{(\lambda_j - E\{\lambda_j\})^2\}$ is independent of any particular transmitted sequence and from figure 1b it is seen to be

$$\begin{aligned} \text{Var}\{\lambda_j\} &= \frac{1}{2} E\left\{ \int_I \int_I \left[\left(n_c(x) \cos \frac{\pi x}{2T} - n_s(x) \sin \frac{\pi x}{2T} \right) a(-x) \cos \frac{\pi x}{2T} - \right. \right. \\ &\quad \left. \left. - \left(n_c(x) \sin \frac{\pi x}{2T} + n_s(x) \cos \frac{\pi x}{2T} \right) a(-x) \sin \frac{\pi x}{2T} \right] \cdot \right. \\ &\quad \cdot \left[\left(n_c(y) \cos \frac{\pi y}{2T} - n_s(y) \sin \frac{\pi y}{2T} \right) a(-y) \cos \frac{\pi y}{2T} - \right. \\ &\quad \left. \left. - \left(n_c(y) \sin \frac{\pi y}{2T} + n_s(y) \cos \frac{\pi y}{2T} \right) a(-y) \sin \frac{\pi y}{2T} \right] dx dy \right\} \end{aligned} \quad (26)$$

Using eq. (22), eq. (23), eq. (24) and trigonometric formulas give

$$\text{Var}\{\lambda_j\} = \frac{N_0}{4} \int_I a^2(t) dt \quad (27)$$

where as before the integral is over the observation interval. This is also equal to the variance for the parallel receiver. Thus, the normalized squared distance, defined by

$$d_j^2 = \frac{E\{\lambda_j\}^2}{\text{Var}\{\lambda_j\}} \cdot \frac{N_0}{E_b} \quad (28)$$

is equal for the two different receivers. This is not a Euclidean distance in signal space, but from an error probability point of view for this receiver it plays the role of a Euclidean distance. Then, also the minimum normalized squared distance is equal, and also given by the same transmitted sequence. This minimum distance is given by

$$d^2 = \min_j \left\{ \frac{2}{T} \frac{\left[\int_I \cos\varphi(t, \alpha_j) a(-t) dt \right]^2}{\int_I a^2(t) dt} \right\} \quad (29)$$

The error probability in estimating the phase node Φ_n is then obtained, by averaging over all possible transmitted waveforms, as a function of the signal to noise ratio E_b/N_0 .

$$P = \frac{1}{m} \sum_{j=1}^m Q \left(\sqrt{d_j^2 \frac{E_b}{N_0}} \right) \quad (30)$$

where m is the number of different transmitted waveforms. E_b is the energy per bit and equals the energy per transmitted symbol E in the binary schemes considered in this paper. N_0 is the one-sided spectral density of the additive white noise. The function $Q(x)$ is the Gaussian error function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} dy \quad (31)$$

This means that, assuming perfect carrier recovery and timing, the performance of the parallel and serial receiver are identical when the filters are chosen according to eq. (18). Now it is also easily verified that

these are the optimum filters for the serial receiver, if $a(t)$ is chosen to be the optimum filter for the corresponding parallel receiver. Using trigonometric formulas on the demodulating frequency in figure 1b and re-ordering the terms gives the receiver of figure 9. This is the block diagram of a parallel receiver. For this parallel receiver it is known that the optimum receiver filter is $a(t)$. From figure 9 it is seen that this leads to

$$\cos \frac{\pi t}{2T} h_1(t) - \sin \frac{\pi t}{2T} h_2(t) = a(t) \quad (32)$$

It is seen that this equation is satisfied by the expressions for $h_1(t)$ and $h_2(t)$ in eq. (18). By this choice it is also seen that the signal in the other quadrature arm is zero, just as for the parallel receiver. However it is possible that the above equation has more solutions. This means that by choosing $h_1(t)$ and $h_2(t)$ according to eq. (18), where $a(t)$ is the optimum filter for the parallel receiver, the optimum filters for the serial receiver are found. Thus the optimum serial and optimum parallel receiver perform equal under perfect synchronization and timing conditions.

Distance calculations for a variety of schemes for the parallel receiver have been made previously and are presented in part II, [3], [4]. From the above it is clear that all these results are also valid for the serial receiver.

IV. SENSITIVITY TO PHASE ERRORS

In this section the performance degradation for a serial receiver due to phase errors will be derived. These results will be compared with the corresponding results for the parallel receiver.

Throughout this section perfect time synchronization will be assumed. Various authors have concluded that a serial receiver to MSK is less sensitive to phase errors than a parallel receiver [7], [11]. If a phase error is present, i.e. $\Phi \neq 0$ in figure 1, this phase error can as well be included in the transmitted phase. In the previous section however, it is shown that for a specific transmitted phase, the performance for the two receivers are identical, whatever phase is being transmitted. Note that the decision variable is Gaussian also when a phase error is present, therefore the previous analysis still holds. This seems to be a contradiction compared to the results quoted above. This is however not the case. Figure 10a and 10b show the detection eyes for a serial receiver to MSK when the phase error is -10 and -45 degrees. These detection eyes for a parallel MSK receiver are shown in figure 11a and 11b. From these figures it is seen that the level of the eye is equal for a specific phase error for the parallel and serial receiver, if the sampling is done according to figure 1. This sampling time is still optimum for the parallel receiver. For the serial receiver however, it is not optimum. By changing the sampling time to the time when the eye opening is as large as possible the performance for the serial receiver is optimized for large signal-to-noise ratios, i.e. the minimum distance is optimized. In general however the optimum sampling time will depend on the error probability for which the optimization is desired. By the above choice however, the error probability will be close to optimum for all practical signal-to-noise ratios. Note that the eye is symmetrical around the ideal sampling time due to phase errors Φ . Thus the change in sampling time is negative for a negative phase error, while it is easily seen to be positive with the same absolute value for a positive phase error. The eye diagrams shown are given for negative Φ .

What is the relationship between the phase error Φ and the change of sampling time ΔT_0 ? It can be verified that the largest opening in the eye of MSK is obtained for the time instant t when the trajectories for the transmitted sequences $\underline{\alpha}_1 = -1, -1, -1, \dots$ and $\underline{\alpha}_2 = 1, 1, 1, \dots$ respectively are equal.

Let us consider the sampling time normally taken place at $t=0$. Then the trajectory in the eye diagram corresponding to the transmitted sequence $\underline{\alpha}_1 = -1, -1, -1, \dots$ is

$$y(t, \underline{\alpha}_1) = \int_{-T+t}^{T+t} \left[\cos(\Phi) \cos^2 \frac{\pi(t-\tau)}{2T} + \sin(\Phi) \frac{1}{2} \cdot \sin \frac{\pi(t-\tau)}{T} \right] d\tau \quad (33)$$

Using trigonometric formulas and integration leads to

$$y(t, \underline{\alpha}_1) = T \cos \Phi \quad (34)$$

Thus this is the straight line seen in the eye diagram.

In the same way the trajectory corresponding to $\underline{\alpha}_2 = 1, 1, 1, \dots$ is seen to be

$$y(t, \underline{\alpha}_2) = \int_{-T+t}^{T+t} \left[\cos\left(\frac{\pi\tau}{T} - \Phi\right) \cos^2 \frac{\pi(t-\tau)}{2T} - \sin\left(\frac{\pi\tau}{T} - \Phi\right) \frac{1}{2} \sin \frac{\pi(t-\tau)}{T} \right] d\tau \quad (35)$$

Again using trigonometric formulas and integration leads to

$$y(t, \underline{\alpha}_2) = T \cos\left(\frac{\pi t}{T} - \Phi\right) \quad (36)$$

ΔT_0 is the time when these two trajectories cross each other, i.e. when

$$\frac{\pi \Delta T_0}{T} + \Phi = \pm \Phi \quad (37)$$

The solutions to this equation are

$$\begin{cases} \Delta T_0 = 0 \\ \Delta T_0 = \frac{2\Phi}{\pi} \cdot T \end{cases} \quad (38)$$

The solution giving the largest eye opening is the second in (38) i.e.

$$\Delta T_0 = \frac{2\Phi}{\pi} \cdot T \quad (39)$$

Thus the change in sampling time is directly proportional to the phase error.

Now the performance of the serial MSK receiver can be compared with the performance of the parallel MSK receiver. It is assumed that the sampling time is chosen optimum in both cases. The performance analysis in the previous section however has to be slightly changed. The decision variables are still Gaussian, but the sampling time is changed. It is easily verified that the distance is given by

$$d_j^2 = \frac{2}{T} \frac{\left[\int_I \cos[\phi(t, \underline{\alpha}_j) - \Phi] a(\Delta I_0 - t) dt \right]^2}{\int_I a^2(t) dt} \quad (40)$$

where $\Delta I_0 = 0$ for the parallel receiver and $\Delta I_0 = \frac{2\Phi}{\pi} \cdot T$ for the serial receiver.

These distances can be used in eq. (30) to calculate the error probability in detecting the phase node sequence when a phase error is present. Figure 12a and 12b show the results for 1REC (MSK) with a serial and a parallel receiver, respectively.

The results for MSK in figure 12a and 12b show that the parallel MSK receiver is much more sensitive to phase errors. With a phase error of 20 degrees the degradation for the serial receiver is about 0.5dB at $P=10^{-6}$, while it is 2.3dB for the parallel receiver. To obtain this large degradation in the serial receiver a phase error as large as 40 degrees can be present.

Does this behaviour remain for transmitted partial response schemes with the MSK, SPAM and asymptotically optimum receivers described in the previous section? Figure 13 shows the detection eye for 3RC with a SMSK receiver when the phase error is -5 degrees and the corresponding detection eye for the PMSK is given in figure 14. It is seen that these receivers for 3RC behave just as the receivers for 1REC. However to find the optimum sampling time in the serial eye is not easy. From measurements in the eye however, we conjecture that the asymptotic optimum is still according to eq. (39), i.e. $\Delta I_0 = \frac{2\Phi}{\pi} \cdot T$, but this has not been verified. Thus in the following this change in sampling time will be used for the serial receivers.

This behaviour for the serial receiver is remained also for other type of receiver filters. Error probabilities for various values of Φ have been calculated and will now be presented. Figure 15a and 15b show the results for transmitted 3RC received in an asymptotically optimum receiver of serial and parallel type, respectively. The observation length is $N_T=9$. For a phase error of 20 degrees the degradation at $P=10^{-6}$ for the serial receiver is about 0.7dB, while it is about 4.7dB for the parallel receiver. Thus the degradation for a given phase error for the serial receiver is only slightly larger than for the 1REC scheme, while the degradation has increased for the parallel receiver compared to 1REC. More results of this type is found in [5].

Figure 16 and 17 summarize the degradation due to phase errors for various schemes. In these figures the degradation in dB relative to BPSK and QPSK is given versus the phase error Φ . The degradations are given at an error probability equal to 10^{-6} in figure 16 and also the asymptotic degradation based on $d_{\min}^2$ in figure 17.

In appendix 1 the bit error probabilities for BPSK and QPSK are derived. It is shown that the bit error probability for BPSK, when a phase error is present, is given by

$$P_{bBPSK} = Q\left(\sqrt{\frac{2E_b}{N_0}} \cdot \cos\Phi\right) \quad (41)$$

where $Q(x)$ is the Gaussian error function given in eq. (31). The bit error probability for QPSK, with a phase error present is given by

$$P_{bQPSK} = \frac{1}{2} \left[Q\left(\sqrt{\frac{4E_b}{N_0}} \cos\left(\frac{\pi}{4} - \Phi\right)\right) + Q\left(\sqrt{\frac{4E_b}{N_0}} \sin\left(\frac{\pi}{4} - \Phi\right)\right) \right] \quad (42)$$

It can easily be concluded that QPSK is much more sensitive to phase errors than BPSK.

First the results at $P=10^{-6}$ are given. Figure 16a shows the results for various schemes with MSK receivers, both serial and parallel, while figure 16b shows the corresponding results with SPAM receivers. In this figure the degradation for the asymptotically optimum 3RC receivers with $N_T=9$ is also given. In both figures the degradation for BPSK and QPSK are given as comparison. It is seen that the serial and parallel receivers with the same type

of filter have the same performance when $\Phi=0$. For larger phase errors however the serial receiver is superior. There is also another advantage for the serial receiver seen in the figures. For the serial receiver the relative degradation is nearly independent of the length in symbol intervals of the transmitted pulse, at least for reasonable short pulses, i.e. $L \leq 4$. For the parallel receiver, on the other hand, the relative degradation is much worse for the longer pulses than for shorter pulses.

For the 1FM5 and 3SRC6 schemes with a serial receiver the degradation results seem a bit strange. This is due to the frequency response pulse shape. The frequency response of these schemes are not strictly positive. They have a $\sin(x)/x$ shape outside the mainlobe which is seen in the eye diagram. The degradation varies of course smoothly, but it is only calculated in a limited number of points and these points are connected with straight lines. These schemes are also more sensitive to phase errors than the RC-schemes. Compare for example the degradations for 3RC and 3SRC6 with SSPAM receivers.

Figure 17a and 17b show the asymptotic degradation due to phase errors. The above conclusions do still hold. The only difference is that the degradation at $\Phi=0$ is larger for the schemes based on longer pulses than it is at $P=10^{-6}$. This will of course influence the degradation also when a phase error is present. The relative degradation due to the phase error however, is about the same as at $P=10^{-6}$ for the various schemes.

By comparing the graphs for MSK and SPAM receivers it can be concluded that the SPAM-type of receiver is more sensitive to phase errors than the MSK-type of receiver both in the parallel and the serial version.

The only results for asymptotically optimum receivers are for 3RC. These results show that the degradation at 10^{-6} is of the same order for these filters as for the SPAM filters. Asymptotically however, it is seen that the degradation for the asymptotically optimum serial receiver is slightly larger than for the SSPAM receiver. For the parallel receiver on the other hand, the degradation is of the same order as for the PSPAM receiver. No results are given for other schemes, but it is expected that the optimum receivers to the other schemes have the same relative degradation. From part II the performance, assuming perfect synchronization, is known and then the degradation is expected to be of the same order as for the SPAM receivers, except perhaps for the asymptotic degradation for the serial receiver. This

might be somewhat larger than for the SSPAM receiver. However in practice this is of minor interest. It is also known that the optimum filters are of the same type as the SPAM filters and therefore it seems natural that the degradation due to phase errors are related to each other.

It can be concluded that, assuming a perfect timing, the sensitivity to phase errors are much smaller for the serial receiver than for the parallel receiver. The SMSK receiver is also less sensitive to phase errors than the SSPAM, but the difference is small. For the parallel receiver however, the PSPAM receiver is more sensitive to phase errors. Note that the SPAM-type of receiver has usually its advantages for smaller error probabilities than 10^{-6} [3], [4].

V. SENSITIVITY TO TIMING ERRORS

In this section we will concentrate on the degradation due to timing errors. Assuming, for a moment, that no phase error is present it is easily seen from the eye diagrams in figure 4, 5, 7 and 8 that the serial receiver is more sensitive to timing errors. For the serial receiver the eye is opened every T seconds and therefore the eye is more narrow than the parallel eye, which is opened every second T only. The error probability can be calculated also for this case. The decision variables are still Gaussian and eq. (40) for the distances does still hold. When no phase error is present, Φ is equal to zero.

For comparisons the sensitivity to timing errors for BPSK is calculated in appendix 2. The bit error probability for BPSK, with a timing error ΔT and a phase error Φ is given by

$$P_b = \frac{1}{2} \left[Q \left(\sqrt{\frac{2E_b}{N_0}} \cos \Phi \right) + Q \left(\sqrt{\frac{2E_b}{N_0}} \cos \Phi \left(1 - 2 \left| \frac{\Delta T}{T} \right| \right) \right) \right] \quad (43)$$

It can be shown that with $\Phi=0$ this result is also valid for QPSK, since the quadrature detection eyes for QPSK are equal to the detection eye for BPSK, when $\Phi=0$. It can be concluded that BPSK is more sensitive to timing errors than to phase errors. Figure 18a and 18b show the error probabilities for 1REC with a SMSK and PMSK receiver respectively, for various values of $\frac{\Delta T}{T}$. For a relative timing error of 0.2 the degradation at 10^{-6} for the PMSK receiver is about 0.4dB but for the SMSK receiver it is about 2.6dB. The degradation for BPSK is about 4.2dB as comparison [5]. The same type of results can be obtained for the RC-schemes with different receivers. It can be concluded that the parallel receiver is superior when the bit time synchronization is not perfect.

Finally some results for a combined phase error and timing error will be given for the different schemes. Figure 19 and 20 show the degradation relative to BPSK/QPSK for a phase error of 10 degrees, versus a relative timing error $\frac{\Delta T}{T}$. Figure 19a and 19b show the degradation at $P=10^{-6}$ and figure 20a and 20b the asymptotic degradation. Note that the serial eye is not symmetric around the asymptotically optimum sampling time, therefore the degradation results are not symmetric for this receiver. The results for 1REC given in figure 18a are the worst case results, i.e. the degradation due to a negative timing error of the same absolute value is smaller than the result in figure 18.

Figure 19a shows the results for MSK receivers to different transmitted schemes. The degradation for BPSK is given as comparison. In figure 19a it is easily seen that the degradation for the SMSK receiver is much larger than for the PMSK receiver. The relative degradation for the SMSK receiver also gets worse for longer pulses, while it remains nearly constant for the PMSK receiver. This is just the opposite of the degradation due to phase errors.

From figure 19 it can be concluded that, given a phase error Φ the SMSK receiver is superior as long as the timing error is not too large. The region where the serial receiver is superior increases slightly for schemes based on longer pulses. For 1REC the region where the SMSK receiver performs better than the PMSK receiver at $P=10^{-6}$ is $-0.13 \leq \frac{\Delta T}{T} \leq 0.07$. The same region for 4RC is $-0.12 \leq \frac{\Delta T}{T} \leq 0.10$. If the maximum absolute relative timing error giving a better performance for the SMSK receiver is compared instead the result is 0.07 for 1REC and 0.10 for 4RC. In this case the difference is larger.

The degradations at $P=10^{-6}$ for the SPAM receiver are given in figure 19b. The degradation for the asymptotically optimum 3RC receiver with $N_T=9$ is also given. By comparing the MSK and SPAM receivers for a particular scheme it can be concluded that the SPAM receiver is more sensitive to timing errors especially the SSPAM receiver. This makes the regions, where the serial receiver performs better than the parallel, smaller.

Figure 20a and 20b show the asymptotic degradations for the serial and parallel receivers of MSK and SPAM type. The above results do still hold, with the difference that the degradation compared to BPSK/QPSK is larger asymptotically than the degradation at $P=10^{-6}$. This shows that the receiver performs better than the minimum distance tells us, for low and intermediate signal-to-noise ratios.

The only results for asymptotically optimum receivers are for 3RC. From the figures it is concluded that the relative degradation due to timing errors for the optimum receiver is of the same order as for the SPAM receivers. The only difference is the asymptotic degradation for the parallel asymptotic receiver, which has a slightly larger relative degradation than the PSPAM receiver. Since the optimum filters are of the same type as the SPAM filters, it is expected that the above conclusions for the optimum 3RC receiver also holds for optimum receivers to other schemes.

The results in figure 20a and 20b also supports the conjecture made in section IV about the asymptotically optimum sampling time for the serial receiver. If the sampling time is not optimum a small timing error would give a better performance and this is not the case for the schemes considered in this paper.

VI. DISCUSSION AND CONCLUSIONS

A serial receiver for CPM signals is considered in this part. This is a receiver for binary modulation schemes with modulation index $h=1/2$. The distance measure introduced in part II, is used for analysing the performance of the receiver. Coherent transmission on the additive white Gaussian noise channel is considered. The minimum distance then gives the asymptotic error probability behaviour. An exact expression for the error probability in detecting the phase node is also given. The error probability analysis is modified to include also receivers with phase errors and timing errors. The degradation due to phase errors and timing errors are given and compared with the degradation for the parallel receiver considered in part II [3], [4]. The transmitted scheme considered in this part are contained among the REC, RC, TFM and SRC schemes.

It is shown that the serial receiver performs equal to the parallel receiver under perfect synchronization conditions. One advantage is that all the decisions on the transmitted symbols can be made from one decision eye. Another advantage is in the sensitivity to phase errors. If the exact phase of the transmitted carrier is not recovered in the receiver, the bit error probability for the parallel receiver increases much faster with the phase error than it does for the serial receiver. If the bit time synchronization is not correct, however, the relation is the opposite. Assuming it is easier to obtain a good bit time synchronization than a correct carrier phase recovery, the serial receiver has advantages compared to the parallel receiver. The filters obtained in [3], [4] can be modified according to the results in this part, and used in the serial receiver. The optimum filters for the serial receiver are also found, and it is shown that they are easily calculated from the optimum filter for the parallel receiver. In this way it is possible to obtain schemes with combined very good spectral and detection properties. These schemes also have only a small degradation due to phase errors.

Previously it is shown that a differential encoder has to be employed for the parallel receiver to give the estimated data sequence from the phase node sequence. This is also the case for the serial receiver. However, if a burst mode transmission is used a unique synch word is used in the beginning of the burst, the serial receiver requires no differential encoding to resolve the data burst while the parallel receiver does. This is due to

the fact that $\alpha_n = -1$ does never change the phase node for the serial receiver while $\alpha_n = +1$ always changes the phase node. This is not true for the parallel receiver.

The serial MSK-type receiver has previously been studied for transmitted MSK [6]-[12]. It is shown in this paper that all the trends reported for serial MSK are also valid for serial partial response schemes.

The general problem of optimizing the choice of transmitter pulse and receiver filters for both the parallel and serial type of receiver remains unsolved. Since the optimum receiver filters are related to each other, it is enough to solve the problem for one type of receiver. The general optimization problem is difficult, since the optimum filter depends on the error probability level. If the joint optimization of transmitter pulse shape and receiver filter is to be solved the problem also must be formulated in terms of bandwidth.

There is also the issue of serial generation of $h=1/2$, binary partial response CPM. This problem is not addressed in this part. However, one approximate technique is to use BPSK plus a suitable bandpass filter followed by a hard limiter or a serial MSK unit [6], [12] (which is a filtered BPSK signal) followed by a linear filter and a hard limiter. For details about similar schemes and comparisons to ideal CPM schemes, see [16].

APPENDIX 1

Derivation of the bit error probabilities for BPSK and QPSK with phase errors

In this appendix, the bit error probabilities for BPSK and QPSK when the phase error in the receiver is Φ , will be derived.

We start with the easiest case, i.e. BPSK. The transmitted signal for BPSK is

$$s(t, \alpha_i) = \sqrt{\frac{2E_b}{T}} \cos(\omega_0 t + \varphi(t, \alpha_i)) \quad (A1.1)$$

where

$$\varphi(t, \alpha_i) = \begin{cases} 0 & \alpha_i = 1 \\ \pi & \alpha_i = -1 \end{cases} \quad iT \leq t < (i+1)T \quad (A1.2)$$

The receiver correlates the signal with $\cos \omega_0 t$ and integrates the correlation over one symbol interval T . Thus the decision variable λ is

$$\lambda = \frac{1}{T} \int_0^T r(t) \sqrt{2} \cos(\omega_0 t + \Phi) dt \quad (A1.3)$$

In signal space we have the signals in figure A1.1.

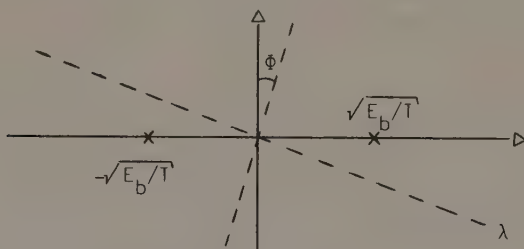


Figure A1.1. Signal space for BPSK.

Additive white Gaussian noise with one-sided spectral density N_0 is assumed. Thus it is easily seen that the variance of λ is

$$\text{Var}\{\lambda\} = \frac{N_0}{2T} \quad (A1.4)$$

If the phase error in the receiver is Φ , the decision boundary is rotated according to figure A1.1. From figure A1.1 it is easily verified that the bit error probability P_b is

$$P_b = \int_0^{\infty} \frac{1}{\sqrt{2\pi \frac{N_0}{2T}}} e^{-\frac{x_1 + \sqrt{\frac{E_b}{T}} \cos \Phi}{2 \frac{N_0}{2T}}} dx_1 = Q\left(\sqrt{\frac{2E_b}{N_0}} \cos \Phi\right) \quad (\text{A1.5})$$

where

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy \quad (\text{A1.6})$$

is the Gaussian error function.

In QPSK or four phase PSK there is four possible transmitted signals. QPSK uses two input bits at a time. Thus the transmitted signal is given by

$$s(t, \alpha_i) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \varphi(t, \alpha_i)) \quad (\text{A1.7})$$

where $\varphi(t, \alpha_i)$ takes one of the values $\frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$ or $\frac{7\pi}{4}$ over the i :th symbol interval and $E = 2E_b$.

In the signal space the signals in figure A1.2 are obtained.

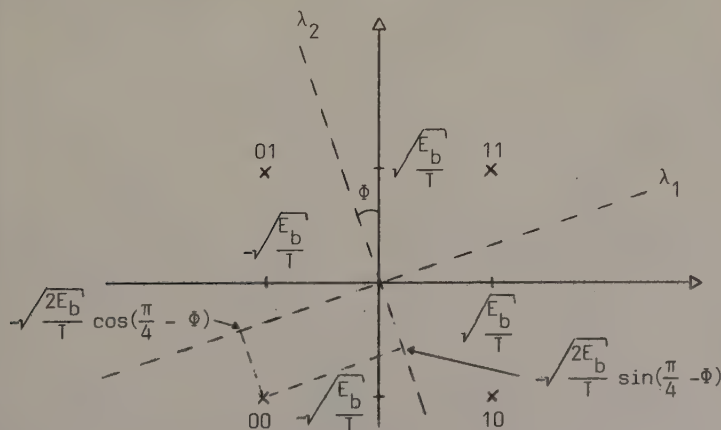


Figure A1.2. Signal space for QPSK.

Just as for BPSK the decision variables are obtained through correlation and integration and is given by

$$\begin{cases} \lambda_1 = \frac{1}{T} \int_0^T r(t) \sqrt{2} \cos(\omega_0 t + \Phi) dt \\ \lambda_2 = \frac{1}{T} \int_0^T r(t) \sqrt{2} \sin(\omega_0 t + \Phi) dt \end{cases} \quad (\text{A1.8})$$

With additive white Gaussian noise λ_1 and λ_2 are independent Gaussian random variables with variance

$$\text{Var}\{\lambda_1\} = \text{Var}\{\lambda_2\} = \frac{N_0}{2T} \quad (\text{A1.9})$$

and mean according to figure A1.2.

When there is a phase error in the receiver, the decision boundaries are changed according to figure A1.2. From figure A1.2 it is seen that the bit error probability is

$$\begin{aligned} P_b = & \frac{1}{2} \int_0^\infty \frac{1}{\sqrt{2\pi \frac{N_0}{2T}}} e^{-\frac{x_1 + \sqrt{\frac{2E_b}{T}} \cos\left(\frac{\pi}{4} - \Phi\right)}^2 \frac{N_0}{2T}} dx_1 \int_{-\infty}^0 \frac{1}{\sqrt{2\pi \frac{N_0}{2T}}} e^{-\frac{x_2 + \sqrt{\frac{2E_b}{T}} \sin\left(\frac{\pi}{4} - \Phi\right)}^2 \frac{N_0}{2T}} dx_2 \\ & + \frac{1}{2} \int_{-\infty}^0 \frac{1}{\sqrt{2\pi \frac{N_0}{2T}}} e^{-\frac{x_1 + \sqrt{\frac{2E_b}{T}} \cos\left(\frac{\pi}{4} - \Phi\right)}^2 \frac{N_0}{2T}} dx_1 \int_0^\infty \frac{1}{\sqrt{2\pi \frac{N_0}{2T}}} e^{-\frac{x_2 + \sqrt{\frac{2E_b}{T}} \sin\left(\frac{\pi}{4} - \Phi\right)}^2 \frac{N_0}{2T}} dx_2 \\ & + \int_0^\infty \frac{1}{\sqrt{2\pi \frac{N_0}{2T}}} e^{-\frac{x_1 + \sqrt{\frac{2E_b}{T}} \cos\left(\frac{\pi}{4} - \Phi\right)}^2 \frac{N_0}{2T}} dx_1 \int_0^\infty \frac{1}{\sqrt{2\pi \frac{N_0}{2T}}} e^{-\frac{x_2 + \sqrt{\frac{2E_b}{T}} \sin\left(\frac{\pi}{4} - \Phi\right)}^2 \frac{N_0}{2T}} dx_2 \end{aligned} \quad (\text{A1.10})$$

By a change of variables this is seen to be

$$P_b = \frac{1}{2} \left\{ Q \left(\sqrt{\frac{4E_b}{N_0}} \cos \left(\frac{\pi}{4} - \Phi \right) \right) + Q \left(\sqrt{\frac{4E_b}{N_0}} \sin \left(\frac{\pi}{4} - \Phi \right) \right) \right\} \quad (A1.11)$$

APPENDIX 2

Derivation of the bit error probability for BPSK with both phase errors and timing errors

In this appendix the bit error probability for BPSK, when the phase error in the receiver is Φ and the timing error is ΔT , will be derived.

From eq. (A1.3) in appendix 1 it is seen that the decision eye is given by the ensemble of $\lambda(t)$, where $\lambda(t)$ is given by

$$\lambda(t) = \sqrt{\frac{E_b}{T}} \int_t^{T+t} \cos(\varphi(\tau, \alpha_1) - \Phi) d\tau \quad (\text{A2.1})$$

This eye is given in figure A2.1.

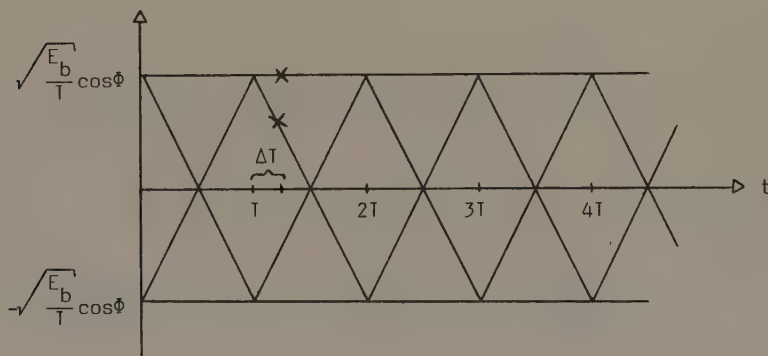
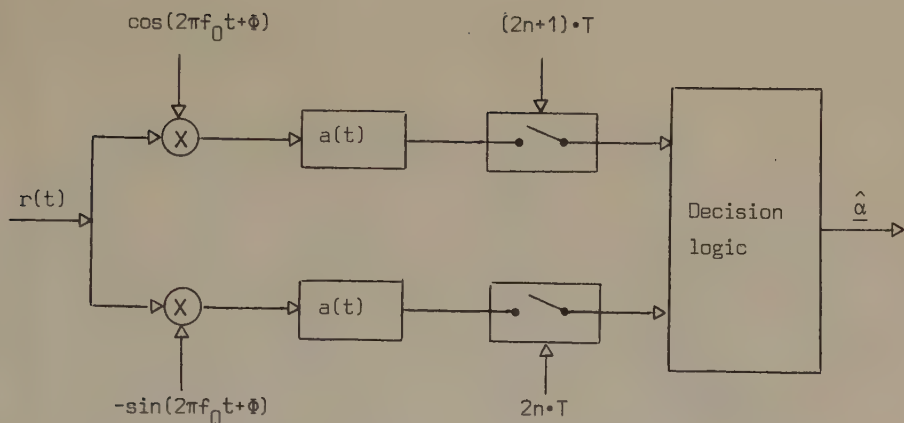


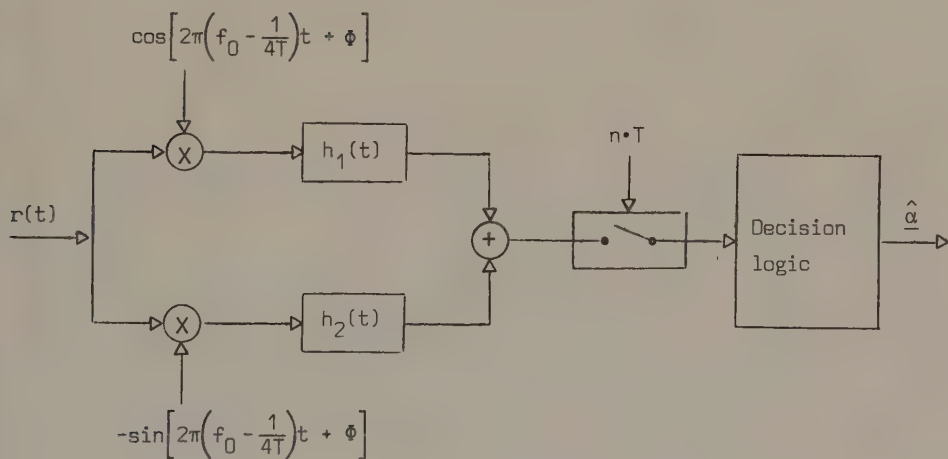
Figure A2.1. Detection eye for BPSK.

From figure A2.1 it is seen that when a timing error ΔT is present one distance is still $\sqrt{2E_b/N_0} \cos \Phi$, while the other is changed to $\sqrt{2E_b/N_0} \cos \Phi \left(1 - 2 \left| \frac{\Delta T}{T} \right| \right)$ where $\left| \frac{\Delta T}{T} \right|$ is the absolute value of $\frac{\Delta T}{T}$. Thus the bit error probability, with a phase error Φ and a timing error ΔT is given by

$$P_b = \frac{1}{2} \left\{ Q \left(\sqrt{\frac{2E_b}{N_0}} \cos \Phi \right) + Q \left(\sqrt{\frac{2E_b}{N_0}} \cos \Phi \left(1 - 2 \left| \frac{\Delta T}{T} \right| \right) \right) \right\} \quad (\text{A2.2})$$



a) Parallel



b) Serial

Figure 1. Receiver structure for MSK-type of receivers to CPM.

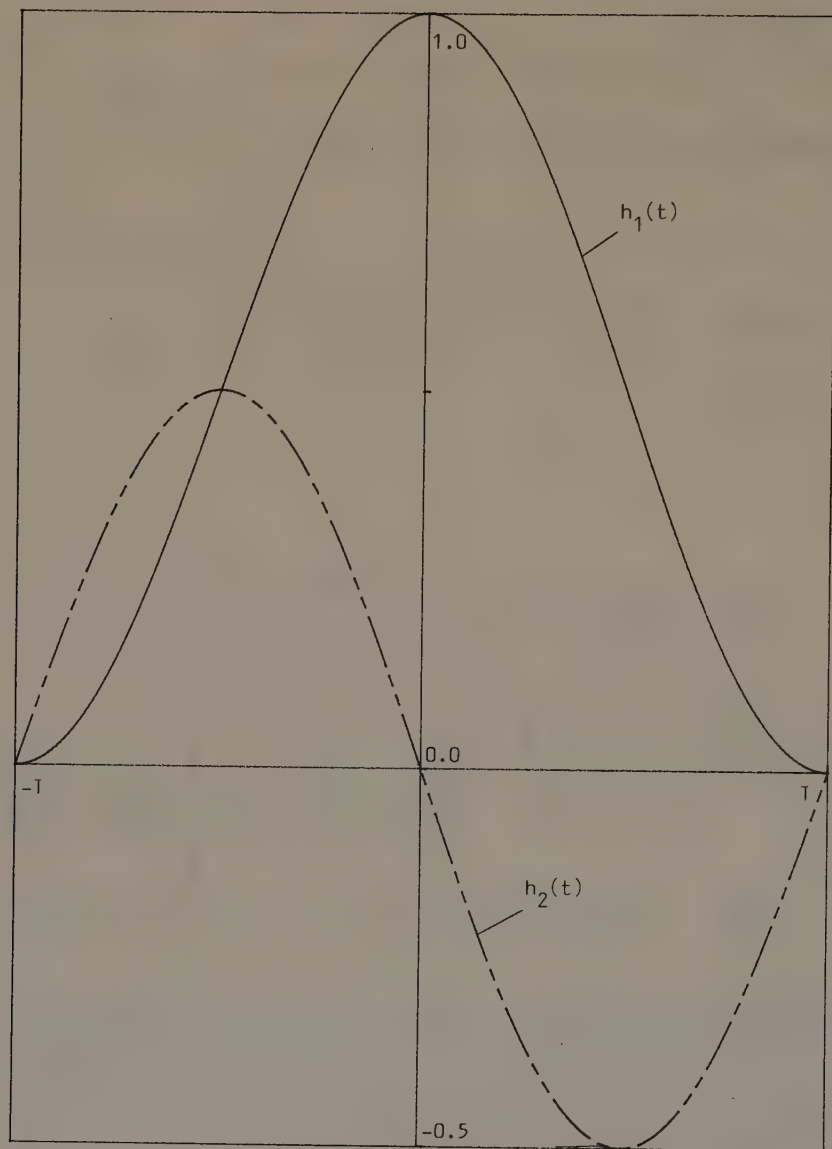


Figure 2. Impulse response of the optimum filters to a serial MSK receiver.

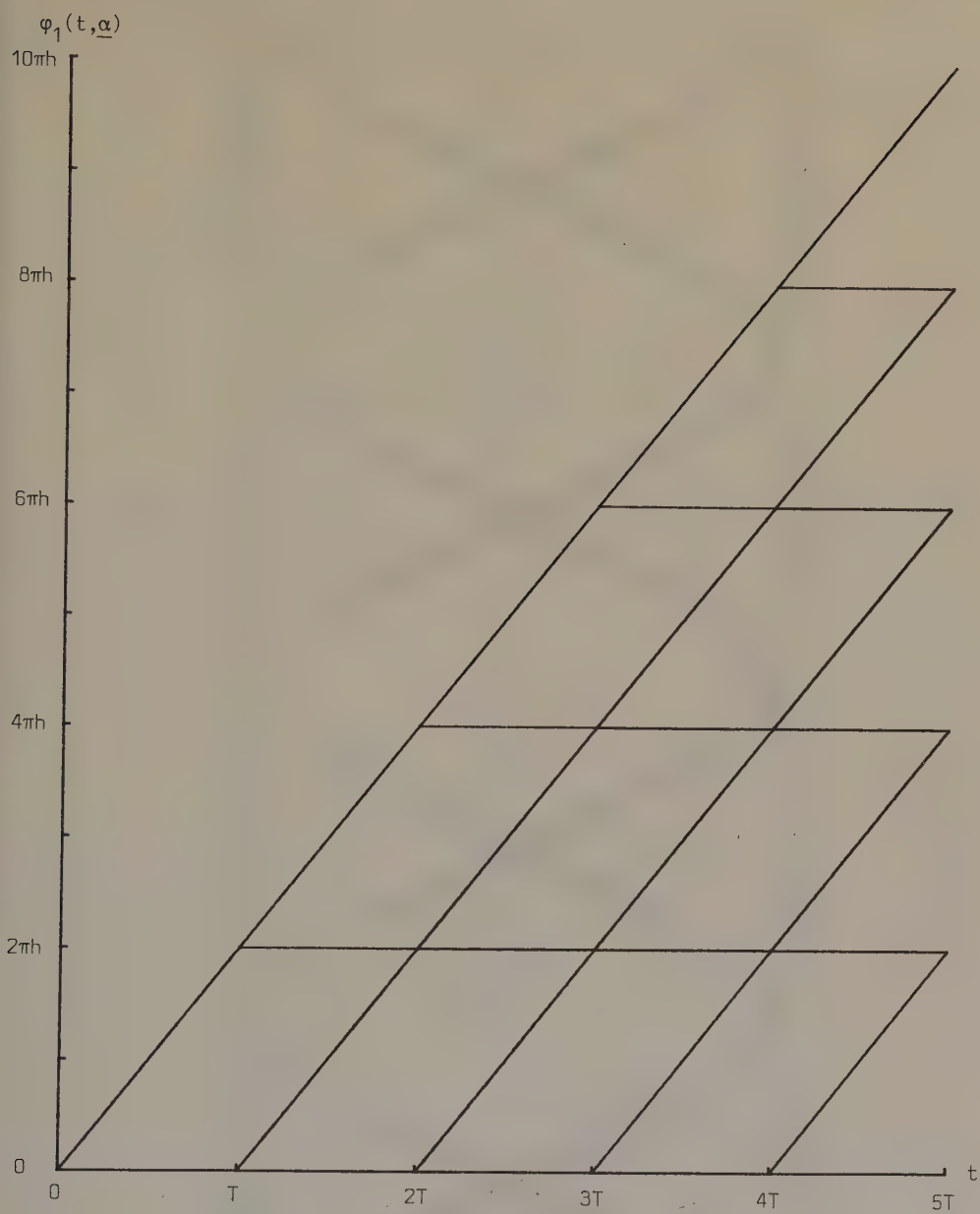


Figure 3. Phase tree for a serial receiver for 1REC.

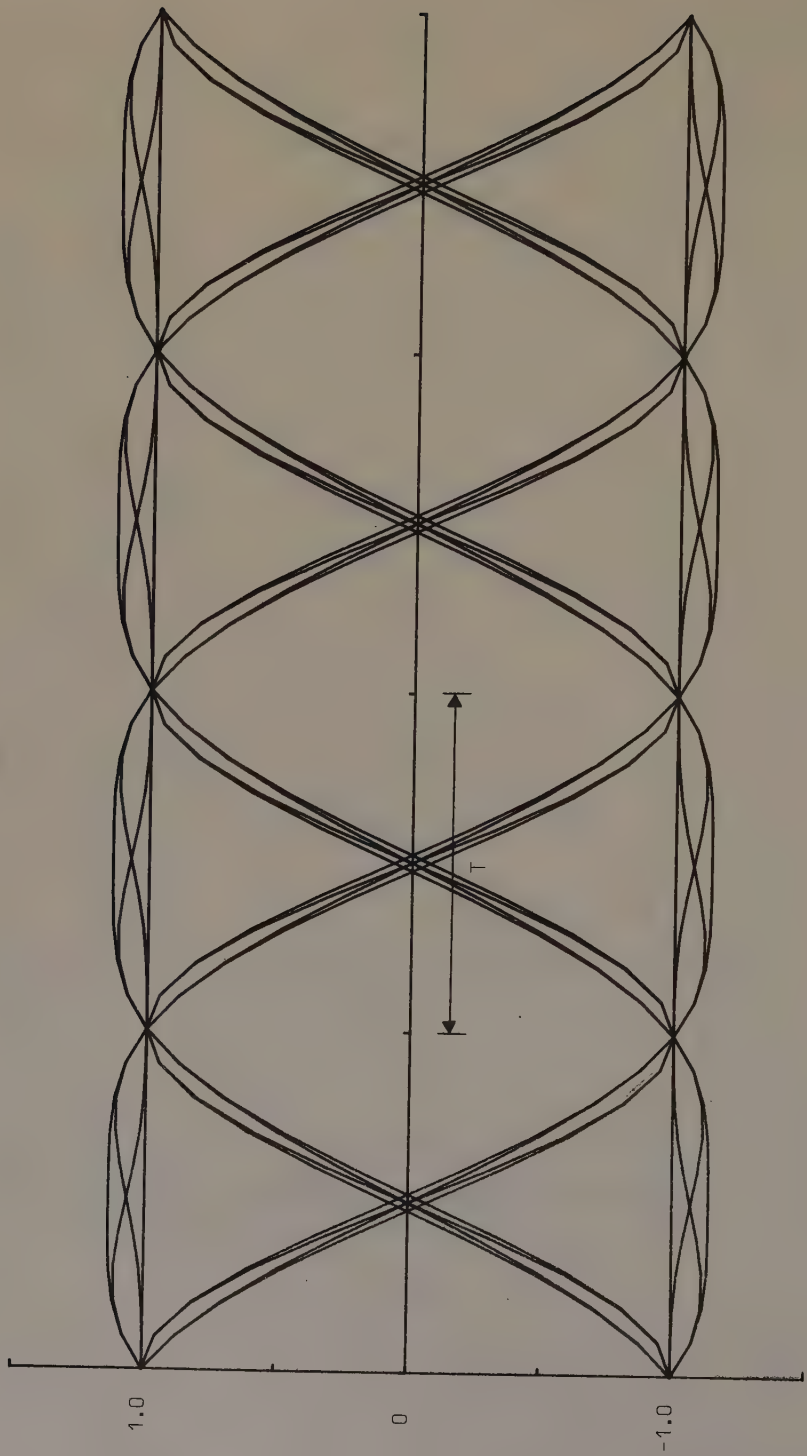


Figure 4. Decision eye for a SMSK receiver, for 1REC.

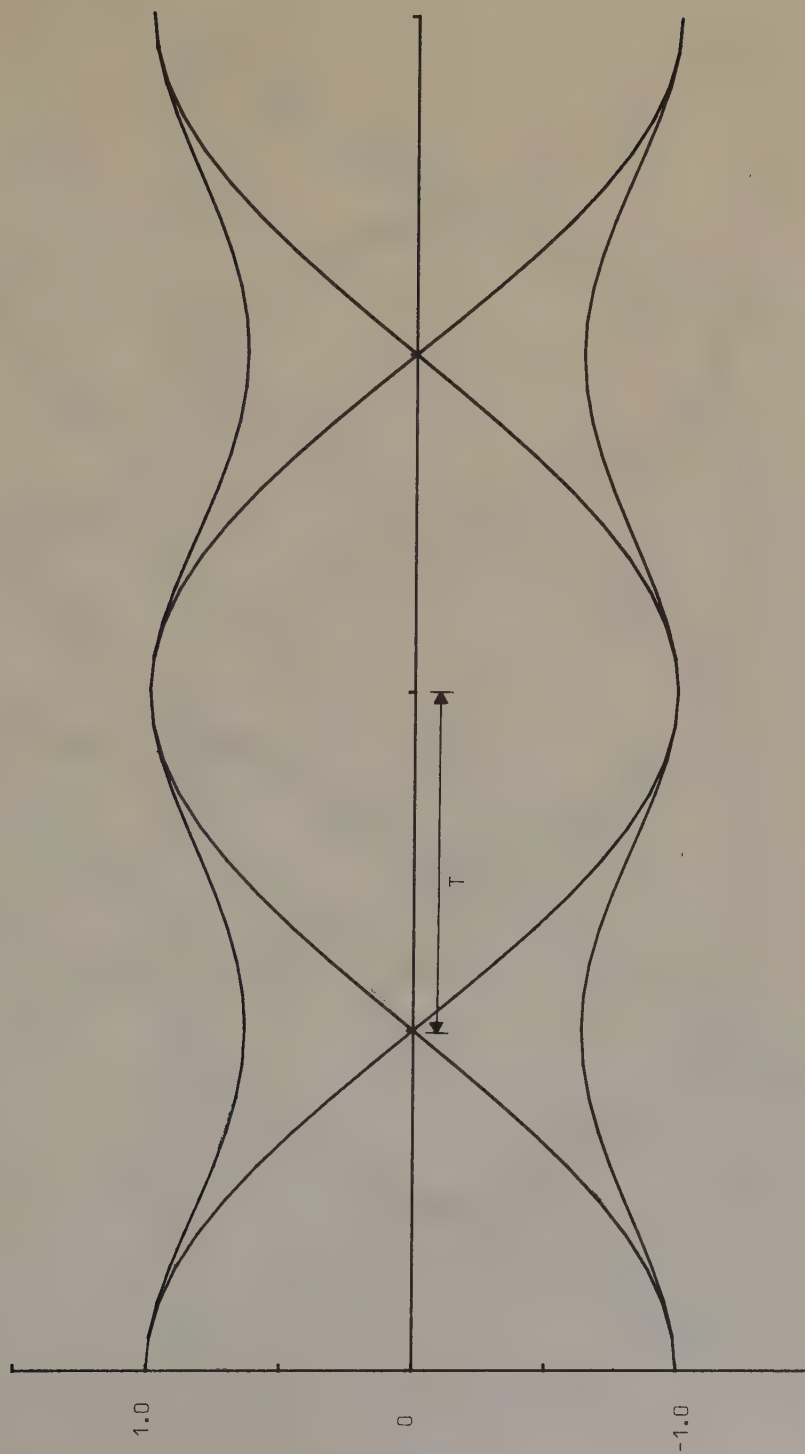


Figure 5. Decision eye for a PMSK receiver, for 1REC.

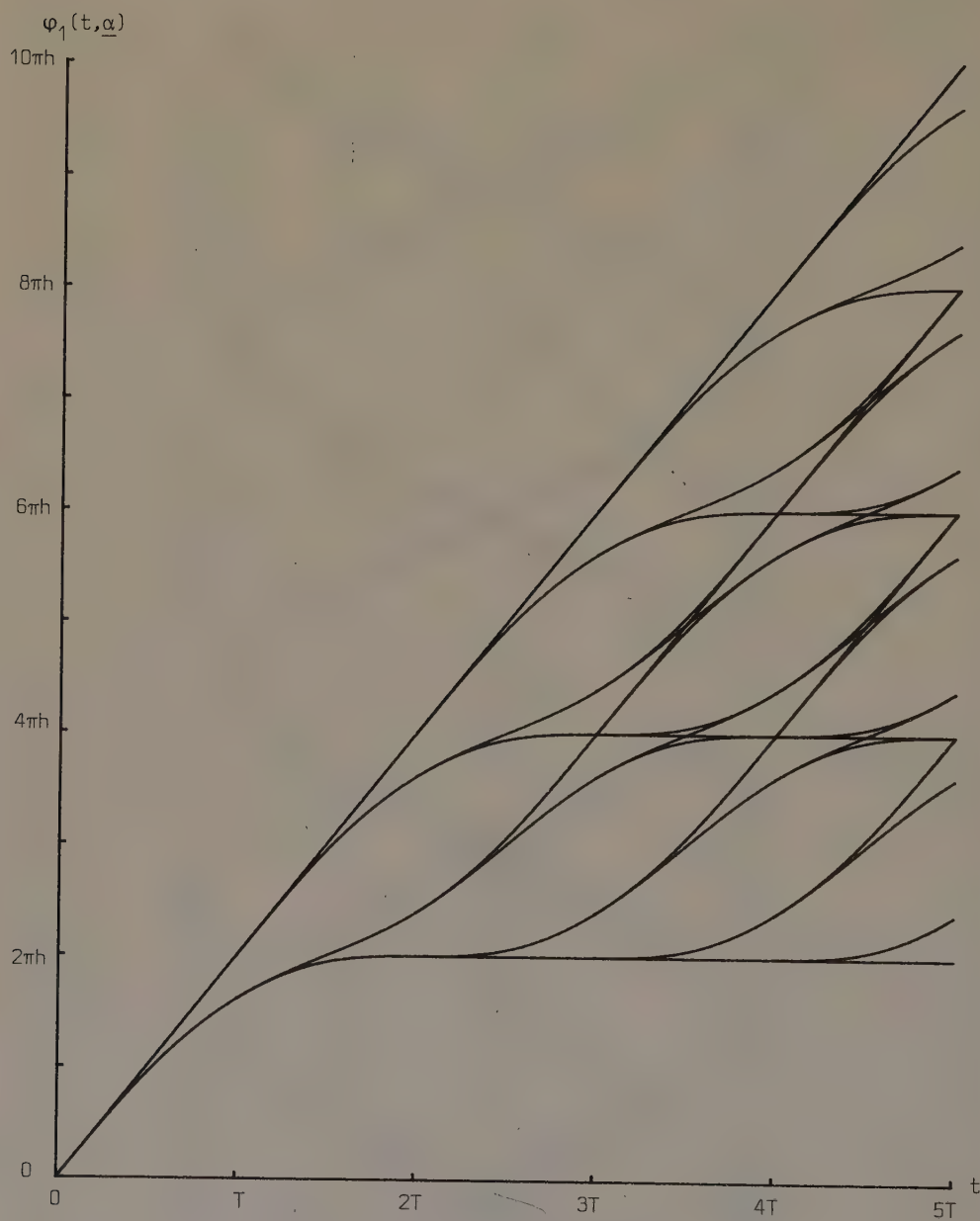


Figure 6. Phase tree for a serial receiver for 3RC.

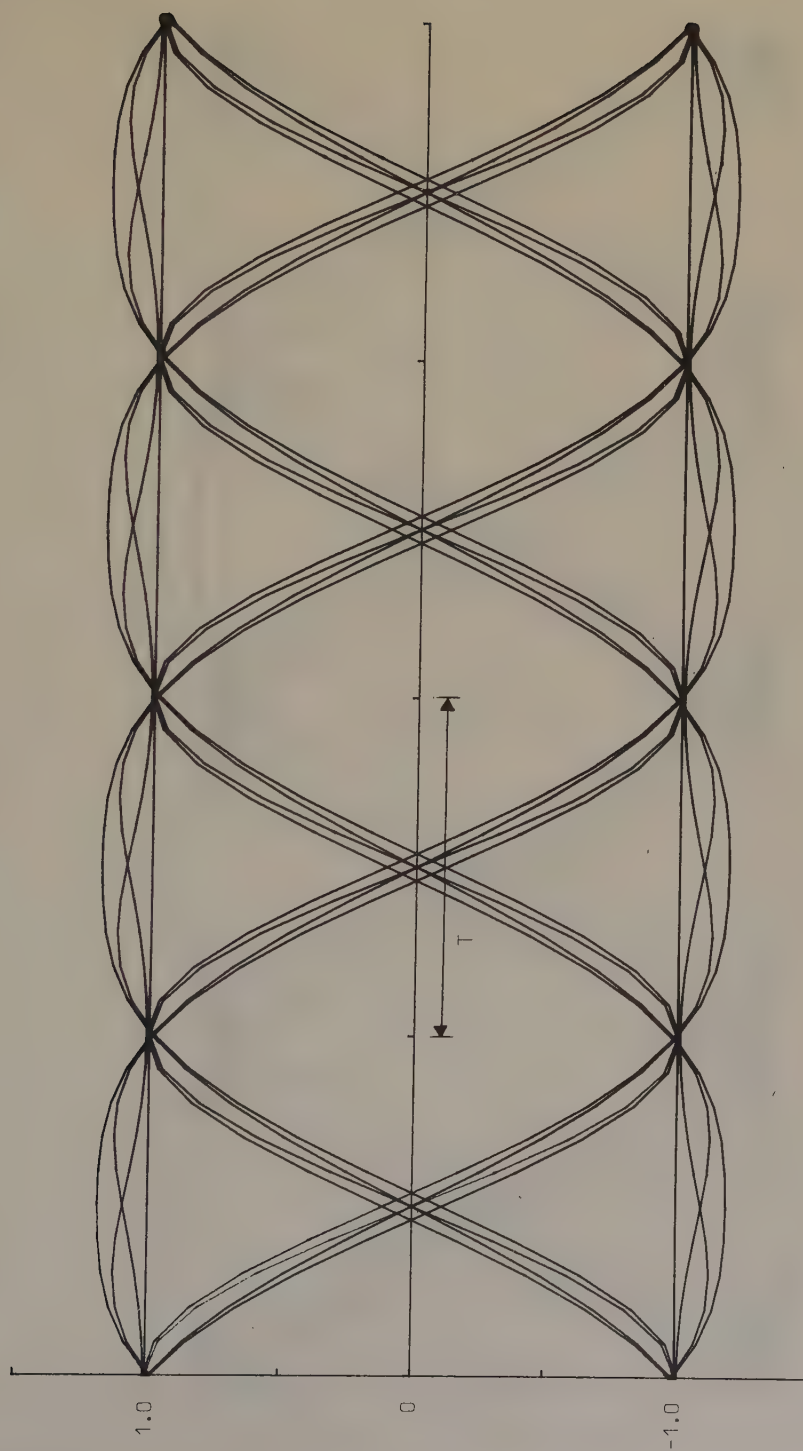


Figure 7. Decision eye for a SMSK receiver for ZRC.

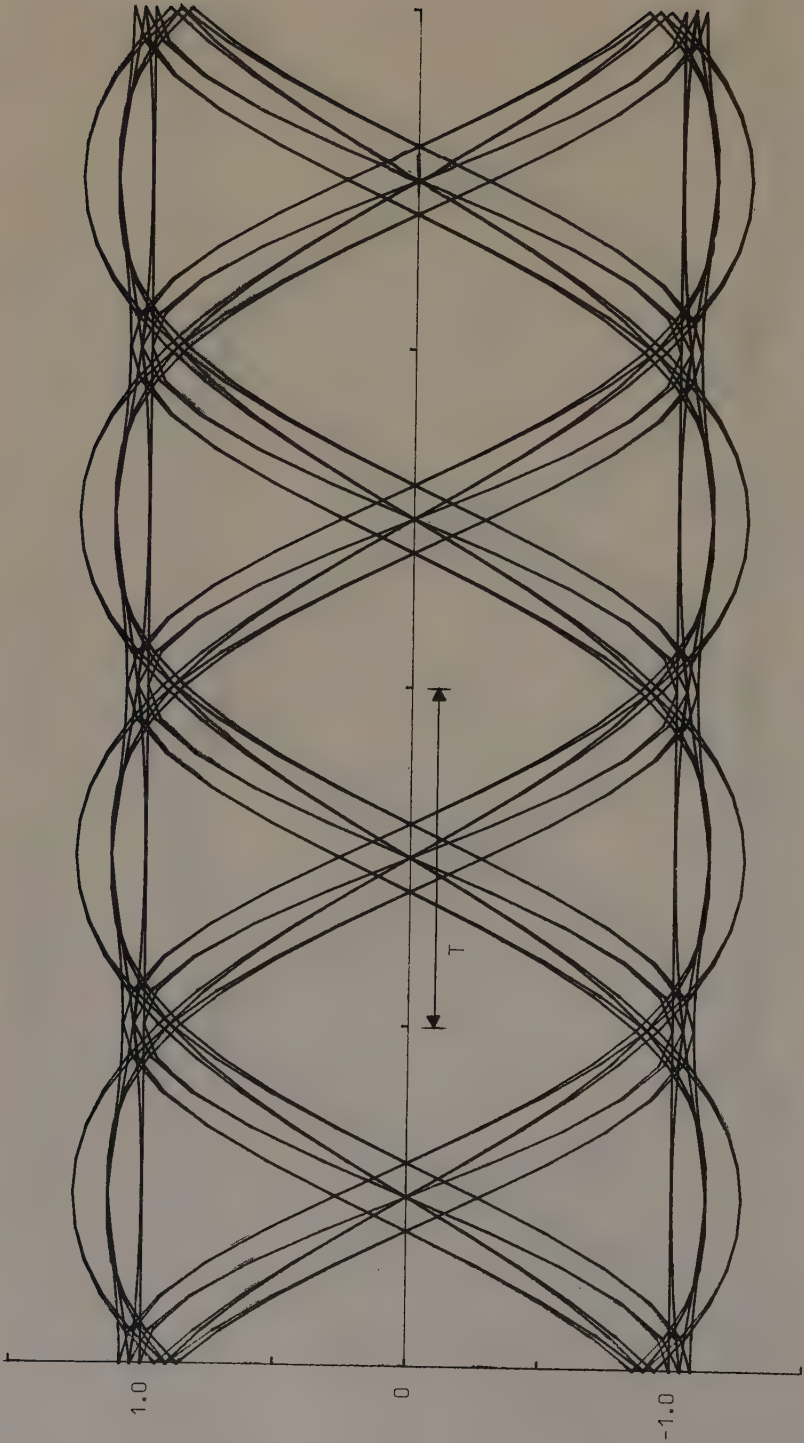


Figure 8. Decision eye for a SMSK receiver for 3RC.

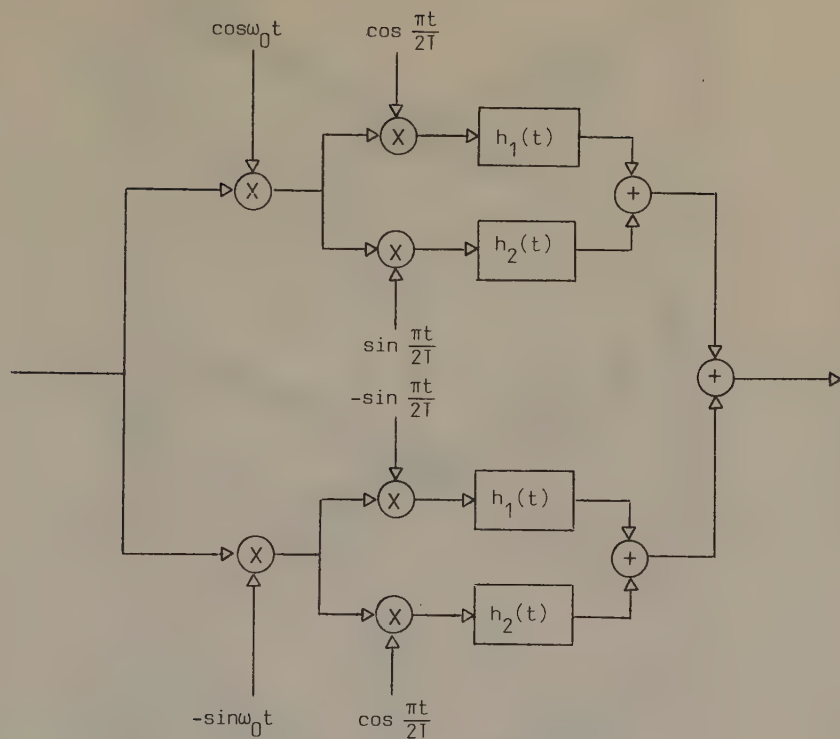


Figure 9. Redrawing of a serial MSK-type receiver ($\Phi=0$).

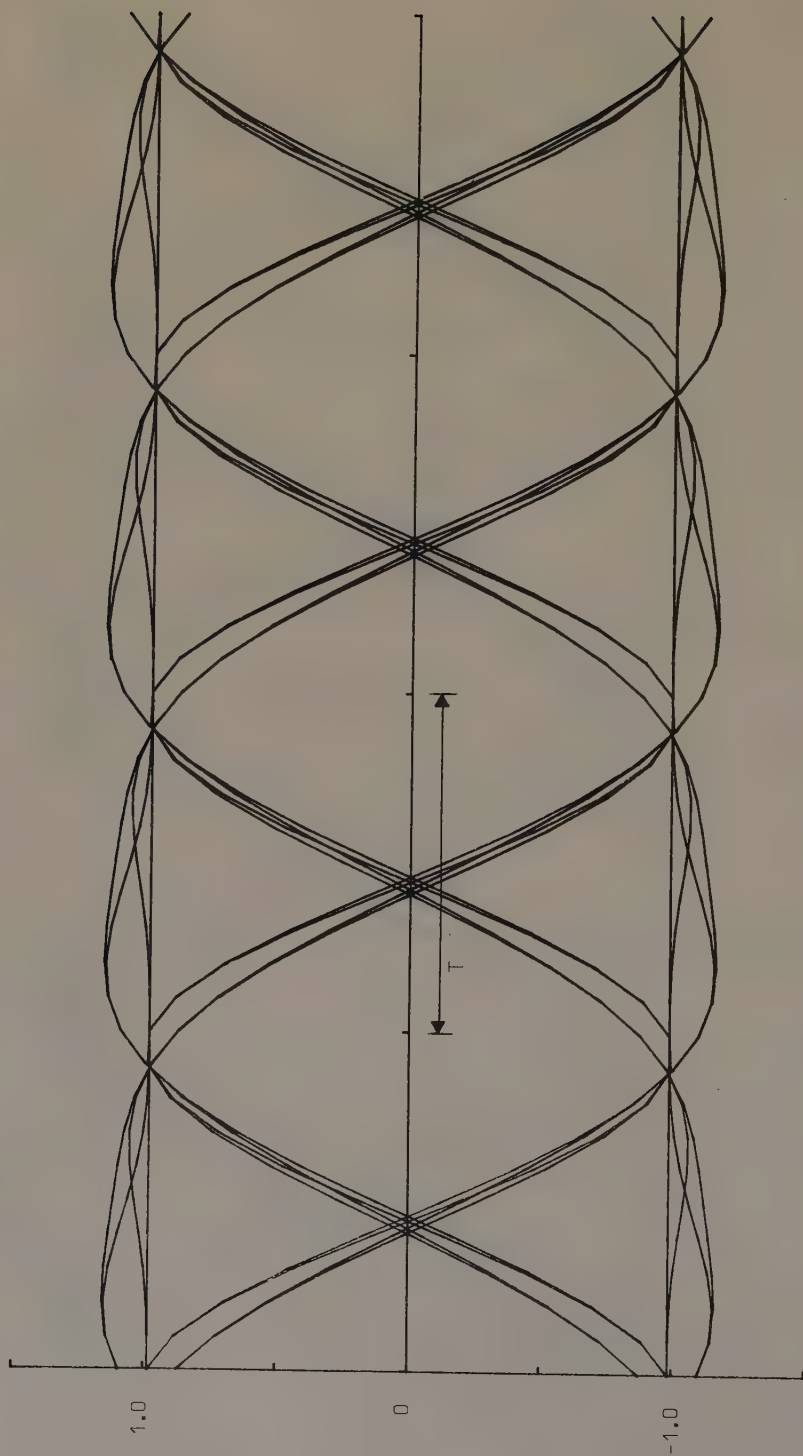


Figure 10a. Decision eye for a SMSK receiver for 1REC, when the phase error is -10 degrees.

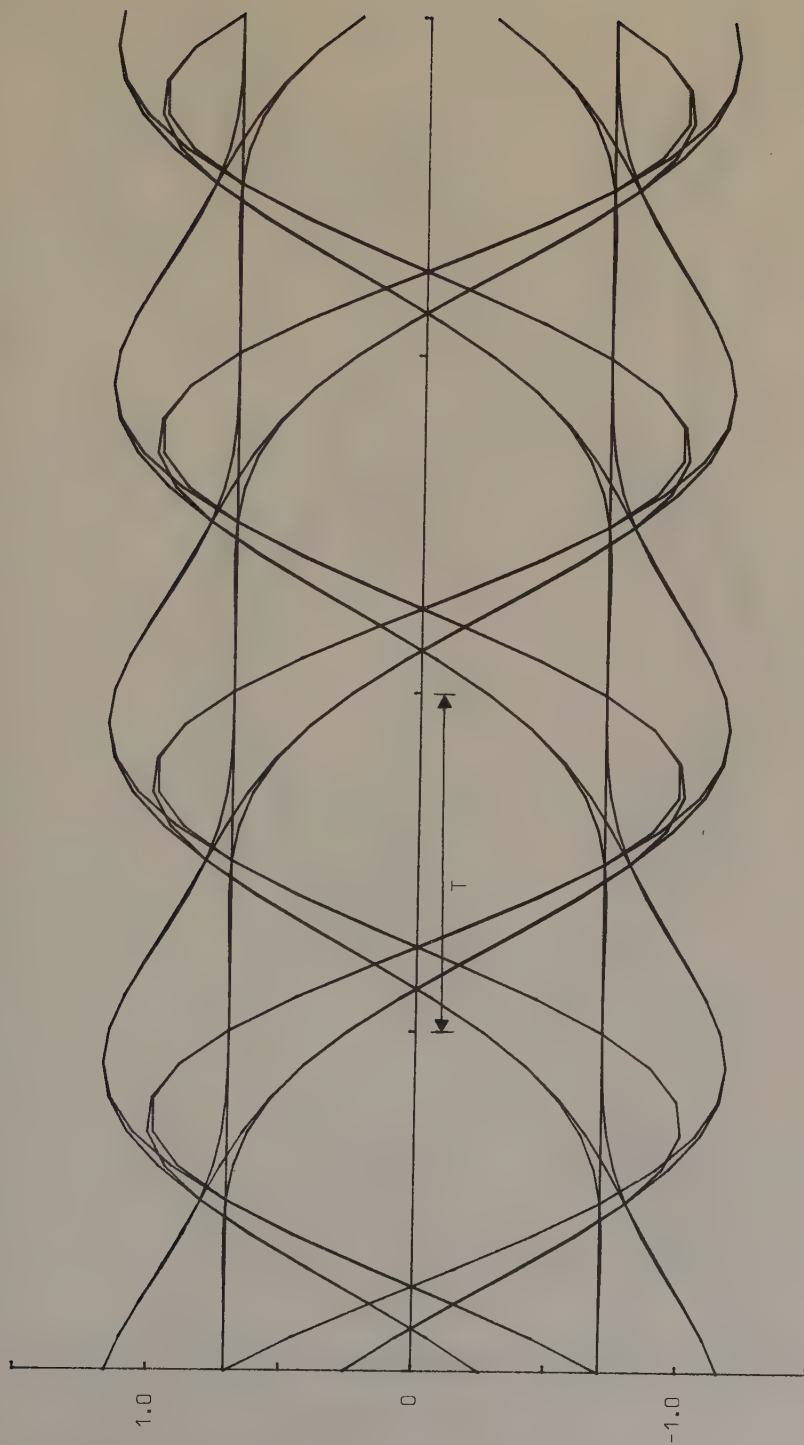


Figure 10b. Decision eye for a SMSK receiver for 1REC, when the phase error is -45 degrees.

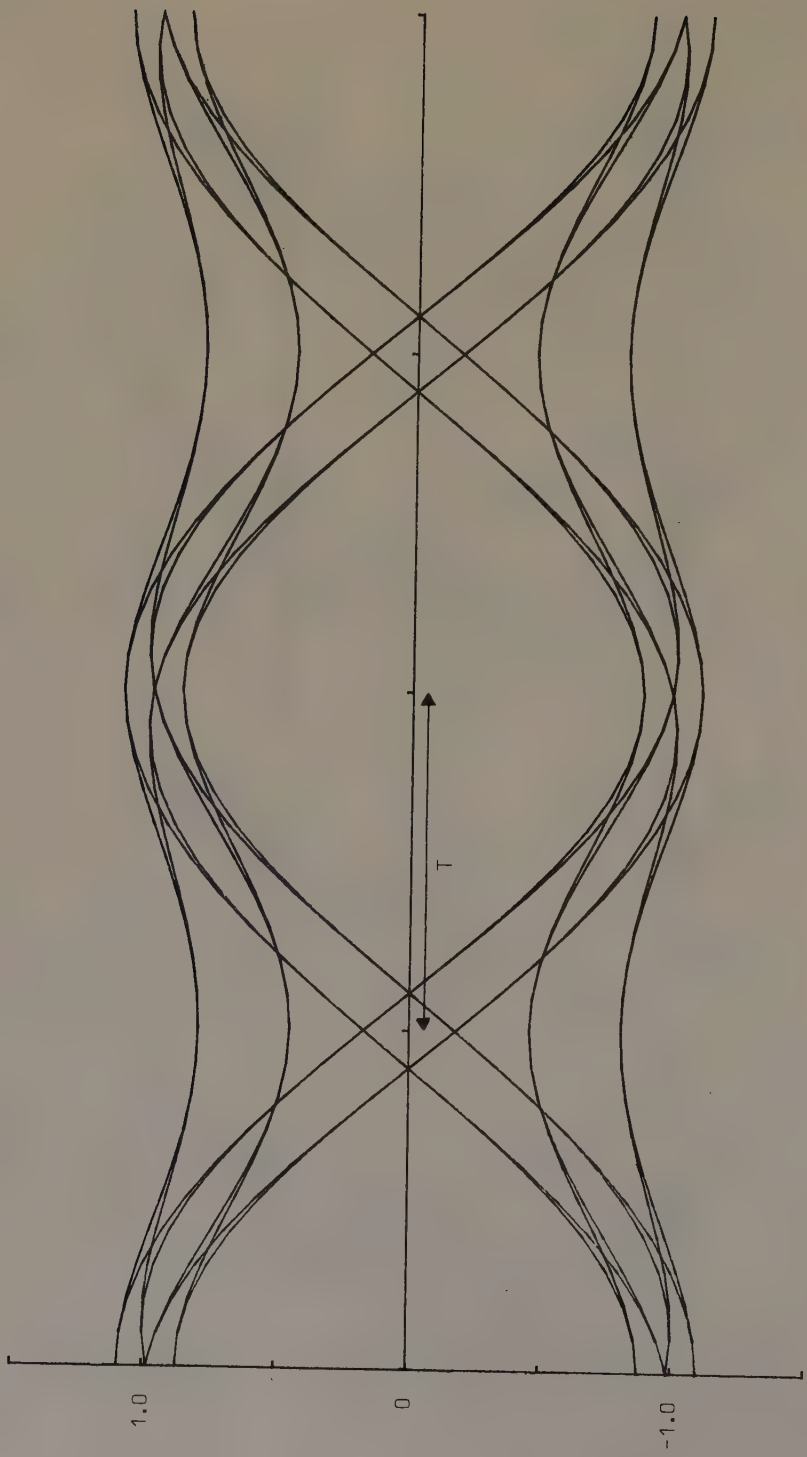


Figure 11a. Decision eye for a PMSK receiver for 1REC, when the phase error is -10 degrees.

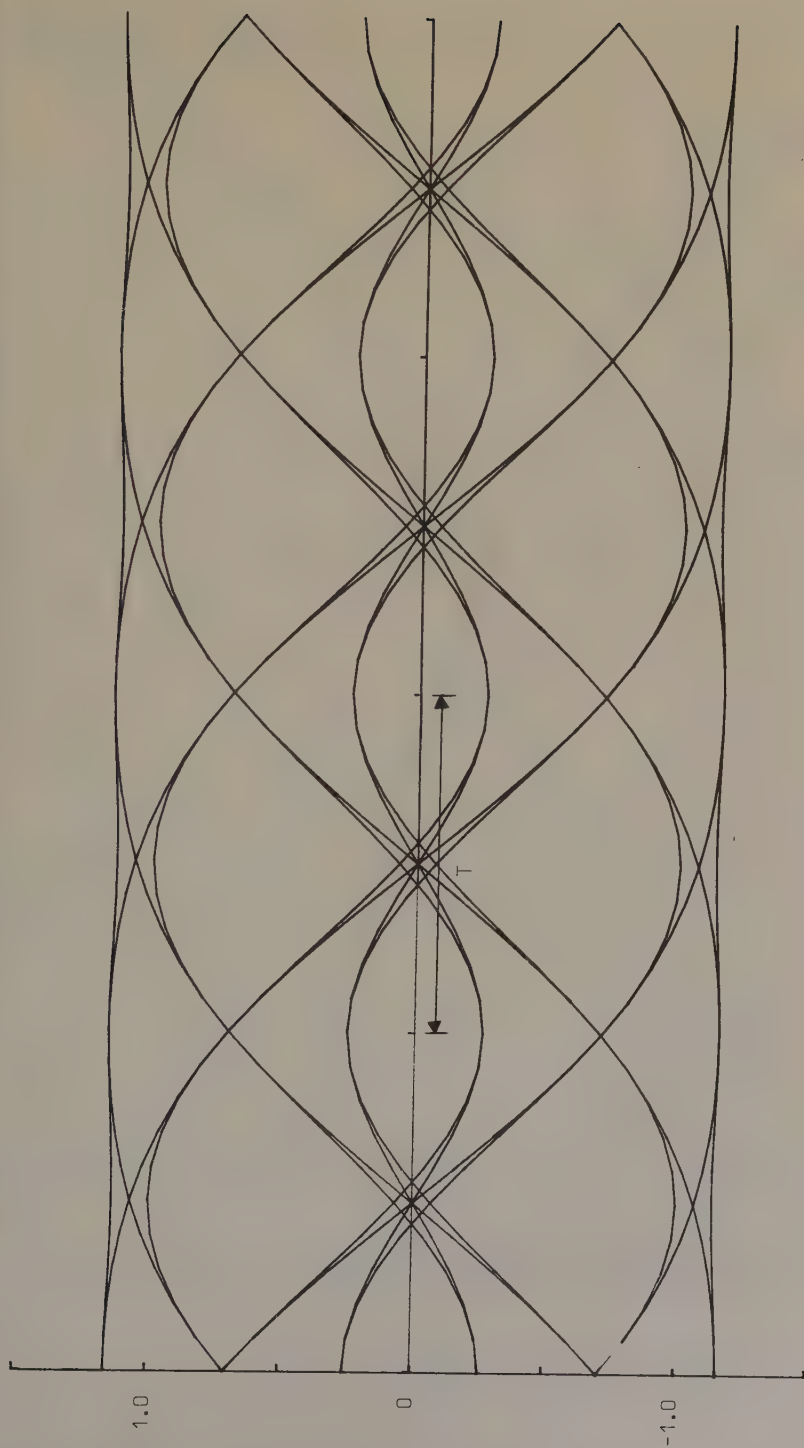


Figure 11b. Decision eye for a PMSK receiver for 1REC, when the phase error is -45 degrees.

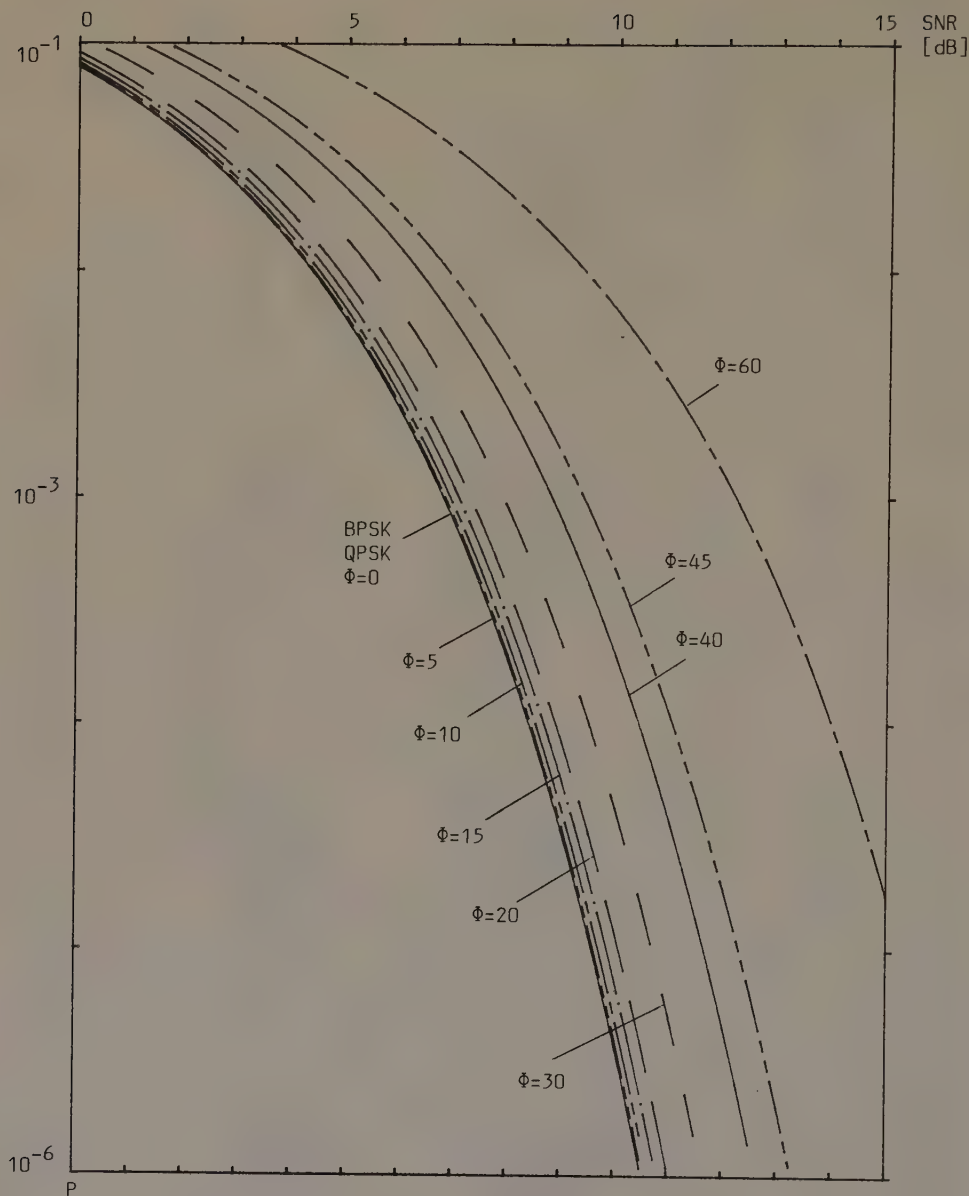


Figure 12a. The error probability for a SMSK receiver for 1REC, for various phase errors Φ .

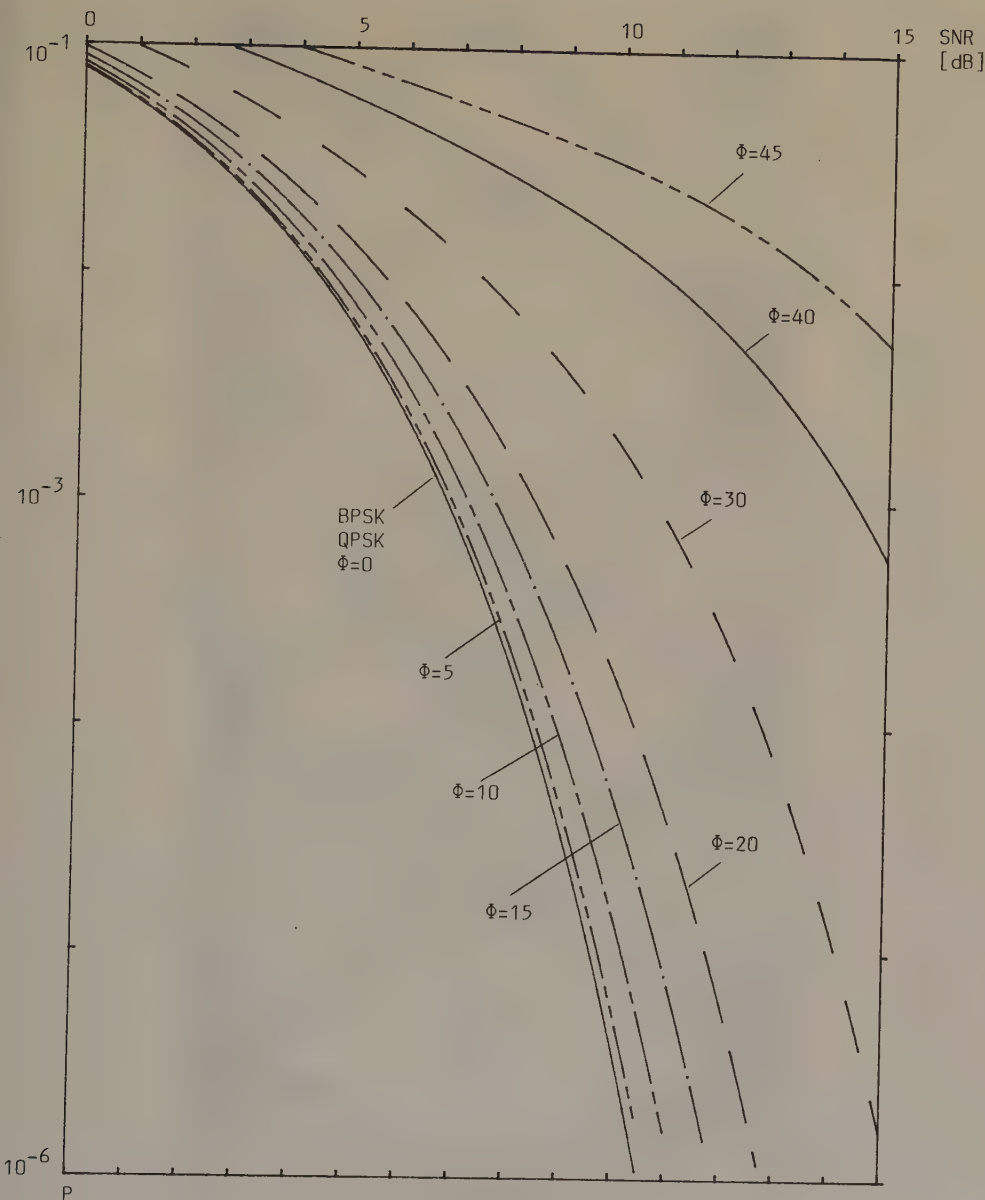


Figure 12b. The error probability for a PMSK receiver for 1REC, for various phase errors Φ .

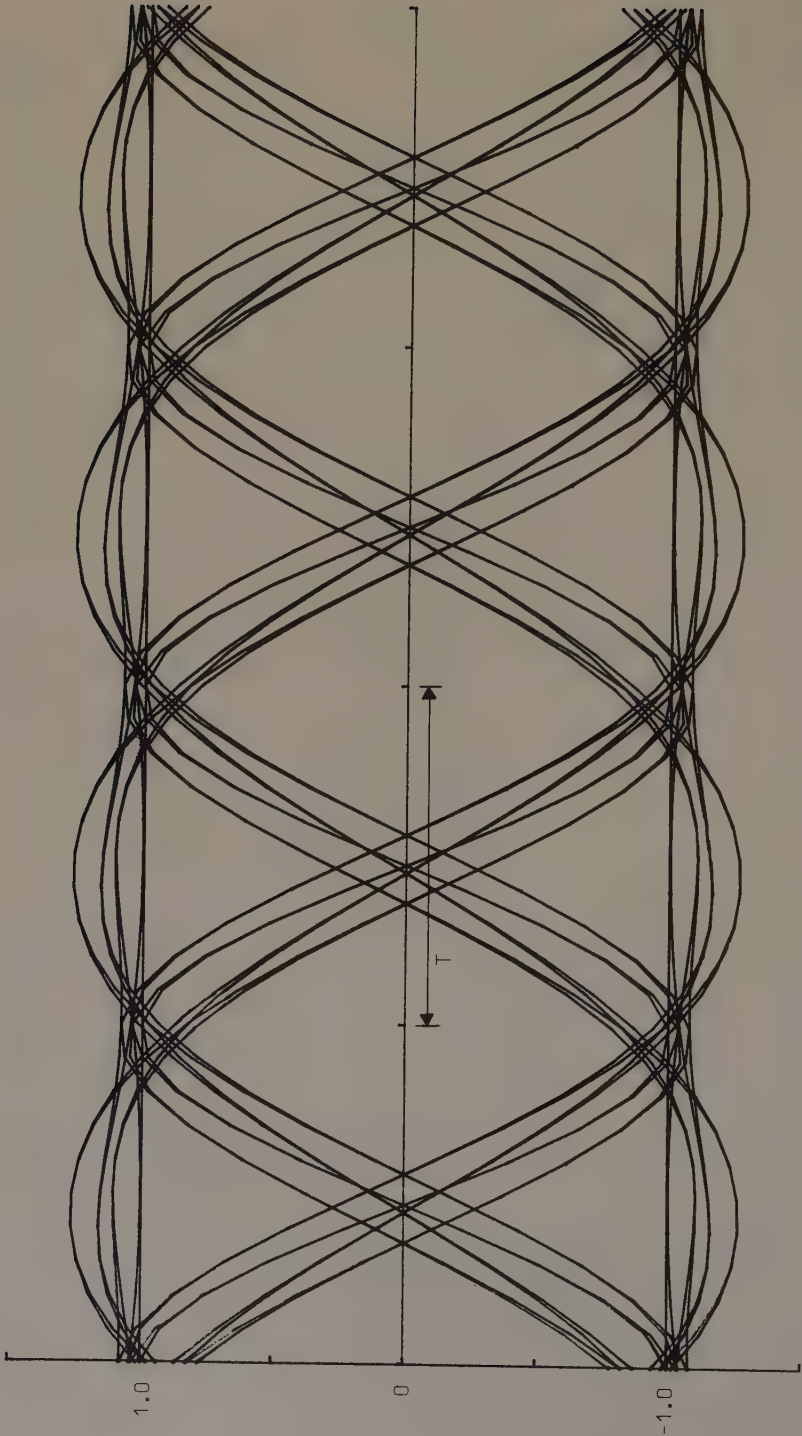


Figure 13. Decision eye for a SMSK receiver for 3RC, when the phase error is -5 degrees.

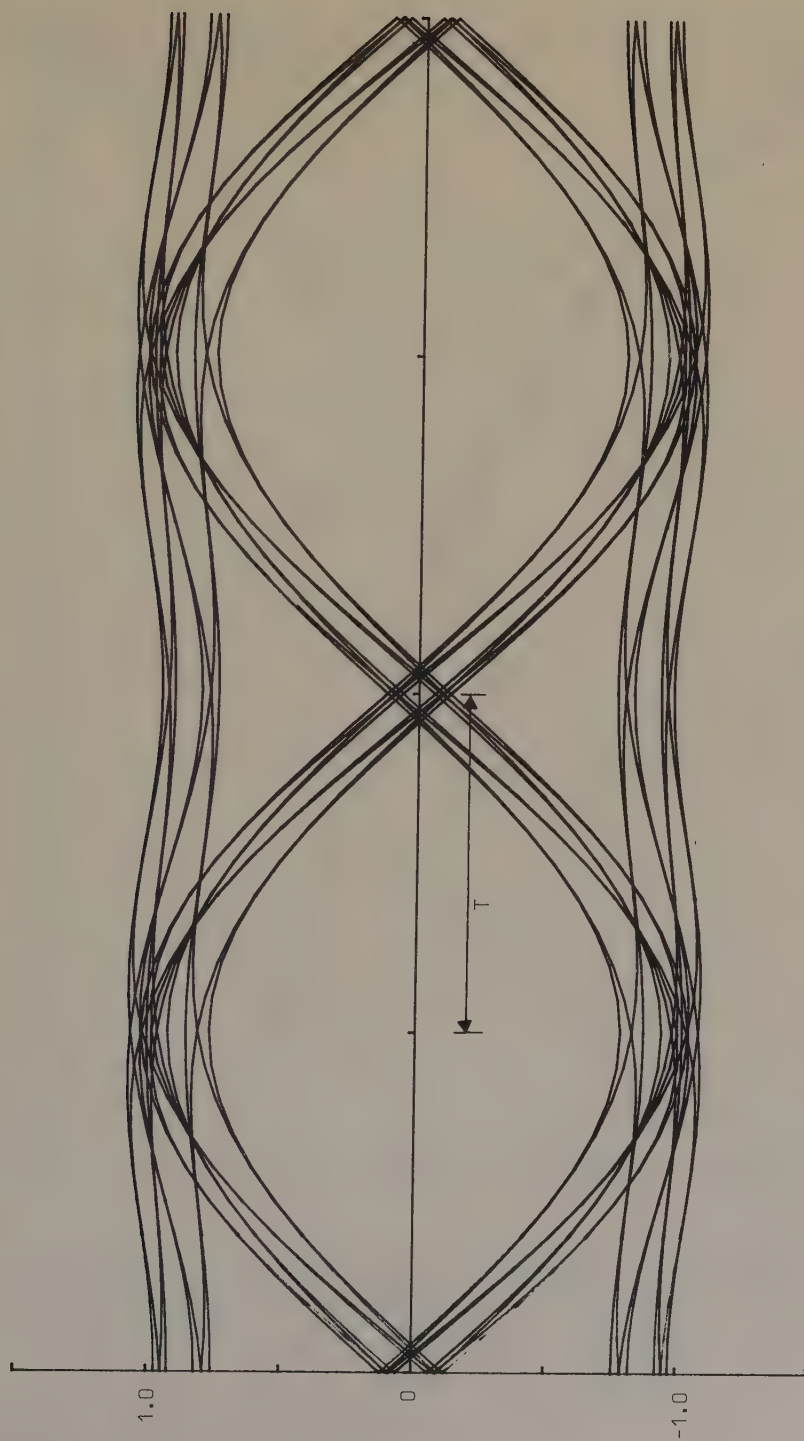


Figure 14. Decision eye for a PMSK receiver for 3RC, when the phase error is -5 degrees.

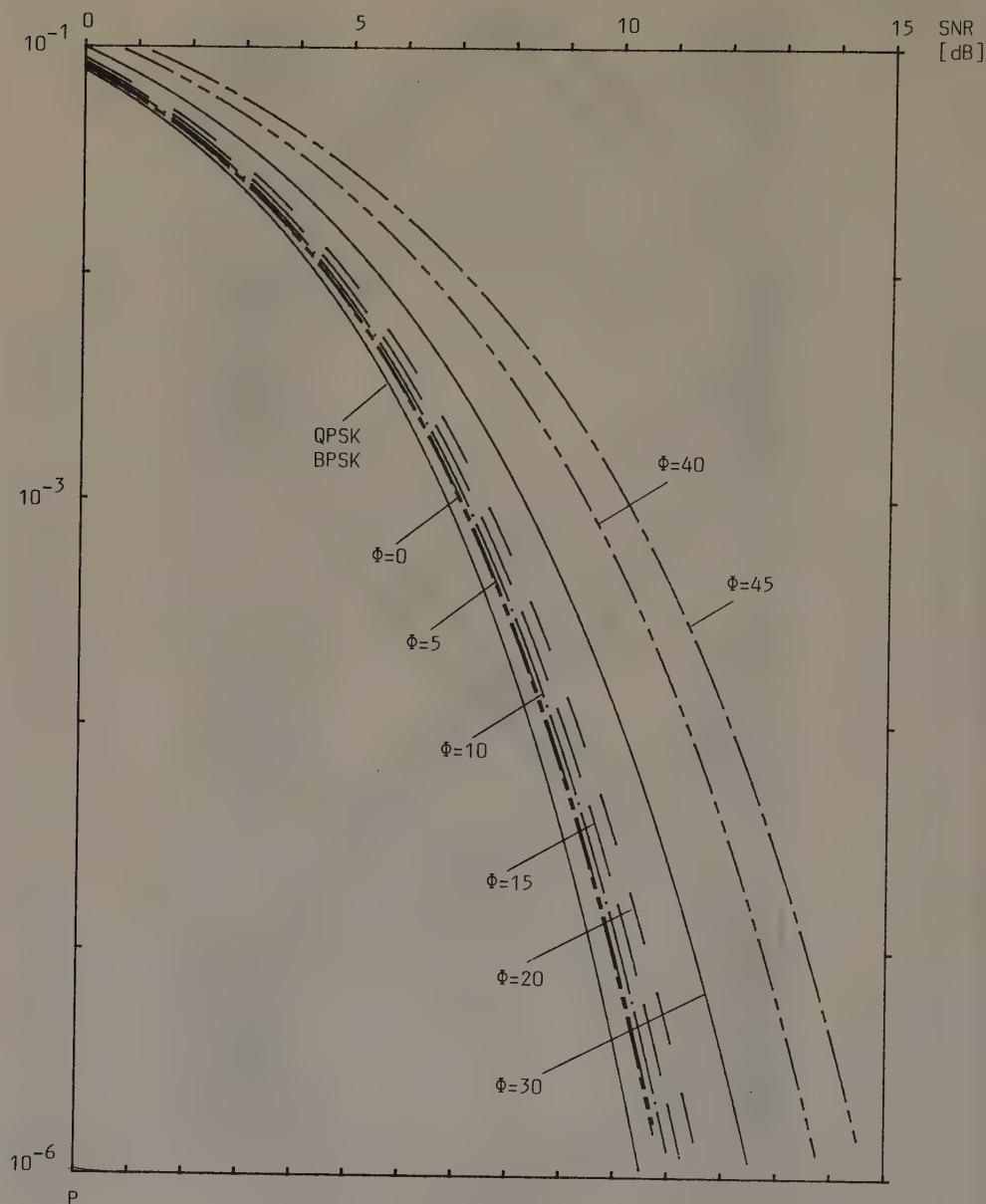


Figure 15a. The error probability for the asymptotically optimum serial receiver with $N_T=9$ for 3RC, for various phase errors Φ .

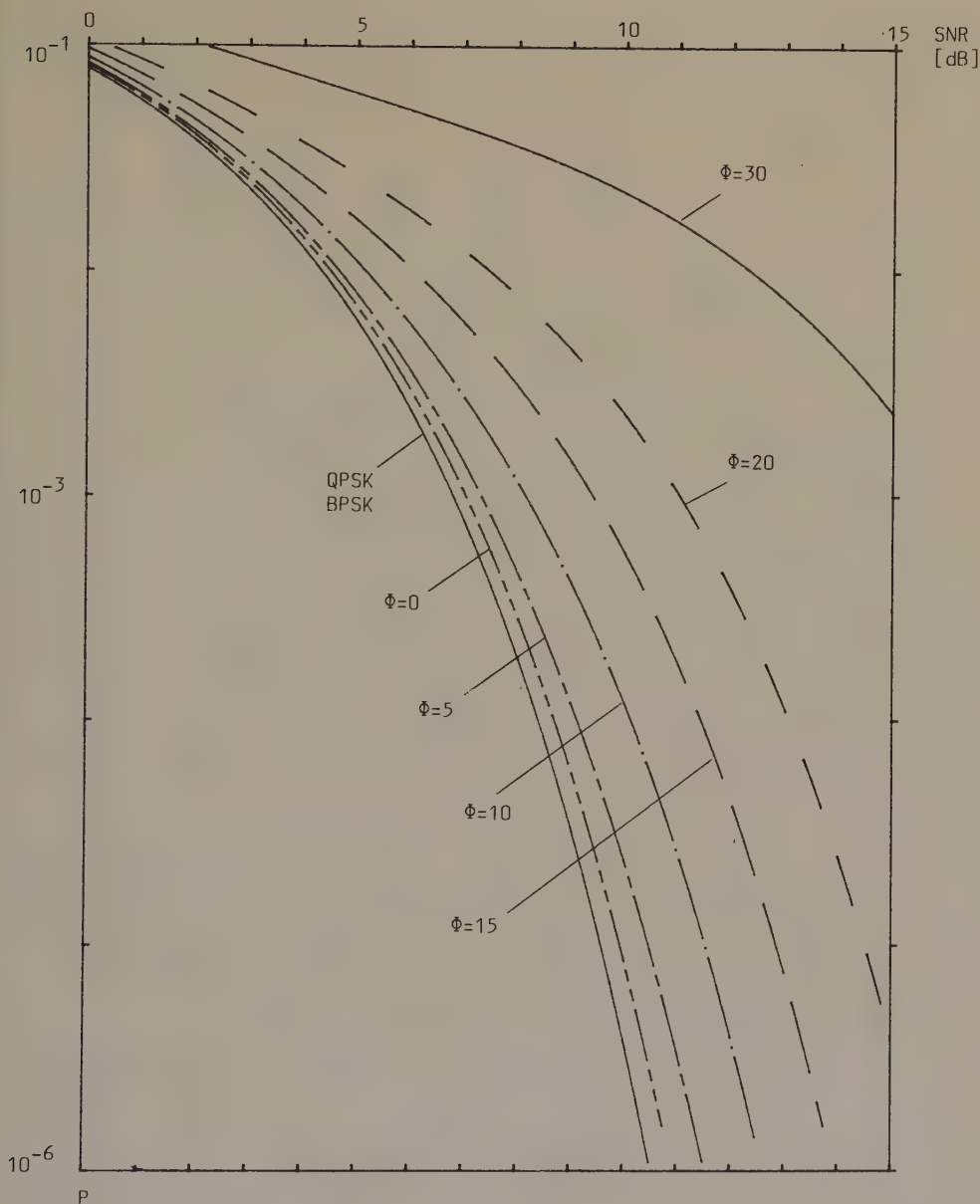


Figure 15b. The error probability for the asymptotically optimum parallel receiver with $N_T=2$ for 3RC, for various phase errors Φ .

degradation

[dB]

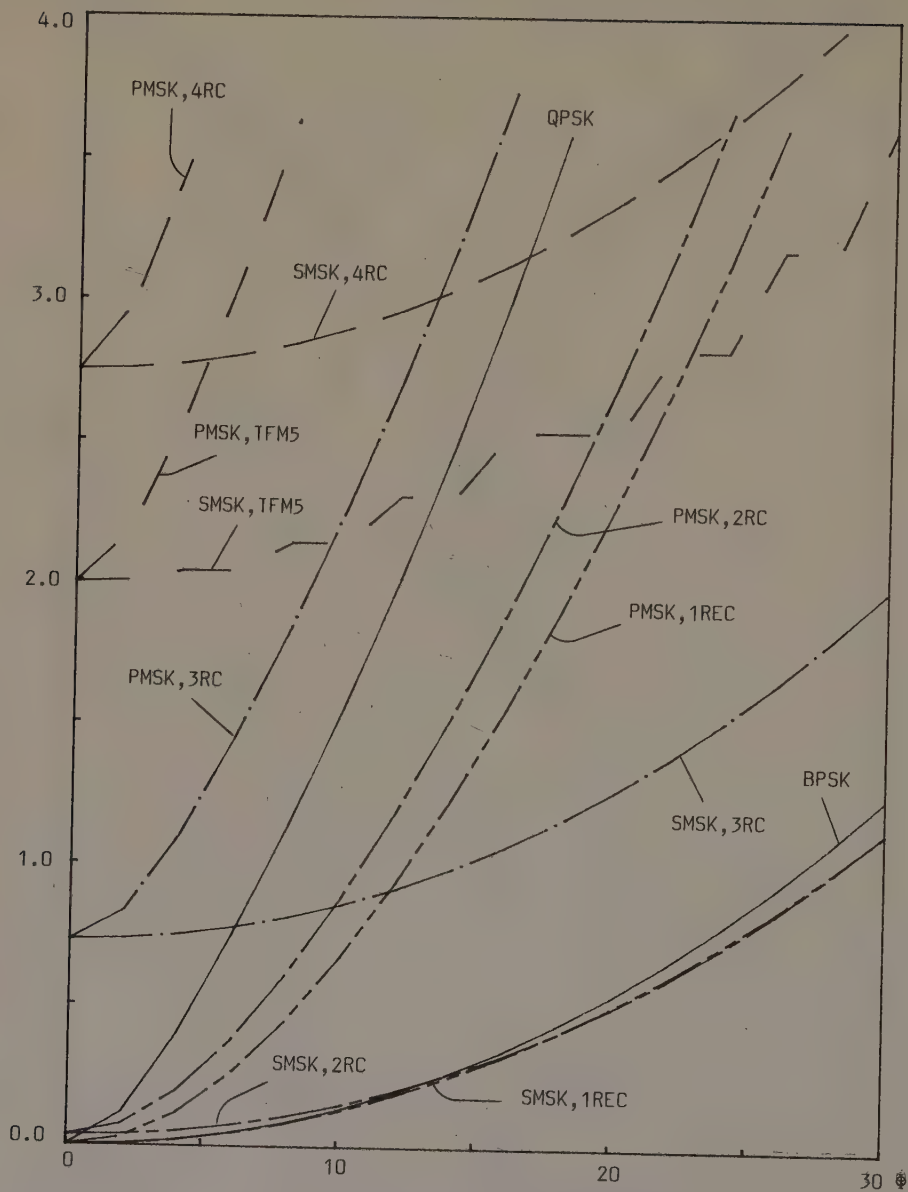


Figure 16a. Degradation relative to ideal BPSK/QPSK at $P=10^{-6}$ versus phase error for various schemes with serial and parallel MSK receivers.

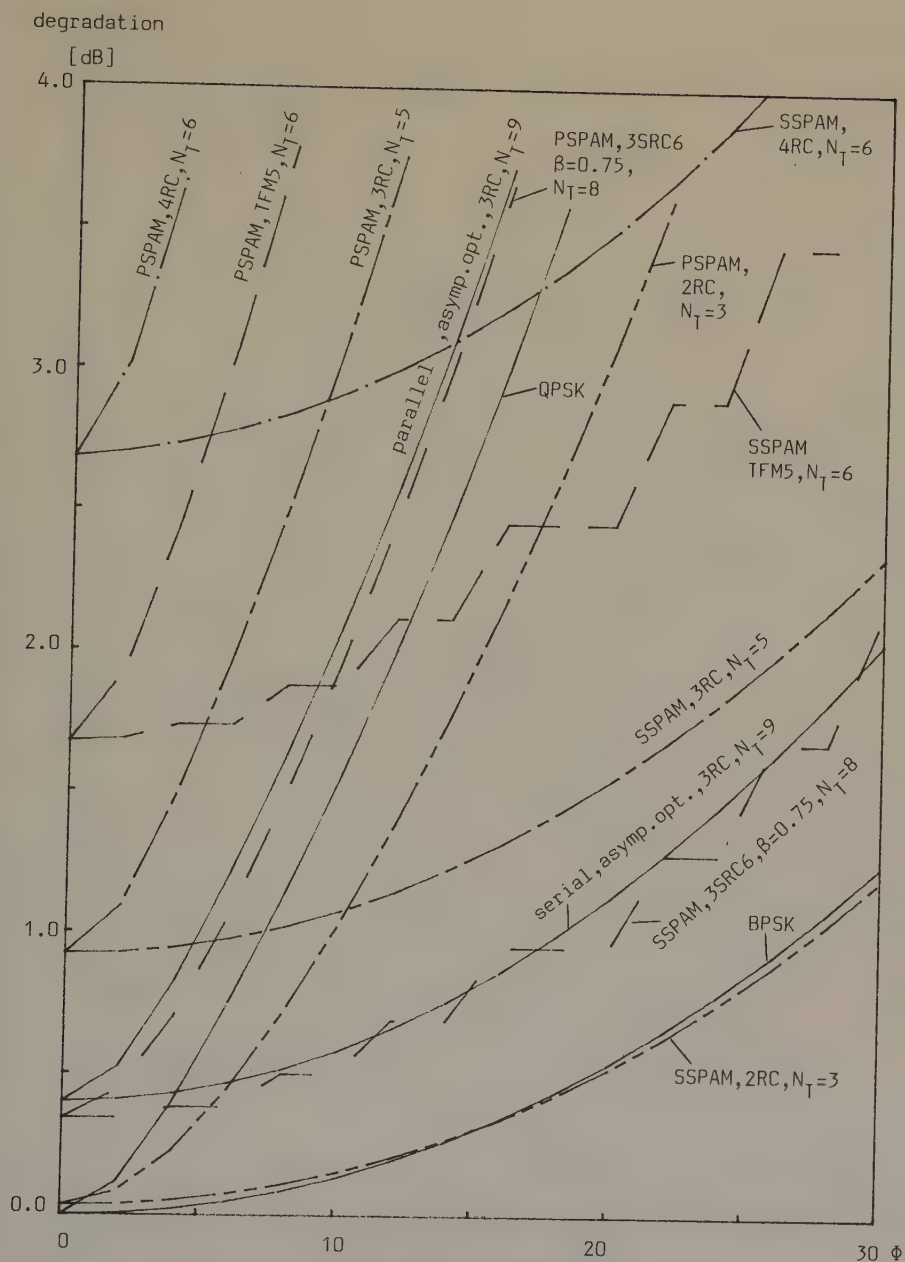


Figure 16b. Degradation relative to ideal BPSK/QPSK at $P=10^{-6}$ versus phase error for various schemes with serial and parallel SPAM receivers. The degradation for an asymptotically optimum receiver for 3RC is also shown.

degradation

[dB]

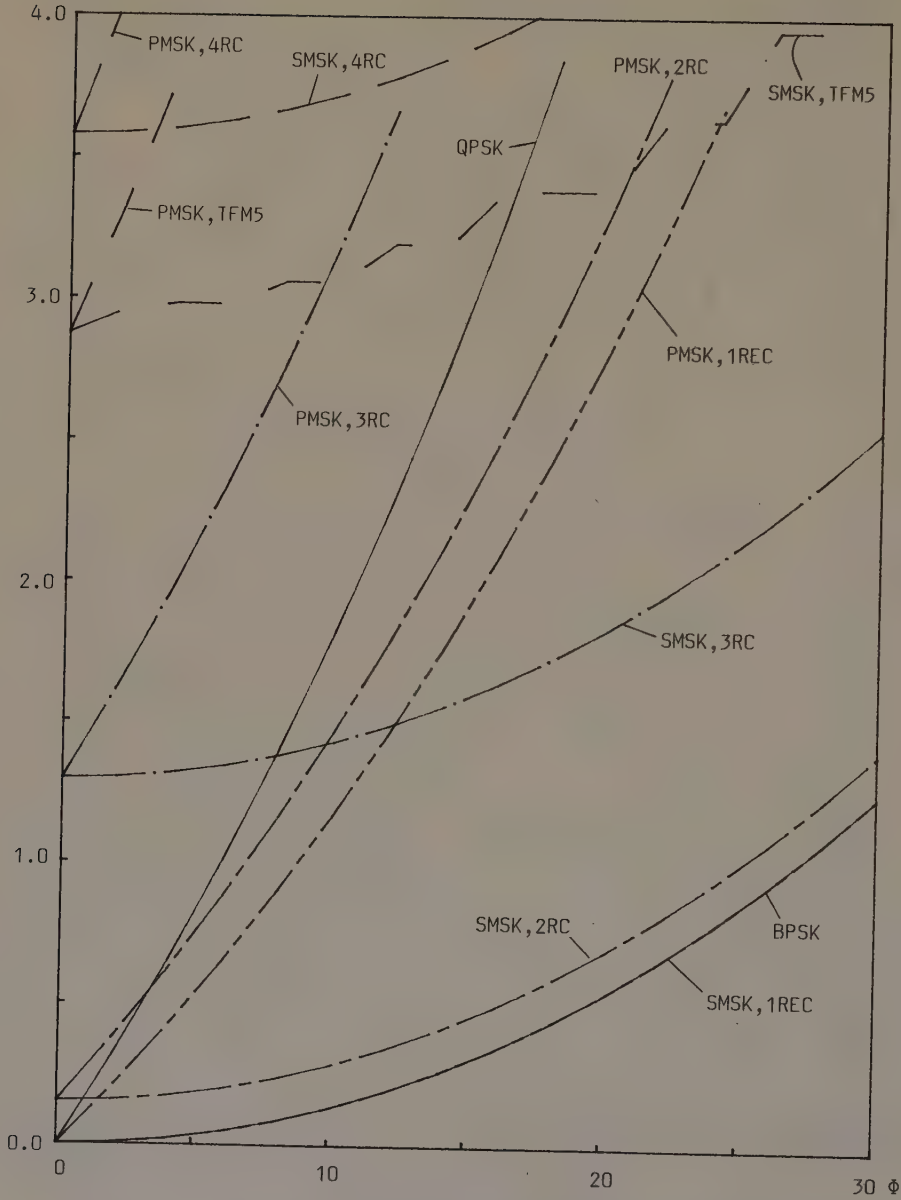


Figure 17a. Asymptotic degradation relative to ideal BPSK/QPSK versus phase error for various schemes with serial and parallel MSK receivers.

degradation

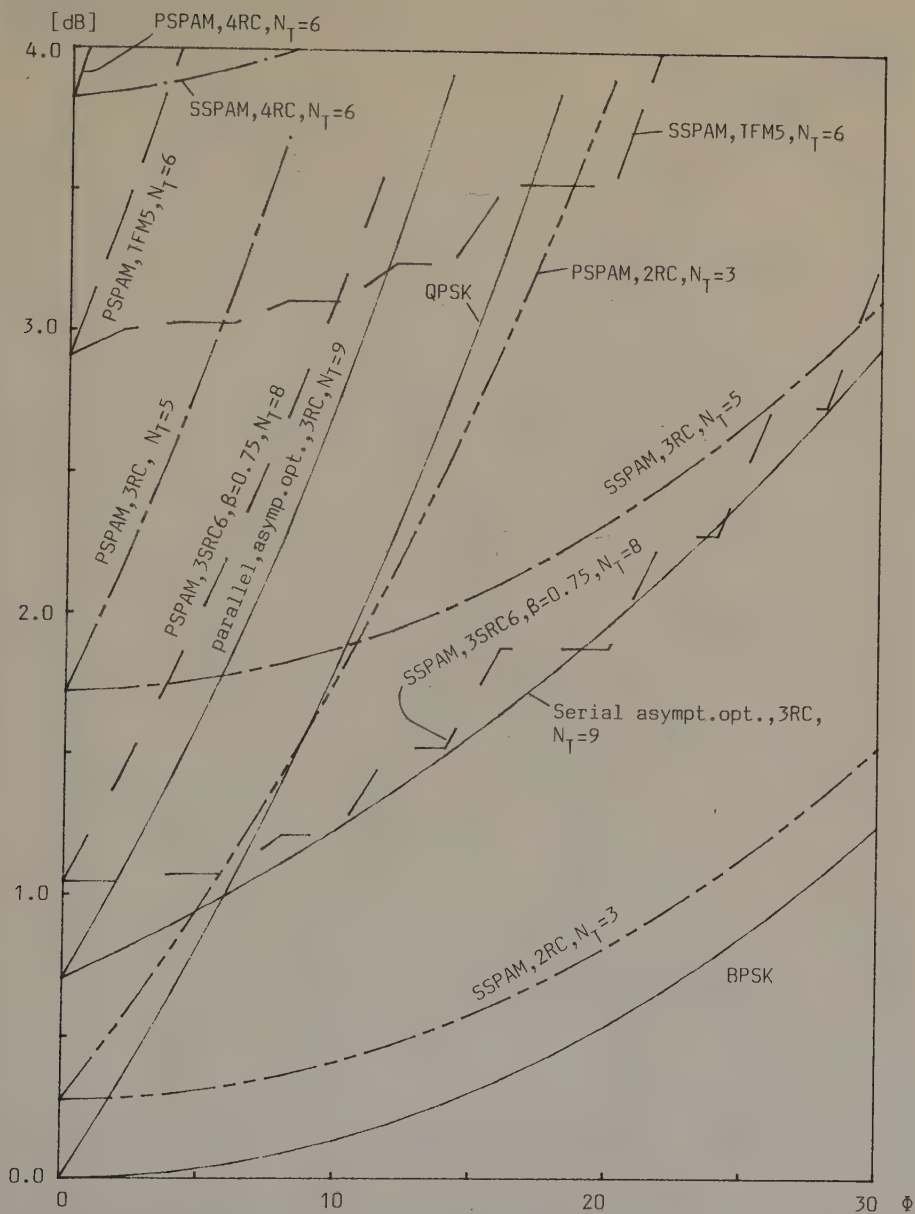


Figure 17b. Asymptotic degradation relative to ideal BPSK/QPSK versus phase error for various schemes with serial and parallel SPAM receivers. The degradation for an asymptotically optimum receiver for 3RC is also shown.

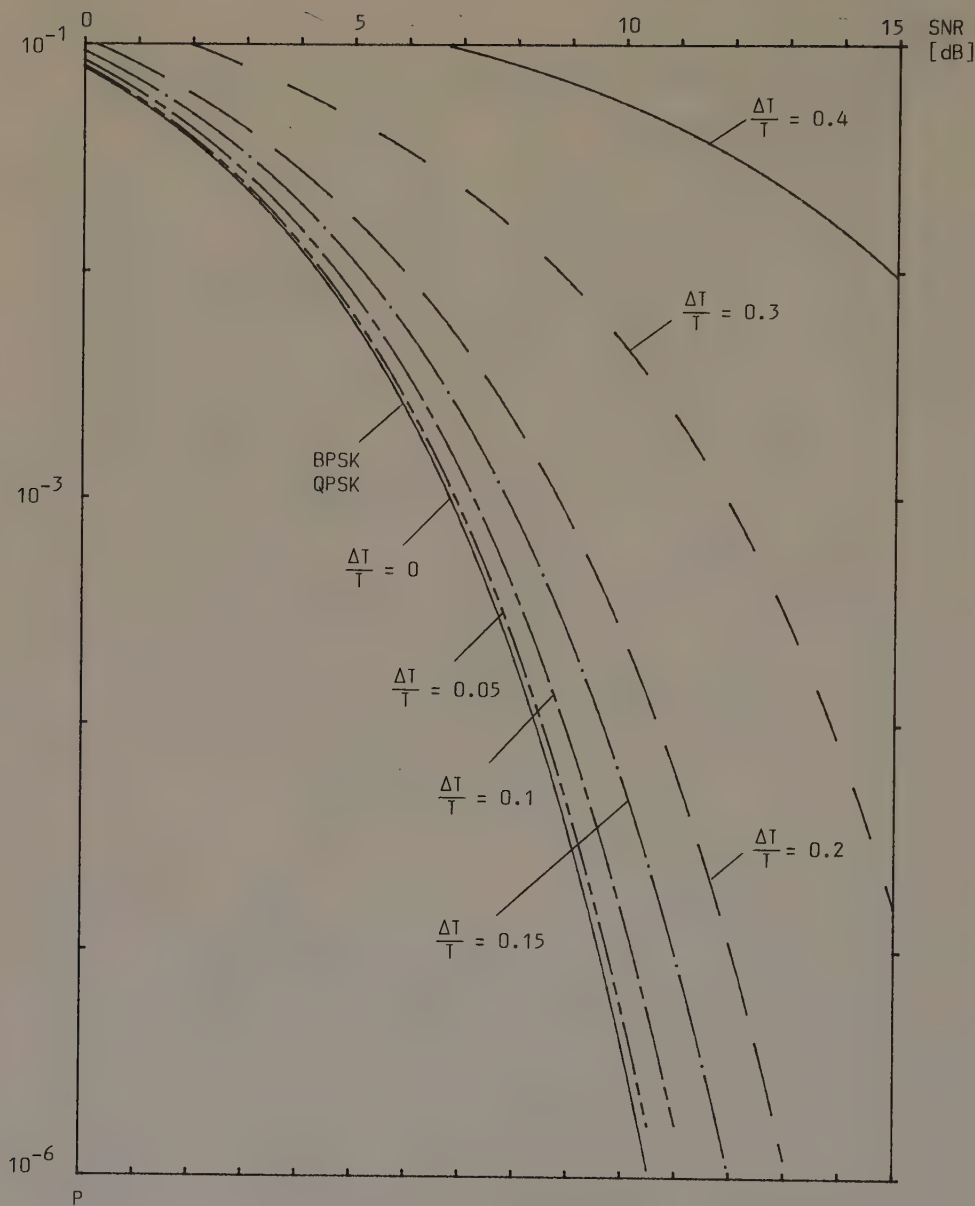
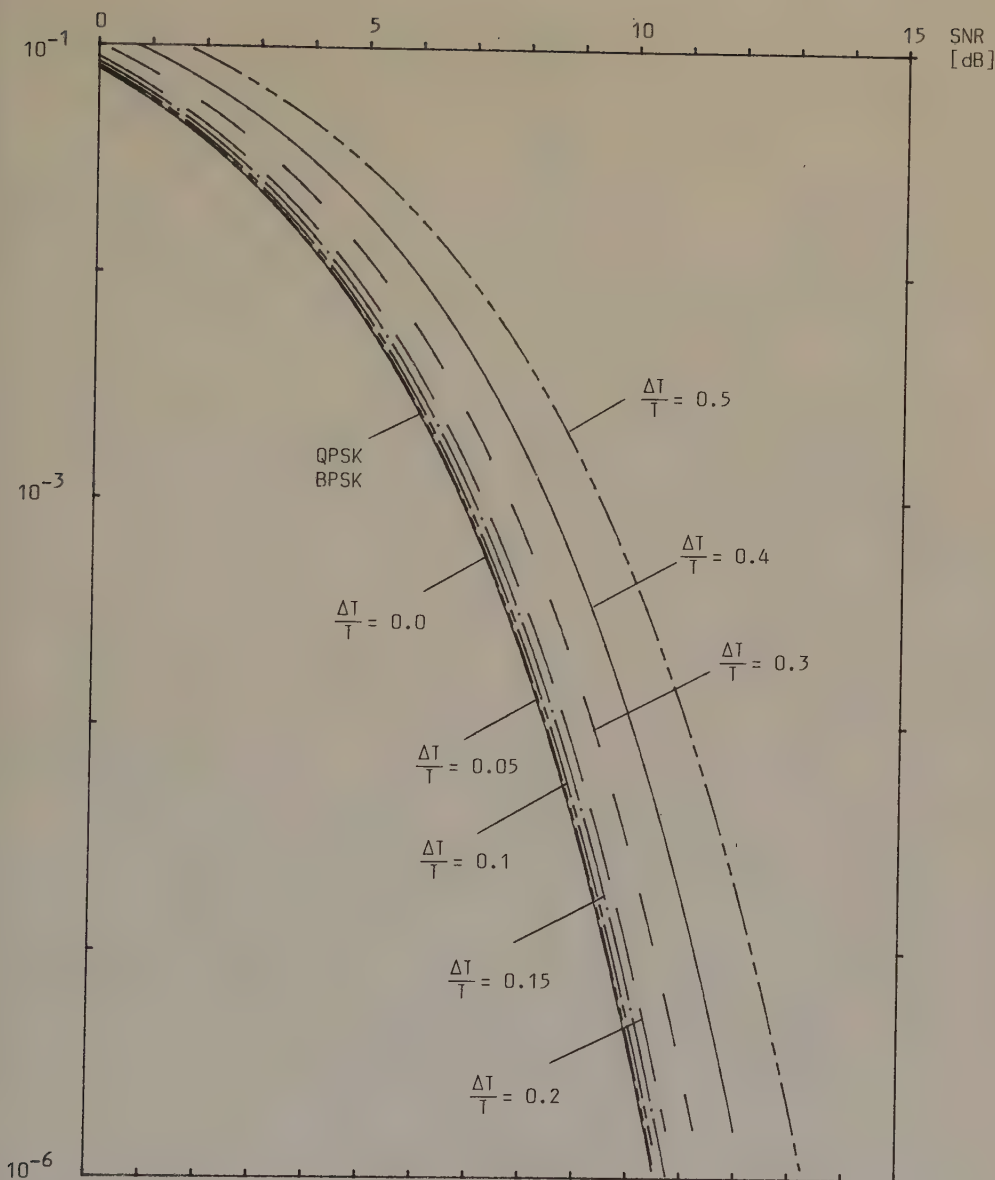


Figure 18a. The error probability for 1REC with a SMSK receiver, for various values of a relative timing error $\frac{\Delta T}{T}$.



P

Figure 18b. The error probability for 1REC with a PMSK receiver for various values of a relative timing error $\frac{\Delta T}{T}$.

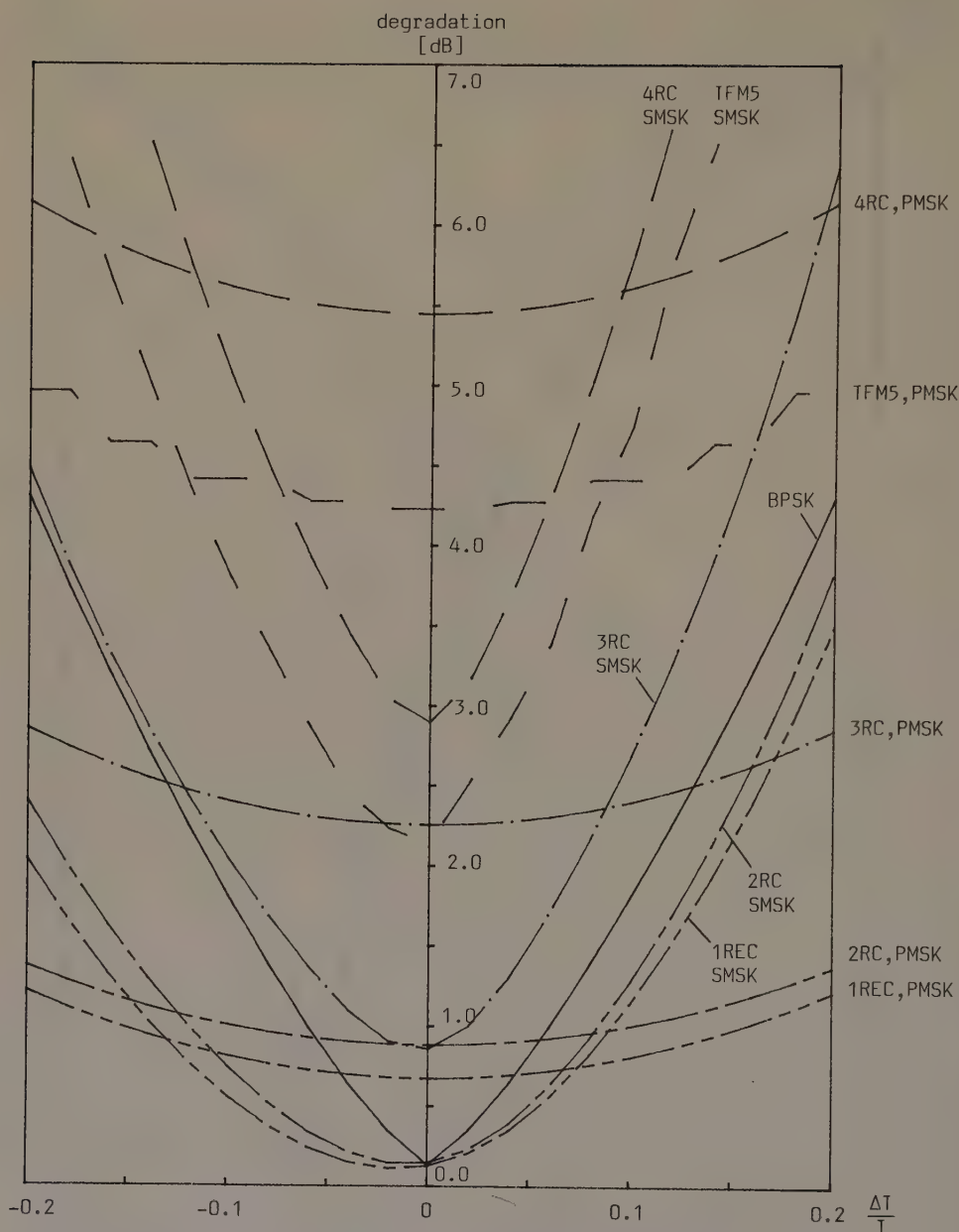


Figure 19a. Degradation at $P=10^{-6}$ relative to ideal BPSK/QPSK, versus a relative timing error, for a phase error of 10 degrees, for various schemes with MSK receivers.

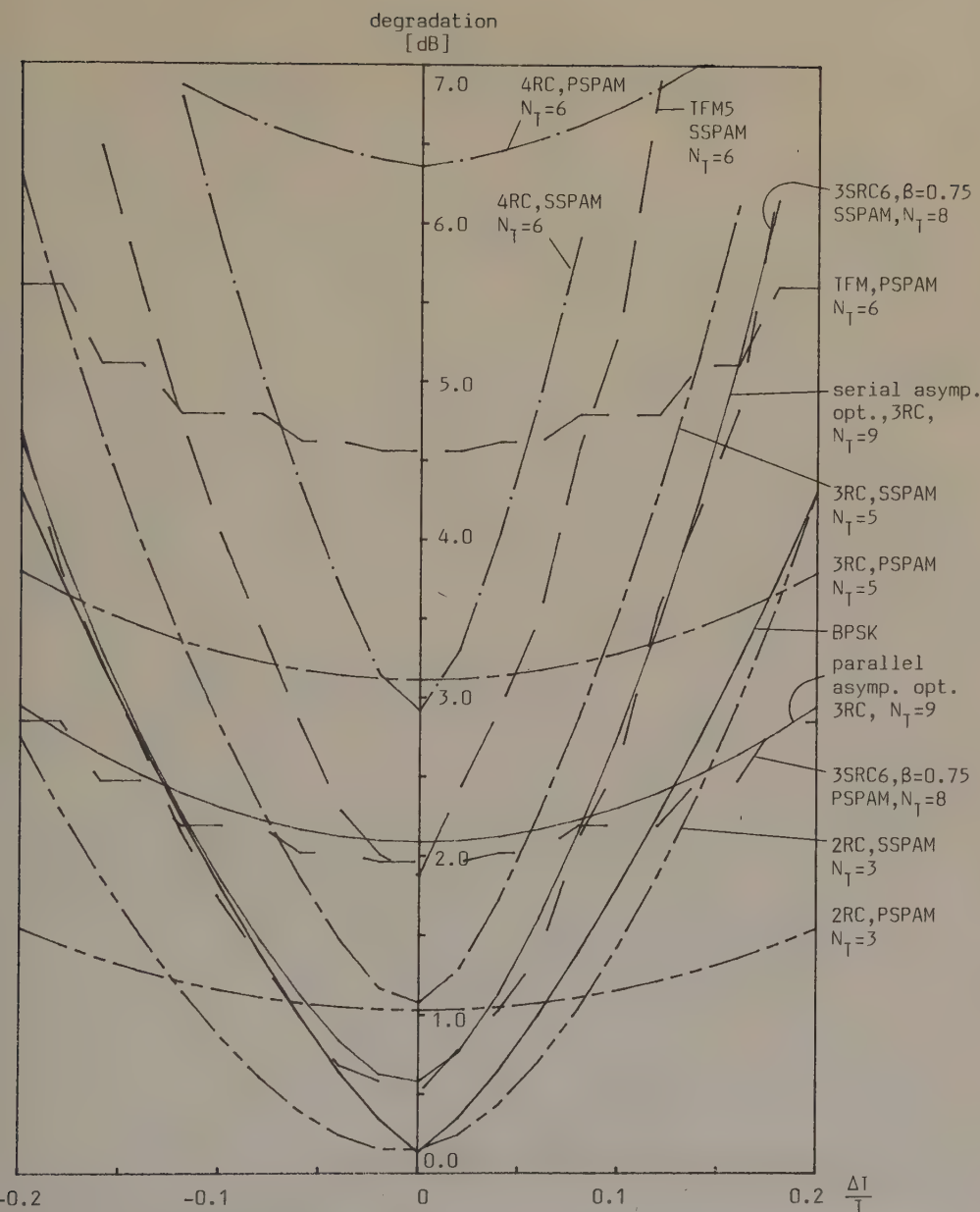


Figure 19b. Degradation at $P=10^{-6}$ relative to ideal BPSK/QPSK, versus a relative timing error, for a phase error of 10 degrees, for various schemes with SPAM receivers. The degradation for an asymptotically optimum receiver for 3RC is also shown.

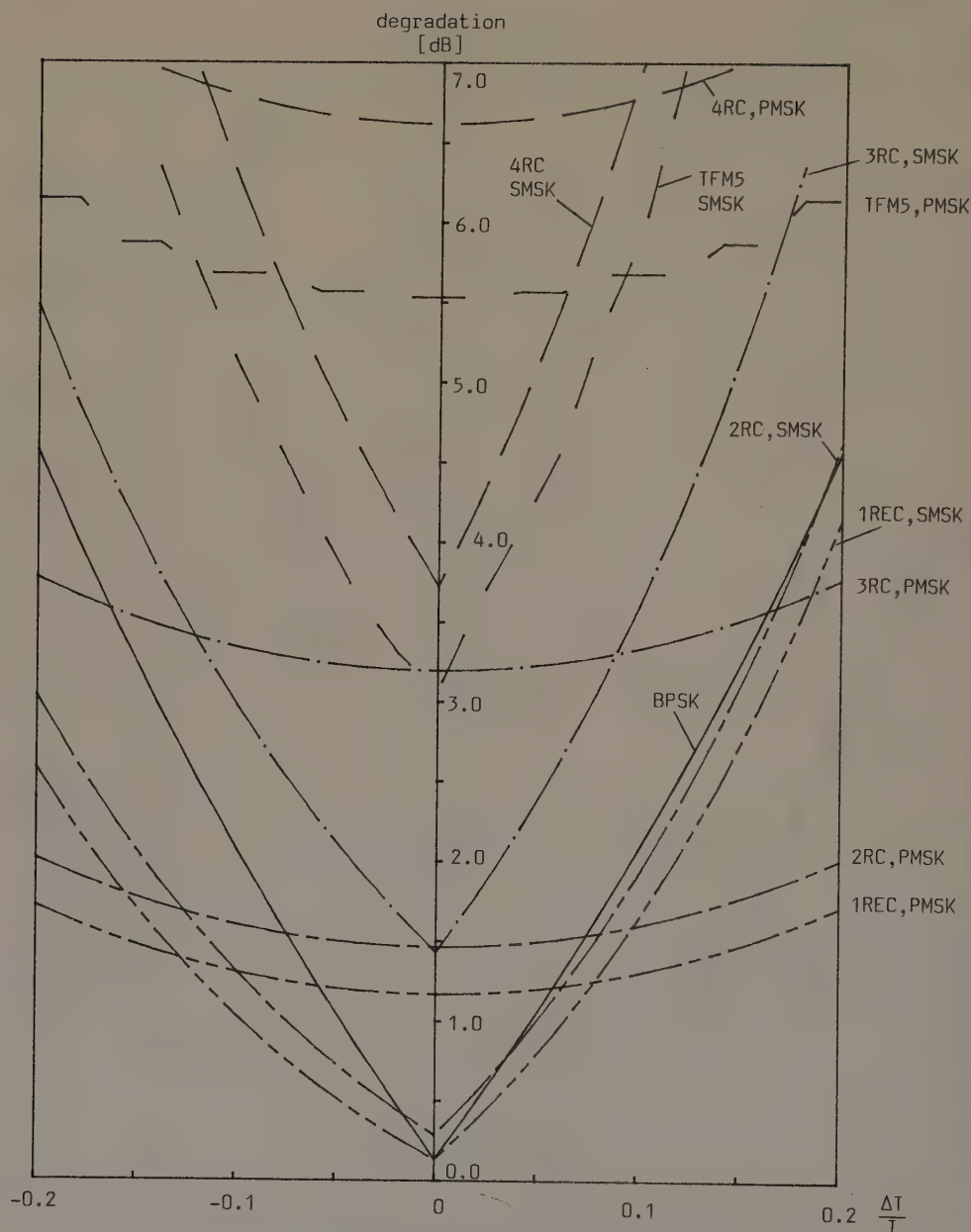


Figure 20a. Asymptotic degradation relative to ideal BPSK/QPSK, versus a relative timing error, for a phase error of 10 degrees, for various schemes with MSK receivers.

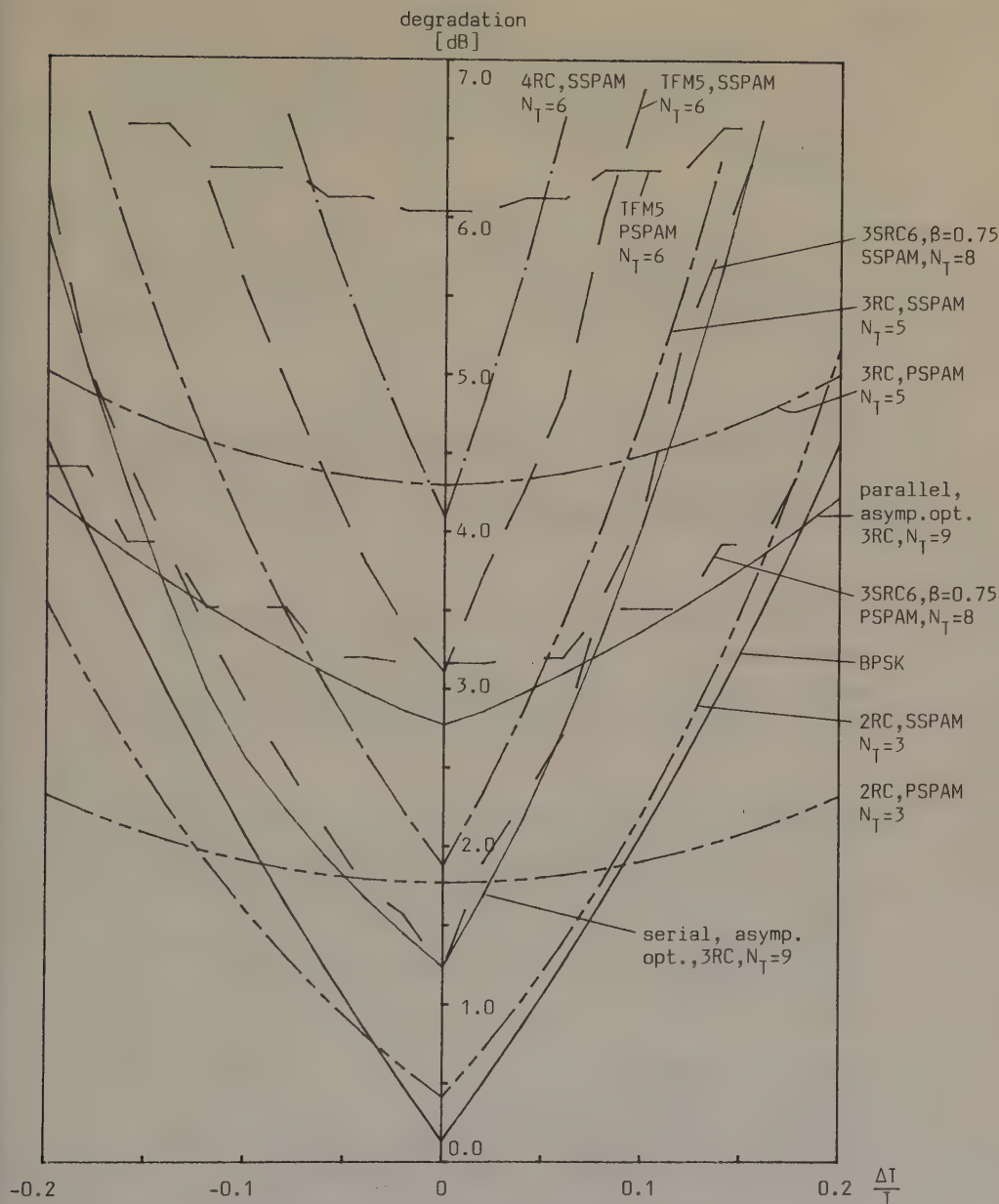


Figure 20b. Asymptotic degradation relative to ideal BPSK/QPSK, versus a relative timing error, for a phase error of 10 degrees, for various schemes with SPAM receivers. The degradation for an asymptotically optimum receiver for 3RC is also shown.

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RECEIVERS FOR CPM

PART IV

AMF RECEIVERS

ABSTRACT

Partial Response Continuous Phase Modulation (CPM) signals can in many cases yield power spectra with narrow mainlobe and also low sidelobes. The optimum receiver for these signals is sometimes complex. In this part the idea of using Average Matched Filter (AMF) receivers are generalized to multiple-symbol Average Matched Filter receivers for partial response schemes. Only coherent detection of signals transmitted over an additive white Gaussian noise channel is considered.

The multiple-symbol AMF receiver can be implemented with fewer filters than the optimum receiver. These receivers are analysed for various partial response CPM signals. The analyse is done under the assumption of error free filter and phase updating. A Euclidean distance is introduced, giving the symbol error probability for large signal-to-noise ratios conditioned on error free updating. Under the above assumptions it is shown that the multiple-symbol AMF receiver performs equally well as the optimum receiver for schemes with low modulation indices and also for larger modulation indices for some schemes. This is the case both for binary and quaternary schemes. A simple updating method is also simulated.

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I. INTRODUCTION

From the previous parts of this thesis it is known that constant envelope digital modulation methods with excellent power spectra can be constructed. A wide class of such schemes are contained in the Continuous Phase Modulation (CPM) schemes. By using partial response techniques and keeping the information carrying phase continuous, the power spectra become very attractive, see also [1], [2].

It has also been shown that schemes with combined good spectral and detection properties can be obtained in the CPM-family. The problem is receiver complexity. The modulation schemes with attractive combined spectra and error probability often require a maximum likelihood sequence detector implemented by means of a Viterbi detector. Such optimum receivers are analysed and simulated in some detail in [2]. These receivers typically consist of a filterbank followed by a Viterbi processor. The number of filters and states (in the Viterbi algorithm) can be large.

In this part we consider the problem of coherent detection of CPM-signals with simple receivers. Osborne and Luntz [6] have derived the optimum coherent receiver for CPFSK, assuming an observation interval of N_T symbols of the CPFSK waveform and producing an optimum decision of one symbol. This receiver is often complex, but it is possible to find a simple receiver optimum for low SNR. This is the average matched filter receiver [3]-[7]. Hirt and Pasupathy [7], [14] have analysed this receiver for some specific full response signal schemes.

We are generalizing this idea of an average matched filter receiver. The Average Matched Filter receiver is introduced also for partial response scheme and various possibilities of choosing the filterbank are investigated. This generalized receiver is called multiple-symbol Average Matched Filter (AMF) receiver. This receiver has mostly fewer filters than the optimum receiver and only a small amount of processing. The receiver makes single symbol decisions. This simplified receiver is of course sub-optimum and the performance is not acceptable for some schemes. It works well only for schemes based on short frequency pulses. A better performance is obtained by increasing the number of receiver filters and match them better to the transmitted signals.

Throughout part IV we will only consider coherent detection assuming perfect synchronization. Furthermore the channel is assumed to be an additive white Gaussian noise channel.

This part is organized in the following manner. Section II defines the receiver principles and the receiver structure. A short background on CPM-signals is also given. Section III contains the performance analysis. Here a distance measure and an upper bound on the error probability using the union bound are derived. In section IV some symmetry properties of the distance is derived. A variety of numerical results in the form of minimum distance results and upper bounds on the symbol error probability are given in section V. Also some simulated error probabilities are given. Both binary and quaternary systems, and also one octal scheme are considered. Section VI presents a comparison of the complexity of the receivers considered in this thesis. Part IV is terminated by a discussion and conclusions section.

II. RECEIVER PRINCIPLES AND RECEIVER STRUCTURE

A short definition of the CPM signaling scheme is repeated in this part [1], [2].

Let $\underline{\alpha} = \dots, \alpha_{n-2}, \alpha_{n-1}, \alpha_n, \alpha_{n+1}, \alpha_{n+2}, \dots$ be an infinitely long sequence of independent M-ary data symbols taking the values $\pm 1, \pm 3, \dots, \pm(M-1)$ with equal probability. The transmitted modulated constant envelope signal is given by

$$s(t, \underline{\alpha}) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_0 t + \varphi(t, \underline{\alpha}) + \varphi_0\right) \quad (1)$$

where the information carrying phase is

$$\varphi(t, \underline{\alpha}) = 2\pi h \sum_{i=-\infty}^{\infty} \alpha_i \cdot q(t-iT) \quad (2)$$

The symbol energy is denoted E , the symbol time T and the carrier frequency f_0 . h is the modulation index and φ_0 is an arbitrary constant phase shift and can be set to zero without loss of generality. The phase response $q(t)$ is defined by

$$q(t) = \int_{-\infty}^t g(\tau) d\tau \quad (3)$$

where $g(t)$ is the frequency response, [1], [2].

The frequency responses considered are the Rectangular pulse (REC) of length L symbol intervals defined by

$$g(t) = \begin{cases} \frac{1}{2LT} & ; \quad 0 < t < LT \\ 0 & ; \quad \text{otherwise} \end{cases} \quad (4)$$

and the Raised Cosine pulse (RC) of length L symbol intervals defined by

$$g(t) = \begin{cases} \frac{1}{2LT} \left(1 - \cos \frac{2\pi t}{LT}\right) & ; \quad 0 \leq t \leq LT \\ 0 & ; \quad \text{otherwise} \end{cases} \quad (5)$$

The received signal is $r(t)$

$$r(t) = s(t, \underline{\alpha}) + n(t) \quad (6)$$

where $n(t)$ is additive white Gaussian noise with one-sided spectral density N_0 . The signal is observed over N_T symbol intervals and the receiver makes a decision on the first symbol. Let this symbol be α_0 . Osborne and Luntz [6] have proposed the Average Matched Filter (AMF) receiver, which is optimum for low signal-to-noise ratios (SNR's), to be used for 1REC. Hirt and Pasupathy have analysed the AMF receiver for various full response schemes [7], [14]. This receiver is a maximum likelihood type receiver consisting of M filters. The impulse response of the filters is the average of all signals having $\alpha_0 = -(M-1)$, $\alpha_0 = -(M-3), \dots, \alpha_0 = (M-1)$ respectively. The output of the filters are sampled every symbol interval and the value of α_0 for the filter giving the maximum likelihood function is taken as an estimate of α_0 .

For partial response schemes this receiver has to be modified. For these schemes the signal depends on the $L-1$ prehistory symbols [2]. The prehistory symbols are defined to be the $L-1$ symbol prior to α_0 , i.e. $\alpha_{-(L-1)}, \dots, \alpha_{-1}$. L is the duration in symbol intervals of the frequency pulse. Thus the average signal is in general nonzero for $-(L-1)T \leq t \leq 0$. In this case there are two possible choices. Still only M filters can be used, one for each value of α_0 , with impulse responses equal to the average over all other symbols, i.e. including the prehistory symbols. These filters are poorly matched to the signals and from [3] it is known that they give a very bad performance. The other possibility is to use one filter set for each prehistory and choose the set according to previous decisions. In this case the impulse response of the filters is the average over all symbols except the prehistory symbols and the decision symbol. This is the receiver we propose in this part.

These filter sets can however be generalized. Instead of having only M filters in each set, M^{N_A} filters can be used. M^{N_A-1} of these filters have the same value of α_0 . The impulse response of each filter is the average of all symbols except $\alpha_{-(L-1)}, \dots, \alpha_0, \dots, \alpha_{N_A-1}$. Consider the transmitted sequence α_{-N_T-1}

$$\underline{\alpha}_{N_T-1} = \dots, \alpha_{-L}, \alpha_{-(L-1)}, \dots, \alpha_{-1}, \alpha_0, \alpha_1, \dots, \alpha_{N_A-1}, \alpha_{N_A}, \dots, \alpha_{N_T-1} \quad (7)$$

The phase in the interval $0 \leq t \leq N_T \cdot T$ corresponding to this sequence does not depend on the symbols α_i , $i \leq -L$ except for a constant phase shift. This constant phase shift gives a constant phase shift in the average signal and thus in the filter. This is just a rotation of the phase and will be neglected for the time being. Now define a prehistory data symbol vector

$$\underline{A}_{-1} = \alpha_{-(L-1)}, \dots, \alpha_{-1} \quad (8)$$

The symbols $\alpha_0, \dots, \alpha_{N_T-1}$ are divided into two vectors

$$\underline{A}_0 = \alpha_0, \dots, \alpha_{N_A-1} \quad (9)$$

and

$$\underline{A}_1 = \alpha_{N_A}, \dots, \alpha_{N_T-1} \quad (10)$$

Thus $\underline{\alpha}_{N_T-1}$ in short notation, can be written

$$\underline{\alpha}_{N_T-1} = \underline{\alpha}_{-L}, \underline{A}_{-1}, \underline{A}_0, \underline{A}_1 \quad (11)$$

When $N_A = N_T$, $\underline{A}_1$ does not exist.

The filterbank consists of a set of filters to every possible prehistory $\underline{A}_{-1}$. The filters in every set are matched to the average of all transmitted waveforms, corresponding to the given prehistory $\underline{A}_{-1}$, and containing the same symbols $\underline{A}_0$ in the beginning of the observation interval. This means that the N_A first symbols are fixed when the average is made. In every set of filters, there is one filter corresponding to every possible sequence $\underline{A}_0$, i.e. a total of M^{N_A} filters in every set. With the new definitions $s(t, \underline{\alpha}_{N_T-1})$ can be written

$$s(t, \underline{\alpha}_{N_T-1}) = s(t, \underline{A}_{-1}, \underline{A}_0, \underline{A}_1, \vartheta) \quad (12)$$

where ϑ is the constant phase given by $\underline{\alpha}_{-L}$, i.e.

$$\vartheta = \pi h \sum_{i=-\infty}^{-L} \alpha_i \quad (13)$$

Thus the average signal given $\underline{A}_{-1}$ and $\underline{A}_0$ is

$$\bar{s}(t, \underline{A}_{-1}, \underline{A}_0, \vartheta) = \frac{1}{m} \sum_{i=0}^{m-1} s(t, \underline{A}_{-1}, \underline{A}_0, \underline{A}_{1i}, \vartheta) \quad (14)$$

where $m = M^{(N_T - N_A)}$ is the total of sequences having a given $\underline{A}_{-1}$ and $\underline{A}_0$. $\underline{A}_{1i}$ is all possible sequences with $N_T - N_A$ M-ary symbols. Index i is an enumeration of these sequences. The impulse response $h(t, \underline{A}_{-1}, \underline{A}_0, \vartheta)$ of the different filters in the bank is

$$h(t, \underline{A}_{-1}, \underline{A}_0, \vartheta) = \bar{s}(N_T \cdot T - t, \underline{A}_{-1}, \underline{A}_0, \vartheta) \quad (15)$$

The filterbank consists of $M^{(L-1)}$ different sets of filters, one set for every possible prehistory. Each set consists of M^{N_A} different filters, one for every possible sequence $\underline{A}_0$. Note that, for the AMF receiver considered in [7], $\underline{A}_0 = \alpha_0$. This means that the receiver consists of a total of $M^{(L-1)} \cdot M^{N_A} = M^{(L-1+N_A)}$ filters. This is the maximum number of filters. For some schemes and modulation indices, some of them might be equal and thus the number of different filters is smaller than $M^{(L-1+N_A)}$.

The receiver chooses one of the sets of filters and computes

$$\ell_j = \int_0^{N_T \cdot T} r(t) \bar{s}(t, \hat{\underline{A}}_{-1}, \hat{\underline{A}}_0, \hat{\vartheta}) dt \quad (16)$$

for all possible sequences $\underline{A}_{0j} = (\alpha_0, \alpha_1, \dots, \alpha_{N_A-1})_j$ in this set and chooses the α_0 , corresponding to the sequence $\underline{A}_{0j}$ giving the largest ℓ_j , as an estimate of α_0 , the decision symbol. Index j is an enumeration of the different sequences $\underline{A}_0$. $\hat{\underline{A}}_{-1}$ and $\hat{\vartheta}$ are estimates of $\underline{A}_{-1}$ and ϑ respectively given by the previous decisions. The receiver model is shown in figure 1.

The constant phase shift ϑ can be handled by multiplications of $\cos \vartheta$ and $\sin \vartheta$. This is easily seen by using (1) and (14).

$$\begin{aligned}
\bar{s}(t, A_{-1}, A_0, \vartheta) &= \frac{1}{m} \sum_{i=0}^{m-1} \sqrt{\frac{2E}{T}} \cos \left[\omega_0 t + \varphi(t, A_{-1}, A_0, A_{1i}) + \vartheta \right] = \\
&= \sqrt{\frac{2E}{T}} \left\{ \cos \omega_0 t \left[A(t, A_{-1}, A_0) \cos \vartheta - B(t, A_{-1}, A_0) \sin \vartheta \right] - \right. \\
&\quad \left. - \sin \omega_0 t \left[B(t, A_{-1}, A_0) \cos \vartheta + A(t, A_{-1}, A_0) \sin \vartheta \right] \right\} \quad (17)
\end{aligned}$$

where

$$A(t, A_{-1}, A_0) = \frac{1}{m} \sum_{i=0}^{m-1} \cos \varphi(t, A_{-1}, A_0, A_{1i}) \quad (18)$$

and

$$B(t, A_{-1}, A_0) = \frac{1}{m} \sum_{i=0}^{m-1} \sin \varphi(t, A_{-1}, A_0, A_{1i}) \quad (19)$$

Thus the filters can be realized in baseband as in figure 2.

Nonuniformly weighted AMF filters

So far only AMF filters where all the signals have equal weight have been considered. In many cases the performance can be improved by using a non-uniformly weighted AMF filter instead. These weights can be optimized to give the best performance at a given error probability. This is a difficult optimization problem, since the optimum weights depend on the error probability level. Any optimization has thus not been done but for some schemes weighted AMF filters have been considered [3] and they improved the performance slightly. These filters have been found by trial and error methods.

III. PERFORMANCE ANALYSIS

In this section, the minimum Euclidean distance and an expression for the symbol error probability of the coherent multiple-symbol AMF receiver are derived. We assume zero-mean additive white Gaussian noise and that the receiver observes the CPM signal over N_T symbol intervals and makes an optimum decision on one symbol. The updating of prehistory $\hat{A}_{-1}$ and constant phase $\hat{\phi}$ will of course affect the symbol error probability in general but in this section they are assumed to be error free, which means that the correct filter set is chosen. Thus $\hat{\phi}$ can be set to zero in the following, without loss of generality. It is easily seen that the decision variable l_1 is a Gaussian random variable, therefore its mean and variance determine the symbol error probability. The distance introduced and the symbol error probability derived in this part are both conditioned on error free updating.

Assume the M-ary sequence $\alpha_{N_T-1} = \underline{A}_{-1}, \underline{A}_0, \underline{A}_1$ is being transmitted. The decision variables l'_j giving a correct decision are given by

$$l'_j = \int_0^{N_T \cdot T} r(t) \cdot \bar{s}(t, \underline{A}_{-1}, \underline{A}_0) dt \quad ; \quad j = 1, 2, \dots, M^{(N_A-1)} \quad (20)$$

where $\alpha'_0 = \alpha_0$. The decision variables $\tilde{l}_j$ giving a wrong decision are given by

$$\tilde{l}_j = \int_0^{N_T \cdot T} r(t) \cdot \bar{s}(t, \underline{A}_{-1}, \tilde{\underline{A}}_0) dt \quad ; \quad j = 1, 2, \dots, (M-1)M^{(N_A-1)} \quad (21)$$

where $\tilde{\alpha}_0 \neq \alpha_0$. The receiver compares $\max_j l'_j$ and $\max_j \tilde{l}_j$ and an error is made if $\max_j \tilde{l}_j$ is greater than $\max_j l'_j$. Thus the symbol error probability when α_{N_T-1} is transmitted is given by

$$P[\epsilon | \alpha_{N_T-1}] = \Pr\{\max_j l'_j < \max_j \tilde{l}_j\} \quad (22)$$

This symbol error probability is difficult to calculate but can be upper bounded by comparing l_1 , the decision variable from the filter $h(t, \underline{A}_{-1}, \underline{A}_0)$, with $\max_j \tilde{l}_j$. Note that l_1 is not greater than $\max_j l'_j$ because l_1 is equal to l'_j for one value of j . Thus

$$P[\varepsilon | \alpha_{N_T-1}] \leq \Pr\{\ell_1 < \max_j \tilde{\ell}_j\} \leq \sum_j \Pr\{\ell_1 < \tilde{\ell}_j\} = \sum_j \Pr\{\ell_1 - \tilde{\ell}_j < 0\} \quad (23)$$

where also the union bound technique is used. Now define $\lambda_j = \ell_1 - \tilde{\ell}_j$ and by using (20) and (21) λ_j is given by

$$\lambda_j = \int_0^{N_T \cdot T} r(t) \cdot [\bar{s}(t, \underline{A}_{-1}, \underline{A}_0) - \bar{s}(t, \underline{A}_{-1}, \tilde{\underline{A}}_0)] dt \quad (24)$$

The probability that $\tilde{\ell}_j$ is greater than ℓ_1 , conditioned that α_{N_T-1} is being transmitted is

$$\Pr\{\tilde{\ell}_j > \ell_1\} = \Pr\{\lambda_j < 0\} = Q\left(\sqrt{d_j^2 \frac{E_b}{N_0}}\right) \quad (25)$$

where d_j^2 is defined as the normalized squared Euclidean distance between ℓ_1 and $\tilde{\ell}_j$. E_b is the energy per bit, i.e. $E_b = E/\log_2 M$ and $Q(x)$ is the Gaussian error function

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \quad (26)$$

d_j^2 is not a Euclidean distance in signal space but behaves like a Euclidean distance in the symbol error probability for large signal-to-noise ratios. d_j^2 is given from the mean of λ_j , conditioned on α_{N_T-1}

$$\bar{\lambda}_j = E\{\lambda_j | \alpha_{N_T-1}\} \quad (27)$$

and the variance of λ_j , which is independent of any particular transmitted α_{N_T-1}

$$\sigma^2 = E\{(\lambda_j - \bar{\lambda}_j)^2\} \quad (28)$$

through the formula

$$d_j^2 = \frac{\bar{\lambda}_j^2}{\sigma^2} \cdot \frac{N_0}{E_b} \quad (29)$$

Now, it is seen that

$$\bar{\lambda}_j = E\{\lambda_j | \alpha_{N_T-1}\} = \int_0^{N_T \cdot T} s(t, \underline{A}_{-1}, \underline{A}_0, \underline{A}_1) \cdot [\bar{s}(t, \underline{A}_{-1}, \underline{A}_0) - \bar{s}(t, \underline{A}_{-1}, \tilde{\underline{A}}_0)] dt \quad (30)$$

where $s(t, \underline{A}_{-1}, \underline{A}_0, \underline{A}_1)$ is given in (1) and $\bar{s}(t, \underline{A}_{-1}, \underline{A}_0)$ is given in (14).

It is easily seen that ($\omega_0 = 2\pi f_0$, $\vartheta = 0$)

$$\bar{s}(t, \underline{A}_{-1}, \underline{A}_0) = \sqrt{\frac{2E}{T}} [\cos \omega_0 t \cdot A(t, \underline{A}_{-1}, \underline{A}_0) - \sin \omega_0 t \cdot B(t, \underline{A}_{-1}, \underline{A}_0)] \quad (31)$$

where $A(t, \underline{A}_{-1}, \underline{A}_0)$ and $B(t, \underline{A}_{-1}, \underline{A}_0)$ are given in (18) and (19) respectively.

The same also holds for $\bar{s}(t, \underline{A}_{-1}, \tilde{\underline{A}}_0)$ and using this gives

$$\begin{aligned} \bar{\lambda}_j = \frac{2E}{T} \cdot \int_0^{N_T \cdot T} \cos[\omega_0 t + \varphi(t, \underline{A}_{-1}, \underline{A}_0, \underline{A}_1)] \cdot \left\{ \cos \omega_0 t \cdot [A(t, \underline{A}_{-1}, \underline{A}_0) - A(t, \underline{A}_{-1}, \tilde{\underline{A}}_0)] - \right. \\ \left. - \sin \omega_0 t \cdot [B(t, \underline{A}_{-1}, \underline{A}_0) - B(t, \underline{A}_{-1}, \tilde{\underline{A}}_0)] \right\} dt \quad (32) \end{aligned}$$

and assuming $\omega_0 T \gg 1$

$$\begin{aligned} \bar{\lambda}_j = \frac{E_b \cdot \log_2(M)}{T} \cdot \int_0^{N_T \cdot T} \left\{ \cos \varphi(t, \underline{A}_{-1}, \underline{A}_0, \underline{A}_1) \cdot [A(t, \underline{A}_{-1}, \underline{A}_0) - A(t, \underline{A}_{-1}, \tilde{\underline{A}}_0)] + \right. \\ \left. + \sin \varphi(t, \underline{A}_{-1}, \underline{A}_0, \underline{A}_1) \cdot [B(t, \underline{A}_{-1}, \underline{A}_0) - B(t, \underline{A}_{-1}, \tilde{\underline{A}}_0)] \right\} dt \quad (33) \end{aligned}$$

It is easily seen, that the variance is given by

$$\sigma^2 = \frac{N_0}{2} \cdot \int_0^{N_T \cdot T} [\bar{s}(t, \underline{A}_{-1}, \underline{A}_0) - \bar{s}(t, \underline{A}_{-1}, \tilde{\underline{A}}_0)]^2 dt \quad (34)$$

and assuming $\omega_0 T \gg 1$ gives

$$\begin{aligned} \sigma^2 = \frac{N_0 E_b \cdot \log_2(M)}{2T} \int_0^{N_T \cdot T} \left\{ [A(t, \underline{A}_{-1}, \underline{A}_0) - A(t, \underline{A}_{-1}, \tilde{\underline{A}}_0)]^2 + \right. \\ \left. + [B(t, \underline{A}_{-1}, \underline{A}_0) - B(t, \underline{A}_{-1}, \tilde{\underline{A}}_0)]^2 \right\} dt \quad (35) \end{aligned}$$

The normalized squared Euclidean distance is then given by

$$d_j^2 = \frac{2}{T} \cdot \log_2(M) \cdot$$

$$\frac{\left\{ \int_0^{N_T \cdot T} [A(t, \underline{A}_{-1}, \underline{A}_0) - A(t, \underline{A}_{-1}, \tilde{\underline{A}}_0)] \cos \varphi(t, \underline{A}_{-1}, \underline{A}_0, \underline{A}_1) + \right.}{N_T \cdot T \int_0^{N_T \cdot T} \left\{ [A(t, \underline{A}_{-1}, \underline{A}_0) - A(t, \underline{A}_{-1}, \tilde{\underline{A}}_0)]^2 + \right.}$$

$$\left. \left. + [B(t, \underline{A}_{-1}, \underline{A}_0) - B(t, \underline{A}_{-1}, \tilde{\underline{A}}_0)] \sin \varphi(t, \underline{A}_{-1}, \underline{A}_0, \underline{A}_1) \right] dt \right\}^2}{\left. + [B(t, \underline{A}_{-1}, \underline{A}_0) - B(t, \underline{A}_{-1}, \tilde{\underline{A}}_0)]^2 \right\} dt} \quad (36)$$

The minimum distance corresponding to an incorrect decision is a measure of the asymptotic error performance [2]. The normalized squared minimum Euclidean distance is given by

$$d_{\min}^2 = \min_{\substack{\underline{A}_{-1}, \underline{A}_0, \underline{A}_1, \tilde{\underline{A}}_0 \\ \alpha_0 \neq \tilde{\alpha}_0}} (d_j^2) \quad (37)$$

From the discussion above it is known that this in fact gives a lower bound on the minimum distance. In many cases however, especially when the filters are short, $\max_j \ell_j^1$ is given by ℓ_1 for large signal-to-noise ratios, i.e. $E\{\ell_1\} = \max_j E\{\ell_j^1\}$ and thus the minimum distance is exact. Note also that there is only one variable giving a correct decision when $N_A=1$ and thus the distance is exact in these cases.

Thus it is seen that the distance have to be calculated for every possible transmitted sequence, including the prehistory. For one specific transmitted sequence the pairwise distance has to be calculated between the correct decision variable and all the variables giving an incorrect decision, given the correct set of filters have been selected. Fortunately some of all these distances are equal and therefore not all of them have to be calculated, as is shown in the next section.

The symbol error probability, conditioned on a specific transmitted sequence $\underline{\alpha}_{N_T-1}$ is upper bounded by, see (23)

$$P[\epsilon|\alpha_{N_T-1}] \leq \sum_j Q\left(\sqrt{d_j^2 \frac{E_b}{N_0}}\right) \quad (38)$$

where d_j^2 is all distances corresponding to an erroneous decision, when α_{N_T-1} has been transmitted (d_j^2 is a function of α_{N_T-1}).

The symbol error probability is then obtained, by averaging over all possible transmitted waveforms, as a function of E_b/N_0 (SNR)

$$P_e = \frac{1}{m_1} \sum_{\alpha_{N_T-1}} P[\epsilon|\alpha_{N_T-1}] \leq \frac{1}{m_1} \sum_{\alpha_{N_T-1}} \sum_j Q\left(\sqrt{d_j^2 \frac{E_b}{N_0}}\right) \quad (39)$$

where the total number of α_{N_T-1} is $m_1 = M^{(N_T+L-1)}$.

Note that for a binary AMF receiver with $N_A=1$ there is only one decision variable corresponding to every α_{N_T-1} giving a wrong decision, and thus the expression for P_e is exact in this case, [3].

IV. PROPERTIES OF THE DISTANCE AND THE UNIFORMLY WEIGHTED AMF FILTER

In this section some properties of the Euclidean distance are derived. It was mentioned in the previous section that quite a large number of distances have to be calculated, to get the minimum distance $d_{\min}^2$. Fortunately some symmetries exist so that not all of them have to be calculated.

The Euclidean distance for the multiple-symbol AMF receiver is independent of the prehistory $\underline{A}_{-1}$, and this means that only one set of filters have to be considered, when $d_{\min}^2$ and the upper bound on the error probability are calculated. To show this, we assume a sequence $\alpha_{N_T-1} = \underline{A}_{-1}, \underline{A}_0, \underline{A}_1$ being transmitted and calculate the distance d_j^2 between the output of the filter $h(t, \underline{A}_{-1}, \underline{A}_0)$ giving the correct decision and the filter $h(t, \underline{A}_{-1}, \tilde{\underline{A}}_0)$ giving a wrong decision, i.e. $\underline{A}_0$ and $\tilde{\underline{A}}_0$ are chosen so that $\alpha_0 \neq \tilde{\alpha}_0$ (θ is assumed to be zero just as in section III). This distance is given in (36).

From (2) we have

$$\begin{aligned} \varphi(t, \alpha_{N_T-1}) &= 2\pi h \sum_{i=-(L-1)}^{N_T-1} \alpha_i \cdot q(t-iT) = \\ &= 2\pi h \sum_{i=-(L-1)}^{-1} \alpha_i \cdot q(t-iT) + 2\pi h \sum_{i=0}^{N_T-1} \alpha_i \cdot q(t-iT) \end{aligned} \quad (40)$$

Now introduce

$$\Theta_p(t, \underline{A}_{-1}) = 2\pi h \sum_{i=-(L-1)}^{-1} \alpha_i \cdot q(t-iT) \quad (41)$$

and

$$\Theta(t, \underline{A}_0, \underline{A}_1) = 2\pi h \sum_{i=0}^{N_T-1} \alpha_i \cdot q(t-iT) \quad (42)$$

When $L=1$ the summation in $\Theta_p(t, \underline{A}_{-1})$ is not defined, and $\Theta_p(t, \underline{A}_{-1})$ is by definition zero.

Using these definitions, gives

$$\varphi(t, \alpha_{N_T-1}) = \Theta_p(t, \underline{A}_{-1}) + \Theta(t, \underline{A}_0, \underline{A}_1) \quad (43)$$

Thus all information about the prehistory is contained in $\Theta_p(t, \underline{A}_{-1})$.

Rewriting $\varphi(t, \alpha_{N_T-1})$ in this way gives

$$\begin{aligned}
 A(t, \underline{A}_{-1}, \underline{A}_0) &= \frac{1}{m} \sum_{i=0}^{m-1} \cos[\theta_p(t, \underline{A}_{-1}) + \theta(t, \underline{A}_0, \underline{A}_{1i})] = \\
 &= \cos\theta_p(t, \underline{A}_{-1}) \frac{1}{m} \sum_{i=0}^{m-1} \cos\theta(t, \underline{A}_0, \underline{A}_{1i}) - \\
 &- \sin\theta_p(t, \underline{A}_{-1}) \frac{1}{m} \sum_{i=0}^{m-1} \sin\theta(t, \underline{A}_0, \underline{A}_{1i}) \quad (44)
 \end{aligned}$$

where $m = M^{(N_T - N_A)}$, since $\theta_p(t, \underline{A}_{-1})$ is independent of summation index i . Rewriting $B(t, \underline{A}_{-1}, \underline{A}_0)$ in the same way gives,

$$\begin{aligned}
 B(t, \underline{A}_{-1}, \underline{A}_0) &= \sin\theta_p(t, \underline{A}_{-1}) \frac{1}{m} \sum_{i=0}^{m-1} \cos\theta(t, \underline{A}_0, \underline{A}_{1i}) + \\
 &+ \cos\theta_p(t, \underline{A}_{-1}) \frac{1}{m} \sum_{i=0}^{m-1} \sin\theta(t, \underline{A}_0, \underline{A}_{1i}) \quad (45)
 \end{aligned}$$

Thus

$$\begin{aligned}
 A(t, \underline{A}_{-1}, \underline{A}_0) - A(t, \underline{A}_{-1}, \tilde{\underline{A}}_0) &= \\
 &= \cos\theta_p(t, \underline{A}_{-1}) \frac{1}{m} \sum_{i=0}^{m-1} [\cos\theta(t, \underline{A}_0, \underline{A}_{1i}) - \cos\theta(t, \tilde{\underline{A}}_0, \underline{A}_{1i})] - \\
 &- \sin\theta_p(t, \underline{A}_{-1}) \frac{1}{m} \sum_{i=0}^{m-1} [\sin\theta(t, \underline{A}_0, \underline{A}_{1i}) - \sin\theta(t, \tilde{\underline{A}}_0, \underline{A}_{1i})] \quad (46)
 \end{aligned}$$

and

$$\begin{aligned}
 B(t, \underline{A}_{-1}, \underline{A}_0) - B(t, \underline{A}_{-1}, \tilde{\underline{A}}_0) &= \\
 &= \cos\theta_p(t, \underline{A}_{-1}) \frac{1}{m} \sum_{i=0}^{m-1} [\sin\theta(t, \underline{A}_0, \underline{A}_{1i}) - \sin\theta(t, \tilde{\underline{A}}_0, \underline{A}_{1i})] + \\
 &+ \sin\theta_p(t, \underline{A}_{-1}) \frac{1}{m} \sum_{i=0}^{m-1} [\cos\theta(t, \underline{A}_0, \underline{A}_{1i}) - \cos\theta(t, \tilde{\underline{A}}_0, \underline{A}_{1i})] \quad (47)
 \end{aligned}$$

The transmitted sequence is $\alpha_{N_T-1} = \underline{A}_{-1}, \underline{A}_0, \underline{A}_1$. Thus using (43) gives

$$\begin{aligned} \cos\varphi(t, \underline{A}_{-1}, \underline{A}_0, \underline{A}_1) &= \cos\theta_p(t, \underline{A}_{-1}) \cos\theta(t, \underline{A}_0, \underline{A}_1) - \\ &- \sin\theta_p(t, \underline{A}_{-1}) \sin\theta(t, \underline{A}_0, \underline{A}_1) \end{aligned} \quad (48)$$

and

$$\begin{aligned} \sin\varphi(t, \underline{A}_{-1}, \underline{A}_0, \underline{A}_1) &= \cos\theta_p(t, \underline{A}_{-1}) \sin\theta(t, \underline{A}_0, \underline{A}_1) + \\ &+ \sin\theta_p(t, \underline{A}_{-1}) \cos\theta(t, \underline{A}_0, \underline{A}_1) \end{aligned} \quad (49)$$

To simplify the equations, we first consider the expression inside the integral in the numerator of d_j^2 . Simplifications give

$$\begin{aligned} &[A(t, \underline{A}_{-1}, \underline{A}_0) - A(t, \underline{A}_{-1}, \tilde{\underline{A}}_0)] \cos\varphi(t, \underline{A}_{-1}, \underline{A}_0, \underline{A}_1) + \\ &+ [B(t, \underline{A}_{-1}, \underline{A}_0) - B(t, \underline{A}_{-1}, \tilde{\underline{A}}_0)] \sin\varphi(t, \underline{A}_{-1}, \underline{A}_0, \underline{A}_1) = \\ &= \cos\theta(t, \underline{A}_0, \underline{A}_1) \frac{1}{m} \sum_{i=0}^{m-1} [\cos\theta(t, \underline{A}_0, \underline{A}_{1i}) - \cos\theta(t, \tilde{\underline{A}}_0, \underline{A}_{1i})] + \\ &+ \sin\theta(t, \underline{A}_0, \underline{A}_1) \frac{1}{m} \sum_{i=0}^{m-1} [\sin\theta(t, \underline{A}_0, \underline{A}_{1i}) - \sin\theta(t, \tilde{\underline{A}}_0, \underline{A}_{1i})] \end{aligned} \quad (50)$$

which is independent of $\underline{A}_{-1}$, i.e. the prehistory. Thus the whole integral in the numerator of d_j^2 is independent of $\underline{A}_{-1}$.

The same derivations on the expression inside the integral in the denominator give

$$\begin{aligned} &[A(t, \underline{A}_{-1}, \underline{A}_0) - A(t, \underline{A}_{-1}, \tilde{\underline{A}}_0)]^2 + [B(t, \underline{A}_{-1}, \underline{A}_0) - B(t, \underline{A}_{-1}, \tilde{\underline{A}}_0)]^2 = \\ &= \left\{ \frac{1}{m} \sum_{i=0}^{m-1} [\cos\theta(t, \underline{A}_0, \underline{A}_{1i}) - \cos\theta(t, \tilde{\underline{A}}_0, \underline{A}_{1i})] \right\}^2 + \\ &+ \left\{ \frac{1}{m} \sum_{i=0}^{m-1} [\sin\theta(t, \underline{A}_0, \underline{A}_{1i}) - \sin\theta(t, \tilde{\underline{A}}_0, \underline{A}_{1i})] \right\}^2 \end{aligned} \quad (51)$$

which is also independent of $\underline{A}_{-1}$, and thus the whole integral is independent of $\underline{A}_{-1}$.

Thus both the numerator and the denominator of d_j^2 are independent of A_{-1} and so is d_j^2 .

The distance has also a sign symmetry, so that the transmitted sequence $\underline{\alpha}$ gives the same set of distances as the sequence $-\underline{\alpha}$. From (50) and (51) it is seen that changing A_0 against $-A_0$ and A_1 against $-A_1$ give the same expressions if $\bar{A}_0$ is changed to $-\bar{A}_0$. Since $\alpha_0 \neq \tilde{\alpha}_0$, also $-\alpha_0 \neq -\tilde{\alpha}_0$. Thus with the above substitution half of the distances required are given. This means that d_j^2 has to be calculated for only the sequences having for example $\alpha_0 > 0$, then we know that the sequences having $\alpha_0 < 0$ give the same set of distances.

With these symmetries we have reduced the number of calculations with a factor $2 \cdot M^{(L-1)}$ where L is the duration in symbol intervals of the frequency response.

Another interesting thing to note, is that the whole set of distances for the AMF receiver with $N_A=1$ and observation length N_T is a subset of the distances for a multiple-symbol AMF receiver with $N_A \geq 2$ but with observation length N_T+N_A-1 . This holds for every type of multiple-symbol AMF. The set of distances for a multiple-symbol AMF receiver with a given number of fixed symbols, say N_A , and observation length N_T is a subset of the distances for any multiple-symbol AMF receiver with more than N_A fixed symbols, say N_{A1} , and observation length $N_T+N_{A1}-N_A$.

This is immediate clear only by looking at the receiver and we only give a binary example in figure 3. There one set of filters are given for an AMF receiver with $N_A=2$. The two upper filters are the filters that give correct decisions if the transmitted signal has $\alpha_0 = -1$. These two filters have the same prehistory $\bar{A}_{-1}$, but also α_0 is the same. This means that by moving one symbol interval to the right with the decision symbol, both the filters have the same prehistory and one fixed symbol. This means that they are the filters of an AMF receiver with $N_A=1$ and an observation interval that is one symbol interval shorter. The same also holds for the two lower filters.

The AMF filters are time limited if $h = \frac{1}{2} (2n-1)$, where n is an integer. This property will now be shown.

From [2] we know that

$$q(t) \equiv 0 \quad t \leq 0 \quad (52)$$

and

$$q(t) = \frac{1}{2} \quad t \geq L \cdot T \quad (53)$$

for the schemes considered in this paper. From the previous sections it is known that if $A(t, \underline{A}_{-1}, \underline{A}_0)$ and $B(t, \underline{A}_{-1}, \underline{A}_0)$ are time limited, then $\bar{s}(t, \underline{A}_{-1}, \underline{A}_0)$ is time limited. $A(t, \underline{A}_{-1}, \underline{A}_0)$ and $B(t, \underline{A}_{-1}, \underline{A}_0)$ are given in (18) and (19) respectively.

From (2) it is seen that the transmitted phase can be written ($\vartheta=0$)

$$\varphi(t, \alpha_{N_T-1}) = 2\pi h \left[\sum_{i=-\infty}^{N_A-1} \alpha_i q(t-iT) + \sum_{i=N_A}^{N_T-1} \alpha_i q(t-iT) \right] \quad (54)$$

Using this in (18) gives

$$\begin{aligned} A(t, \underline{A}_{-1}, \underline{A}_0) = \frac{1}{m} \sum_{k=0}^{m-1} \left\{ \cos \left(2\pi h \sum_{i=-\infty}^{N_A-1} \alpha_i q(t-iT) \right) \cos \left(2\pi h \sum_{i=N_A}^{N_T-1} \alpha_{i,k} q(t-iT) \right) - \right. \\ \left. - \sin \left(2\pi h \sum_{i=-\infty}^{N_A-1} \alpha_i q(t-iT) \right) \sin \left(2\pi h \sum_{i=N_A}^{N_T-1} \alpha_{i,k} q(t-iT) \right) \right\} \quad (55) \end{aligned}$$

where $m = M^{(N_T-N_A)}$. The two first factors are constant, independent of k , which gives

$$\begin{aligned} A(t, \underline{A}_{-1}, \underline{A}_0) = \cos \left(2\pi h \sum_{i=-\infty}^{N_A-1} \alpha_i q(t-iT) \right) \frac{1}{m} \sum_{k=0}^{m-1} \cos \left(2\pi h \sum_{i=N_A}^{N_T-1} \alpha_{i,k} q(t-iT) \right) - \\ - \sin \left(2\pi h \sum_{i=-\infty}^{N_A-1} \alpha_i q(t-iT) \right) \frac{1}{m} \sum_{k=0}^{m-1} \sin \left(2\pi h \sum_{i=N_A}^{N_T-1} \alpha_{i,k} q(t-iT) \right) \quad (56) \end{aligned}$$

Now

$$\sum_{k=0}^{m-1} \sin \left(2\pi h \sum_{i=N_A}^{N_T-1} \alpha_{i,k} q(t-iT) \right) = 0 \quad : \quad t \geq N_A \cdot T \quad (57)$$

because, to every sequence $(\alpha_{N_A}, \alpha_{N_A+1}, \dots, \alpha_{N_T-1})$ giving a phase $\varphi_1(t)$, there is always a sequence $(-\alpha_{N_A}, -\alpha_{N_A+1}, \dots, -\alpha_{N_T-1})$ giving the phase $-\varphi_1(t)$. Since

$$\sin \varphi_1(t) + \sin(-\varphi_1(t)) = \sin \varphi_1(t) - \sin \varphi_1(t) = 0 \quad (58)$$

the sum in (57) is also zero.

$$\sum_{k=0}^{m-1} \cos \left(2\pi h \sum_{i=N_A}^{N_T-1} \alpha_{i,k} q(t-iT) \right) = 0 \quad (58)$$

if $h = \frac{1}{2} (2n-1)$, where n is an integer, and $t \geq (L+N_A)T$ because to every sequence $(\alpha_{N_A}, \alpha_{N_A+1}, \dots, \alpha_{N_T-1})$ giving a phase $\varphi_1(t)$, there is always a sequence $(\alpha_{N_A}, -\alpha_{N_A+1}, \dots, -\alpha_{N_T-1})$ giving a phase $\varphi_2(t)$, so that $\varphi_1(t)$ and $\varphi_2(t)$ are symmetric around $\pi/2 (2n-1)$, for $t \geq (L+N_A)T$ and therefore $\cos \varphi_1(t) + \cos \varphi_2(t) = 0$, $t \geq (L+N_A)T$. The proof for $B(t, \underline{A}_{-1}, \underline{A}_0)$ is analogous. This means that even weighted AMF-filters have a maximum length of $L+N_A$ symbol intervals, when $h = \frac{1}{2} (2n-1)$, and especially when $h = \frac{1}{2}$. This is not true for weighted AMF-filters in general.

Figure 4 shows the average signals building up the AMF-filter for a 3RC pulse and modulation index $h = \frac{1}{2}$ and prehistory $\underline{A}_{-1} = 1, 1$.

V. NUMERICAL RESULTS

Euclidean distances have been calculated for both binary, quaternary and octal schemes. All distances given are normalized with the energy per bit E_b . The results are given in graphs where $d_{\min}^2$ is given versus modulation index h . The upper bound on the Euclidean distance for the optimum receiver is shown in some distance graphs as comparison. The maximum on $d_{\min}^2$ for AMF receivers with fewer symbols fixed and given observation lengths is also shown as comparison.

For some selected modulation indices the error probability is given. Note that these error probabilities are exact for binary schemes with $N_A=1$ otherwise they are upper bounds. Simulated error probabilities are also shown. The updating for the simulated receiver is very simple. $\underline{A}_{-1}$ is estimated from the previous $L-1$ decisions and Φ is estimated from all the other previous decisions. This is the only method considered but others might improve the performance.

The normalized squared Euclidean distance for AMF receivers to binary 1REC is shown in figure 5, 6 and 7, for various N_A and N_T values. The upper bound for the optimum receiver, and maximum on $d_{\min}^2$ for AMF receivers with smaller N_A are shown as comparison.

The distance for the AMF receiver with $N_A=1$ does not reach the upper bound for the optimum receiver except for $h=1/2$. Note that the distance not always grows with an increase in observation length. The AMF receiver with $N_A=2$ has a distance that reaches the upper bound in several regions of h . Note that the distance for an AMF receiver with $N_A=N_T$ is equal to the distance for the optimum receiver with observation length N_T . Also when $N_A=2$ the performance is bad in some regions. This performance can be improved by choosing $N_A=4$.

Figure 8, 9 and 10 show error probabilities and simulations for some selected schemes. The simulated results differ from the calculated due to the updating of phase and prehistory. The loss is however small except for $h=0.6$, where the updating seems to degrade the performance more.

Distance results for AMF receivers to binary 2RC are shown in figure 11, 12 and 13 for various N_A and N_T values. Both the $N_A=1$ and $N_A=2$ AMF receivers never reach the upper bound of the optimum receiver even if the distance is close for $N_A=2$. For 2RC $N_A \geq 3$ is needed to reach the bound.

Otherwise the trends are similar to the 1REC scheme. Calculated and simulated error probabilities are given in figure 14 for $h=0.6$. These results have the same tendency as the 1REC results for $h=0.6$.

For 3RC $N_A=3$ is needed to reach the upper bound of the optimum receiver as seen in figure 15, 16 and 17. These figures show the distance for AMF receivers with $N_A=1$, $N_A=2$ and $N_A=4$. For modulation indices larger than 0.5 $N_A=3$ is needed to give a good distance. In some regions an even larger N_A value is needed. Figure 18 and 19 show calculated and simulated error probabilities for some schemes. It is seen that the simulated error probabilities sometimes are very large. This is due to the updating. If the receiver makes an error, the wrong filters are used for the next decision. This can cause the receiver to loose the track and not find the way back again. This shows that the updating of phase and prehistory does not work for schemes based on longer pulses.

Figure 20, 21 and 22 show the distance for AMF receivers to quaternary 1REC. $N_A=2$ is needed to reach the upper bound and for most regions of h , $N_T=N_A$ gives the best performance. Figure 23 shows calculated and simulated error probabilities for $h=0.3$. The loss due to the updating of phase is quite large, around 2dB for $N_A=2$.

Figure 24, 25 and 26 give the same results for quaternary 2RC. For low h -values $N_T = N_A+1$ gives the best performance while the N_T -value giving best performance for larger h -values varies. N_A equal to 3 is needed to reach the bound. Figure 27 shows error probabilities for $h=0.3$. Still the tendency is the same as for 1REC above but the simulated error probabilities are somewhat unstable.

Finally figure 28 and 29 give the distances for octal 1REC with AMF receivers. For low h -values a good performance can be obtained while the distances are far from the bound for larger h -values.

VI. RECEIVER COMPLEXITY AND RECEIVER COMPARISON

In part I of this thesis [8], [9] another type of simplified receiver is given and analysed, i.e. the reduced-complexity Viterbi receiver. This receiver is of maximum likelihood estimation type and consists of a filterbank and a Viterbi processor. In this section the complexity of the AMF receiver will be compared with the complexity of the Viterbi receiver. The optimum Viterbi receiver is a special case of the reduced-complexity receiver and will also be compared.

The complexity of the Viterbi receiver is given by the number of filters $F = 2 \cdot M^L$ and the number of states $S = p \cdot M^{L-1}$, where p is the number of phase states [2]. The complexity of the AMF receiver is given by the number of filters per filterset $F_F = 4 \cdot M^{N_A}$, if L is greater than 2 otherwise $F_F = 4 \cdot M^{N_A}/2$ if $L=1$, the number of sets $F_S = M^{L-1}/2$ and the number of comparisons $S_C = M^{N_A}$.

The complexity does not depend on the modulation index for the AMF receiver, while it does for the Viterbi receiver. Therefore the comparison is given for different h -values. The complexity given does not include the complexity of the updating. In section V a simple updating method is described and simulated but for some schemes this method gives unacceptable performance. Thus a more complex updating is needed for some schemes, which increases the total complexity for the AMF receiver.

Note that there is an even simpler receiver for $h=1/2$ schemes considered in part II and part III [10]–[13]. This is the MSK-type receiver, which consists of only 2 filters and simple decision logic. The loss for large signal-to-noise ratios compared to the optimum Viterbi receiver is also given and compared for the various receivers. The loss for the AMF receiver is calculated under the assumption of correct phase and prehistory updating.

Table 1 shows the complexity of binary 1REC. In this case there is no reduced-complexity Viterbi receiver since $L=1$. If a small loss can be tolerated, then the $N_A=1$ AMF receiver is to be preferred for h -values less than around 0.5. For larger h -values or if no degradation can be accepted, the Viterbi receiver is to be preferred. In this case the number of filters is less for the Viterbi receiver and also the number of comparisons. However the number of comparisons for the Viterbi receiver depends on the h -value. If an h -value giving many phase states is chosen, then the number of comparisons will be large and in these cases the AMF receiver with $N_A=3$ is to be preferred.

h	Opt. Viterbi		AMF, $N_A=1$				AMF, $N_A=3$			
	S	F	F_F	F_S	S_C	loss	F_F	F_S	S_C	loss
1/3	6	4	4	1	2	0.4	16	1	8	0
1/2	4	4	4	1	2	0	16	1	8	0
2/3	3	4	4	1	2	1.3	16	1	8	0
3/4	8	4	4	1	2	2.3	16	1	8	0

Table 1. Complexity of 1REC receivers.

The complexity of the 2RC scheme is summarized in table 2. Now also one reduced-complexity Viterbi receiver is considered. This receiver is to be preferred if a small degradation in performance can be tolerated. The degradation for the reduced-complexity Viterbi receiver is much smaller for most h-values compared to the $N_A=1$ AMF receiver, while the complexity is of the same order. If no degradation is tolerated the Viterbi receiver is to be used because the complexity is smaller than for the $N_A=3$ AMF receiver, if the number of phase states are not too large.

h	Opt. Viterbi		Reduced Viterbi			AMF, $N_A=1$				AMF, $N_A=3$			
	S	F	S	F	loss	F_F	F_S	S_C	loss	F_F	F_S	S_C	loss
1/3	12	8	6	4	<0.1	8	1	2	0.32	32	1	8	<0.1
1/2	8	8	4	4	0.10	8	1	2	<0.1	32	1	8	0
2/3	6	8	3	4	0.12	8	1	2	1.5	32	1	8	<0.1
3/4	16	8	8	4	0.15	8	1	2	2.3	32	1	8	0

Table 2. Complexity of 2RC receivers.

Table 3 shows the complexity of 3RC receivers. For 3RC there are two different reduced-complexity Viterbi receivers. In this case one of these two is to be preferred if a performance degradation can be accepted. The choice between the two depends on the degradation to be accepted. From the simulations it is known that the AMF receivers have a tendency to loose the track for 3RC and therefore the Viterbi receivers should be chosen. The gain for the optimum Viterbi receiver is very small compared

to the reduced-complexity Viterbi receiver with $L_R=2$. This makes it hard to claim to use the optimum receiver instead of the reduced-complexity receiver.

h	Opt. Viterbi		Reduced Viterbi $L_R=2$			Reduced Viterbi $L_R=1$			AMF, $N_A=1$				AMF, $N_A=3$			
	S	F	S	F	loss	S	F	loss	F_F	F_S	S_C	loss	F_F	F_S	S_C	loss
1/3	24	16	12	8	<0.1	6	4	0.8	8	2	2	0.9	32	2	8	0
1/2	16	16	8	8	<0.1	4	4	0.7	8	2	2	0.4	32	2	8	0
2/3	12	16	6	8	<0.1	3	4	0.8	8	2	2	2.1	32	2	8	1.7
3/4	32	16	16	8	<0.1	8	4	0.9	8	2	2	2.6	32	2	8	1.6

Table 3. Complexity of 3RC receivers.

The complexity of 4RC is summarized in table 4. Also for 4RC the AMF receiver has a tendency to loose track and therefore the same conclusions as for 3RC holds for 4RC. The choice between the reduced-complexity receivers depends on the degradation to be accepted. If a performance close to optimum is wanted, the reduced-complexity receiver with $L_R=3$ is to be preferred.

h	Opt. Viterbi		Reduced Viterbi $L_R=3$			Reduced Viterbi $L_R=2$			AMF, $N_A=1$				AMF, $N_A=3$			
	S	F	S	F	loss	S	F	loss	F_F	F_S	S_C	loss	F_F	F_S	S_C	loss
1/3	48	32	24	16	<0.1	12	8	0.4	8	4	2	1.4	32	4	8	<0.1
1/2	32	32	16	16	<0.1	8	8	0.4	8	4	2	1.4	32	4	8	0.1
2/3	24	32	12	16	<0.1	6	8	0.4	8	4	2	1.1	32	4	8	0.7
3/4	64	32	32	16	<0.1	16	8	0.4	8	4	2	2.0	32	4	8	1.6

Table 4. Complexity of 4RC receivers.

Table 5 and 6 give the complexity of quaternary 1REC and 2RC. For both of these schemes a very large number of filters is needed in the AMF receiver to obtain an acceptable performance. Therefore the Viterbi receiver or for 2RC the reduced-complexity Viterbi receiver is to be preferred. The choice between the Viterbi receivers for 2RC depends on the degradation tolerated.

h	Opt. Viterbi		AMF, $N_A=1$				AMF, $N_A=3$			
	S	F	F_F	F_S	S_C	loss	F_F	F_S	S_C	loss
1/4	8	8	8	1	4	3.0	128	1	64	0
1/3	6	8	8	1	4	2.6	128	1	64	0.7
1/2	4	8	8	1	4	3.0	128	1	64	3.0

Table 5. Complexity of quaternary 1REC.

h	Opt. Viterbi		Reduced Viterbi $L_R=1$			AMF, $N_A=1$				AMF, $N_A=3$			
	S	F	S	F	loss	F_F	F_S	S_C	loss	F_F	F_S	S_C	loss
1/4	32	32	8	8	0.4	16	2	4	4.5	256	2	64	<0.1
1/3	24	32	6	8	0.5	16	2	4	3.9	256	2	64	0.3
1/2	16	32	4	8	0.6	16	2	4	2.9	256	2	64	2.9

Table 6. Complexity of quaternary 2RC.

The MSK-type receiver is a simple receiver for $h=1/2$ schemes. If a small degradation in performance is accepted, this is the receiver to choose. It consists of only two filters and simple decision logic. The asymptotic loss compared to the optimum Viterbi receiver is very small for 2RC, about 0.16dB for 3RC and about 0.91dB for 4RC.

VII. DISCUSSION AND CONCLUSIONS

In this part the multiple-symbol Average Matched Filter (AMF) concept is used to get simplified sub-optimum receivers for coherent partial response CPM. A distance measure has been derived for analysing the receiver performance under the assumption of error free filter updating. Upper bounds on the symbol error probability have also been calculated under the above assumption. Simulations of the receiver performance including the updating of phase and filter have been performed. Comparisons of complexity compared to other receivers are also given.

The degradation in minimum Euclidean distance for this sub-optimum multiple-symbol AMF receivers compared to the optimum Viterbi receiver is shown in figure 30, for the binary $h=0.5$ case. The minimum Euclidean distance is given for the RC family as a function of the frequency pulse duration L . The minimum Euclidean distance for the weighted AMF receivers with $N_A=1$, considered in [3], are also shown, and comparisons with the simpler offset quadrature receivers considered in part II [10], [11] are also made. Some points for the Tamed Frequency Modulation (TFM) scheme [3], [13] with an AMF receiver are also given. It is shown that for high SNR's the different type of AMF receivers perform mainly inbetween the optimum Viterbi receiver and the offset quadrature receiver. Also shown is that the AMF receiver should not be used for $h=0.5$ schemes. In this case the MSK-type receiver is much simpler with a performance which is better than an AMF receiver with $N_A=1$. It is also shown that the minimum Euclidean distance is increased by matching filters to sequences of symbols in the AMF receiver. This of course increases the number of filters which is given by $M^{(L-1+N_A)}$. Some of these filters might be equal, so the number of filters can be smaller.

It is shown in [3] that it is possible to slightly increase the minimum distance for the AMF receiver with $N_A=1$ by modifying the receiver filters, i.e. to give unequal weights to the different signals, giving the average filter. This has not been studied for the AMF receivers with $N_A \geq 2$, but it should also be possible to improve the performance for these receivers. Any optimizations have not been done.

The AMF receiver concept with decision feedback updating of the prehistory and phase based on previous decisions, gives good asymptotic performance (large SNR) for short pulses and for schemes with low h -values. Filter length (impulse response length) is an important parameter when the dis-

tance is to be optimized. The distance decrease with the filter length for some modulation indices.

The performance is degraded by the phase and prehistory updating. However the degradation is small for schemes based on short pulses, i.e. L less than 3. For the more complex schemes the AMF receiver has a tendency to loose the track, which causes an unacceptable performance. For the quaternary schemes the performance is acceptable only for the 1REC schemes.

It is also shown that the complexity of some AMF receivers is of the same order as the complexity of the reduced-complexity Viterbi receiver. Of course this depends on the modulation index. If the h -value is chosen such that the number of phase states in the Viterbi receiver is large, the complexity of this receiver is large and the AMF receiver is to prefer. It is however difficult to compare complexity, because it depends highly on the implementation of the receivers. One advantage for the AMF receiver is little processing. The number of metric calculations and comparisons in the Viterbi receiver are time consuming, while the filtering in the AMF receiver can be done parallel and much faster even if the number of filters is large.

The complexity of the AMF receiver might however be reduced using the same ideas as for the reduced-complexity Viterbi receiver. This means that an AMF receiver for a less complex scheme is used also for the more complex schemes. This idea has not been examined for AMF receivers but the relative degradation is expected to be about the same as for the reduced-complexity Viterbi receivers. The idea of reducing the complexity also makes the problem of updating prehistory less since this updating works well for short pulses.

The simulations indicate that the simple phase and prehistory updating considered in this part does not work well if L greater than 2. One idea to improve the updating could be to use a Viterbi detector for updating the phase. This will however increase the complexity of the AMF receiver.

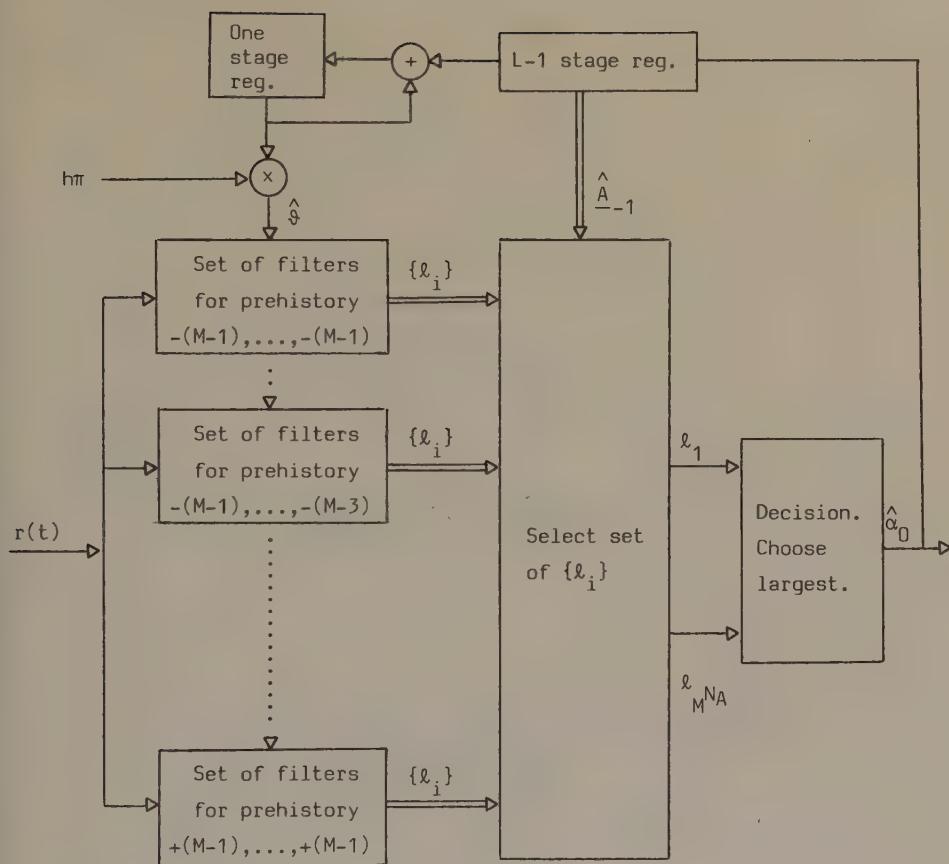


Figure 1. Receiver model for an AMF receiver.

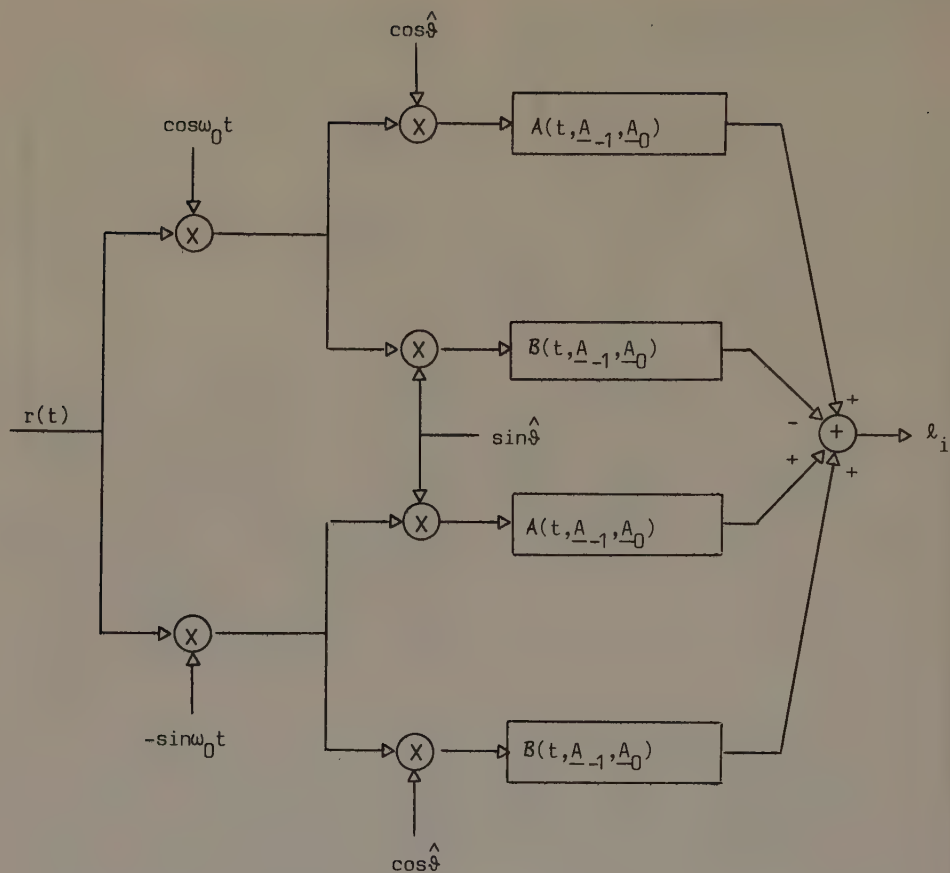


Figure 2. Baseband filter for an AMF receiver.

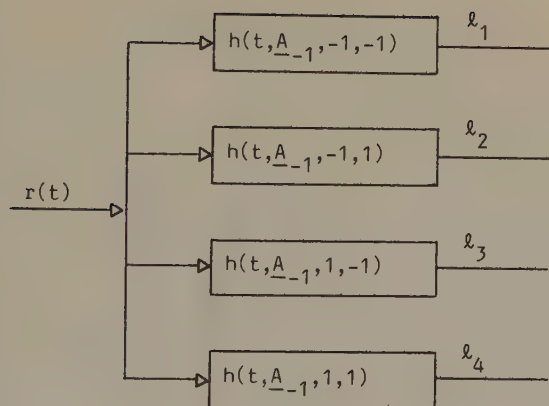


Figure 3. One set of filters for an AMF receiver with $N_A=2$.

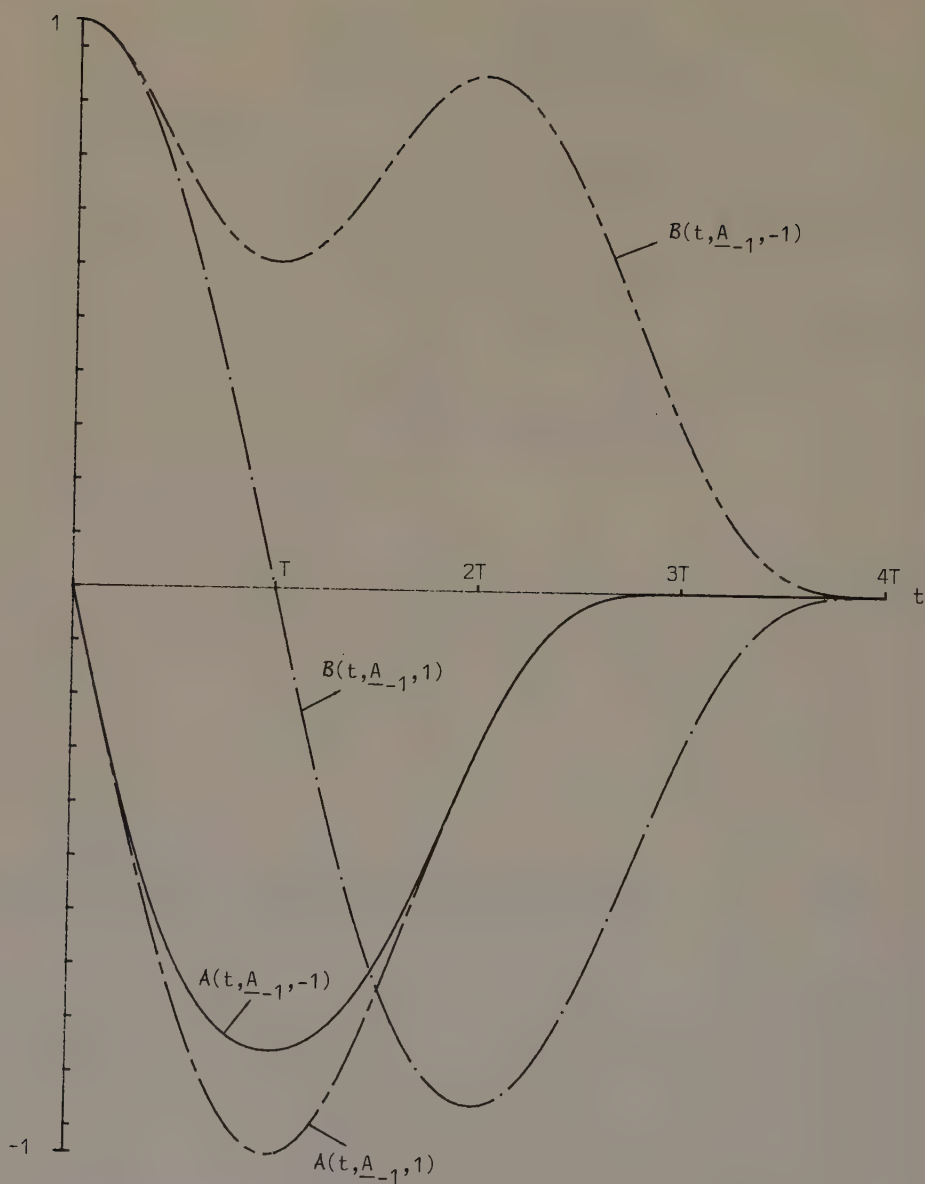


Figure 4. The average signals $A(t, \underline{A}_{-1}, \underline{A}_0)$ and $B(t, \underline{A}_{-1}, \underline{A}_0)$ for a transmitted 3RC-pulse, when the prehistory $\underline{A}_{-1}$ is $+1, +1$.

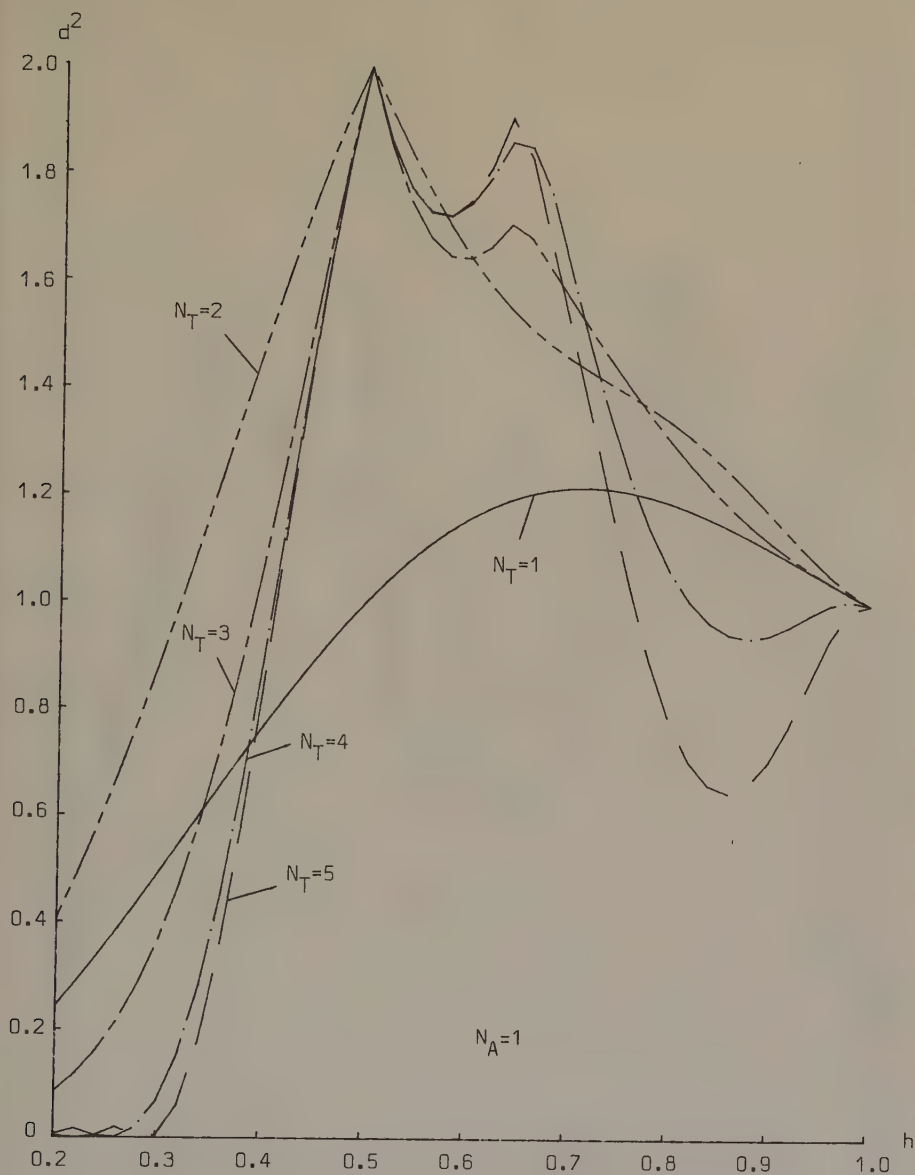


Figure 5. Minimum normalized squared Euclidean distance for a binary 1REC AMF receiver with $N_A=1$ as a function of h and N_T .

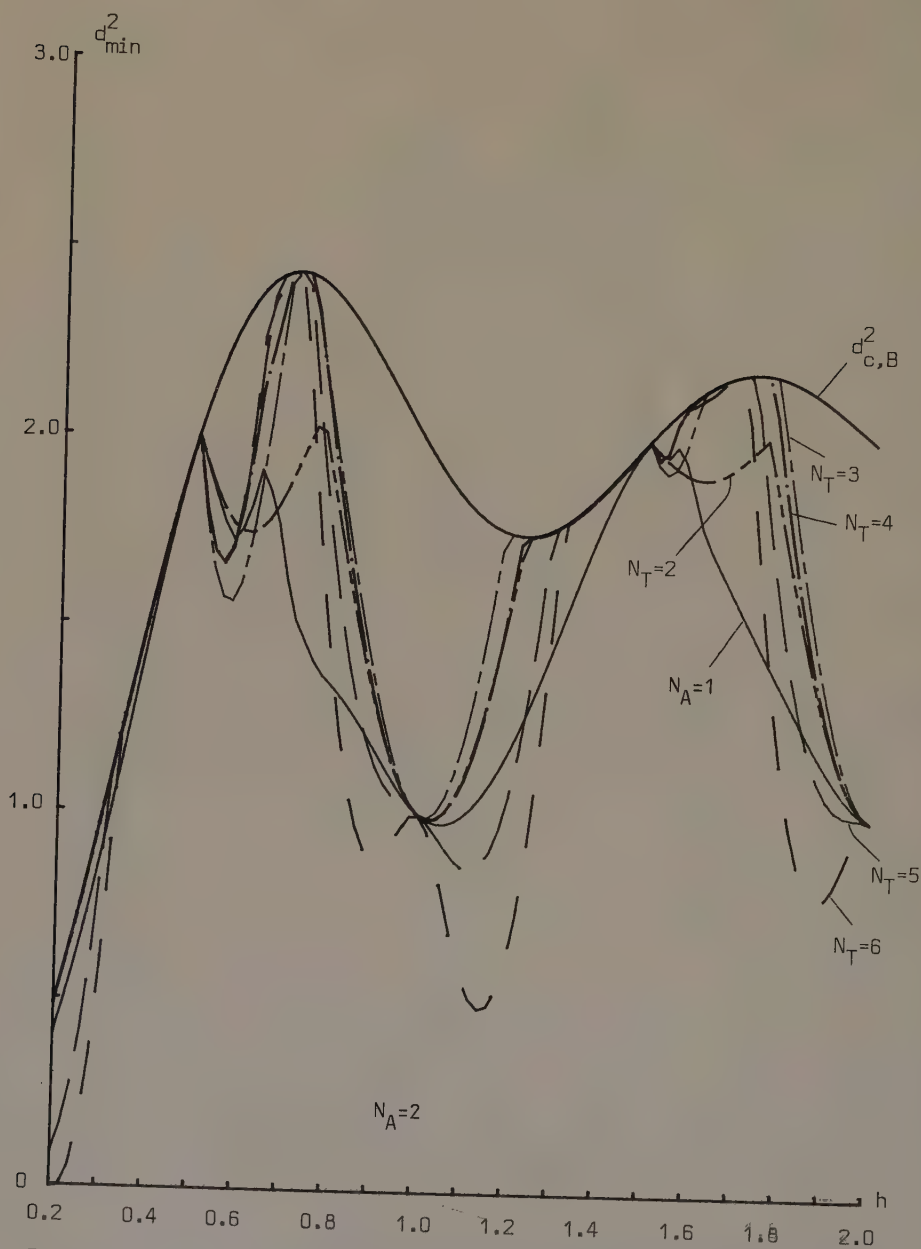


Figure 6. Minimum Euclidean distance for a binary 1REC AMF receiver with $N_A=2$ as a function of h and N_T . The upper bound $d_{c,B}^2$ for the optimal receiver, and maximum on $d_{\min}^2$ for an AMF receiver with $N_A=1$ and $N_T \leq 5$ are shown as comparison.

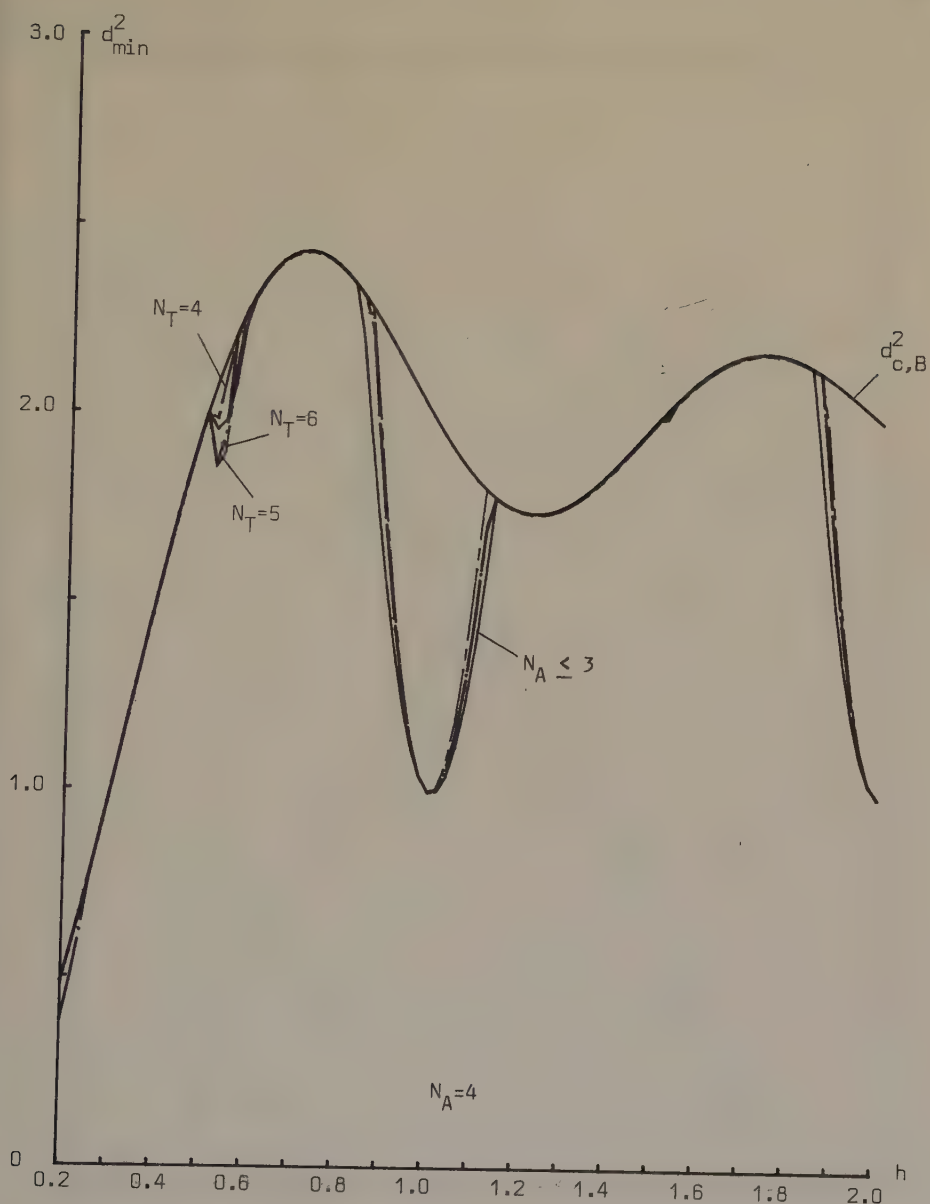


Figure 7. Minimum Euclidean distance for a binary 1REC AMF receiver with $N_A=4$ as a function of h and N_T . The upper bound $d_{c,B}^2$ for the optimal receiver, and maximum on $d_{\min}^2$ for the AMF receivers with $N_A \leq 3$ are shown as comparison.

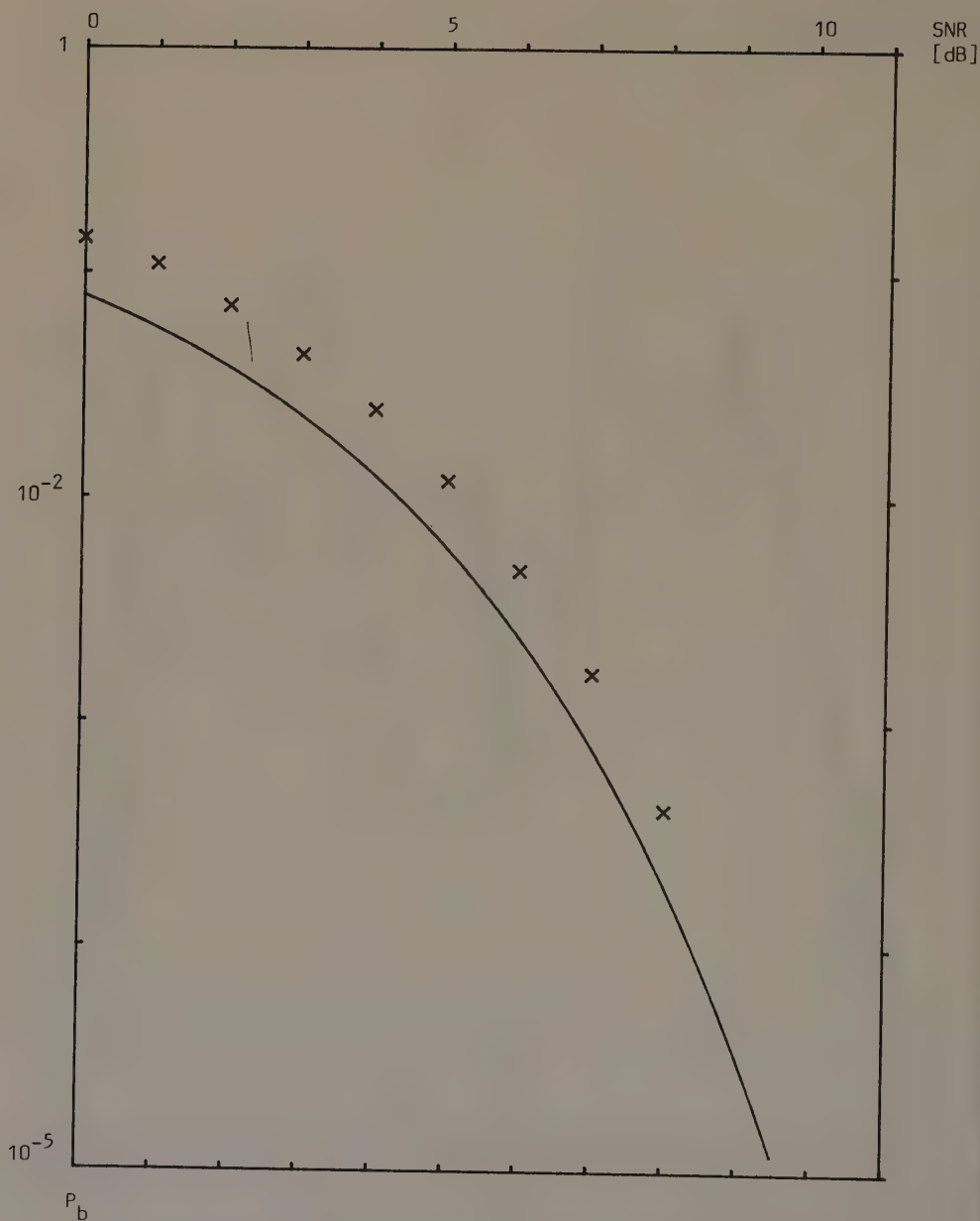


Figure 8. Simulated (x) and calculated error probability for 1REC with $h=1/2$. The receiver is an AMF receiver with $N_A=1$ and observation length $N_T=2$ symbol intervals.

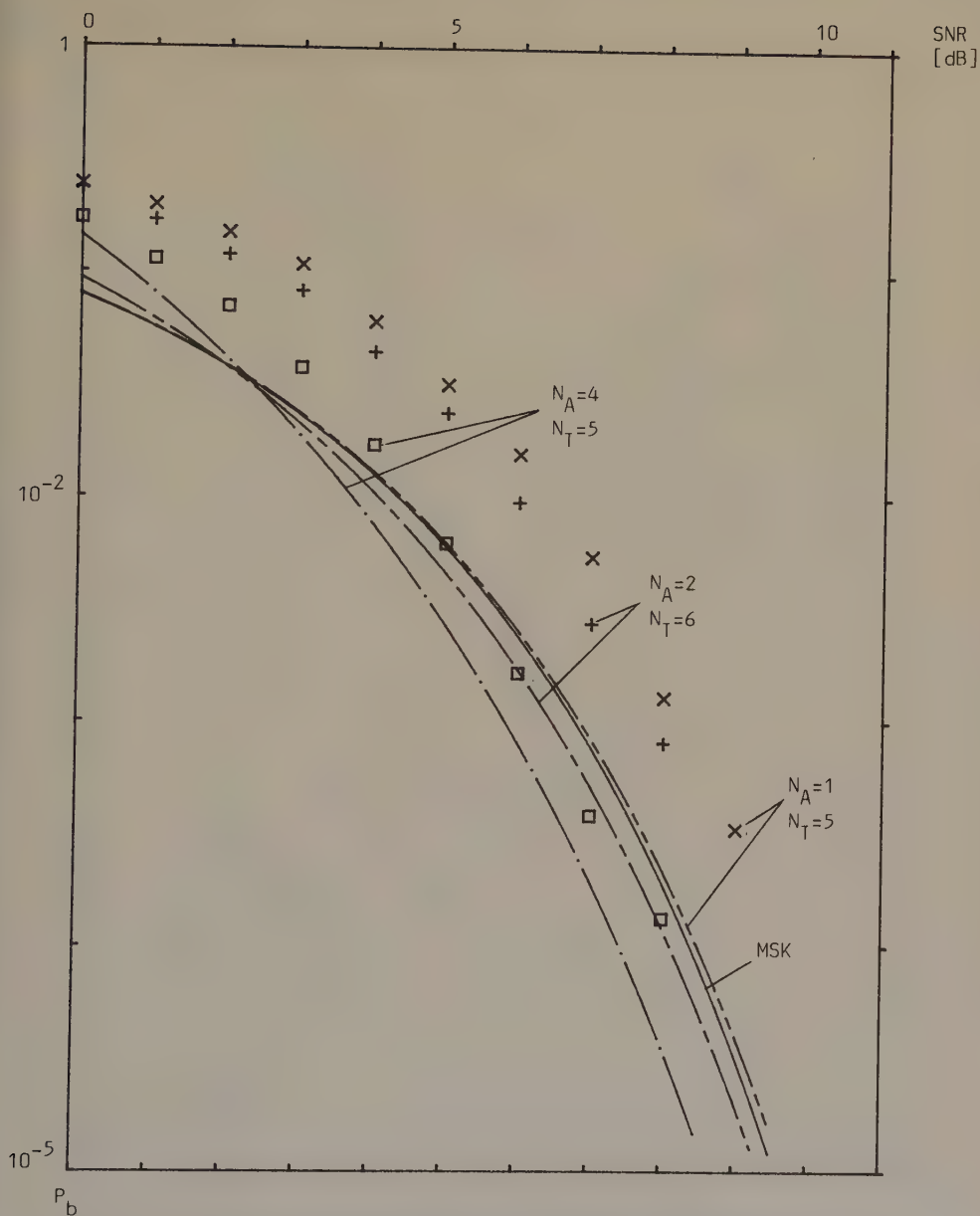


Figure 9. Simulated and calculated error probabilities for 1REC with $h=0.6$. The receivers are given above. The calculated results for $N_A \geq 2$ are upper bounds.

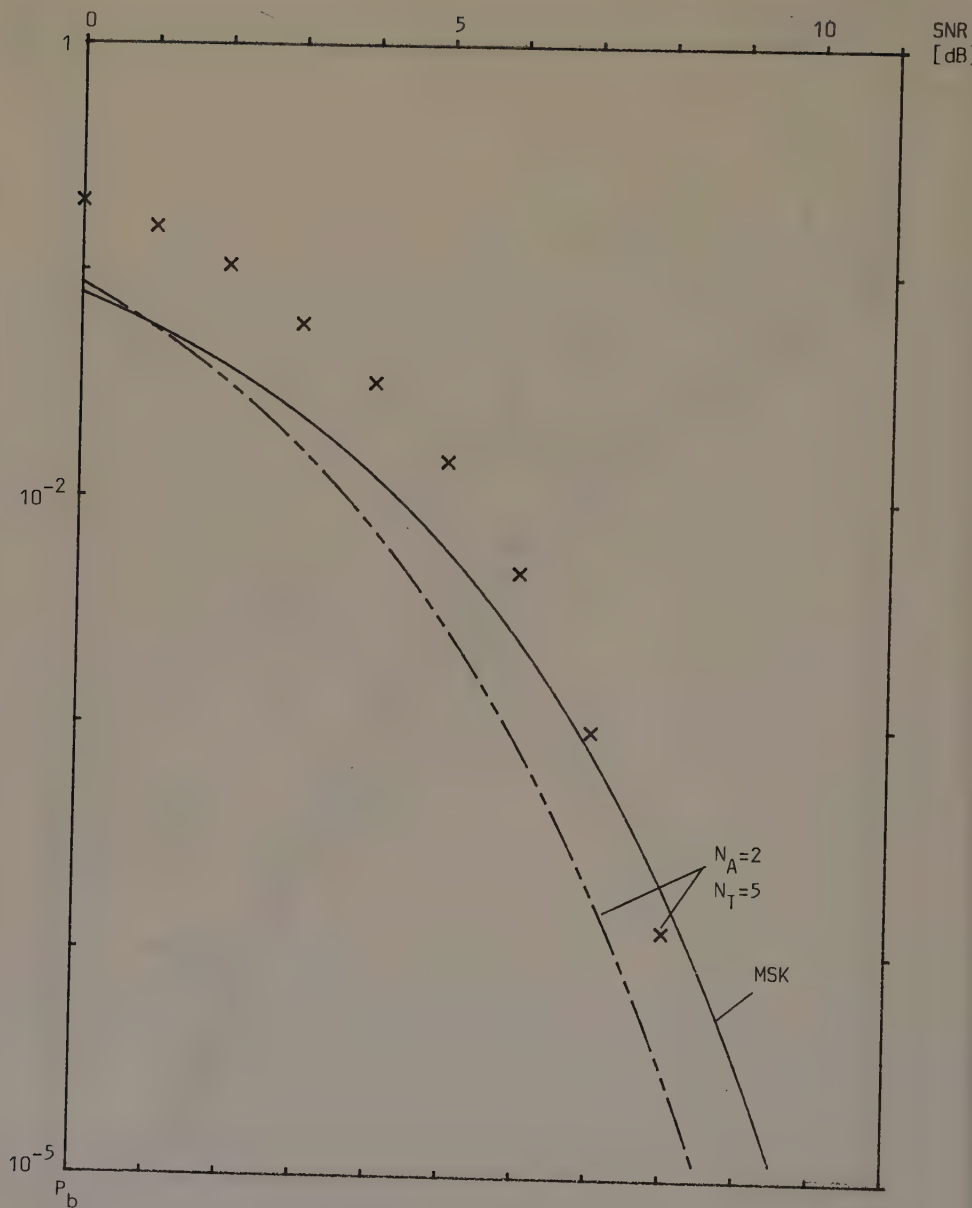


Figure 10. Simulated error probability and calculated upper bound for 1REC with $h=0.7$. The receiver is given above.

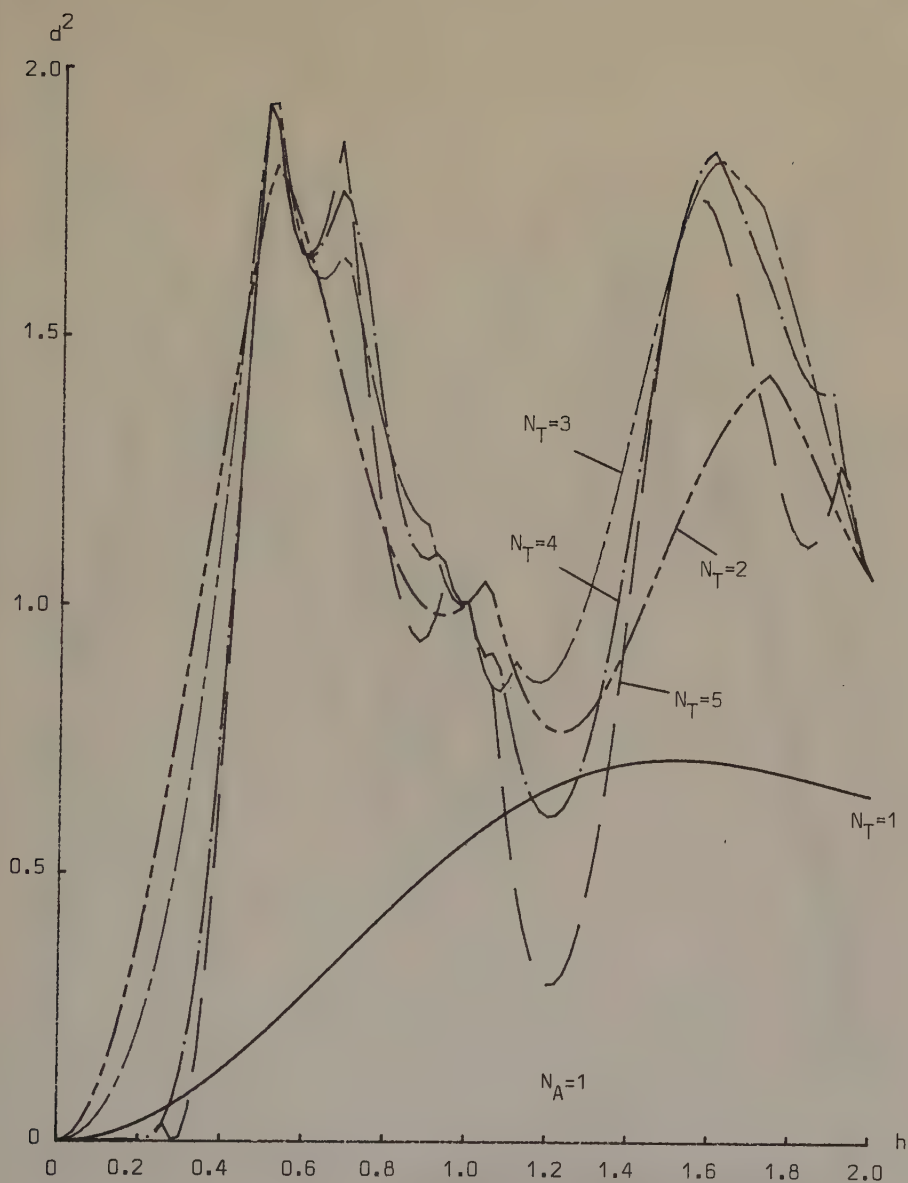


Figure 11. Minimum normalized squared Euclidean distance for a binary 2RC AMF receiver with $N_A=1$ as a function of h and N_T .

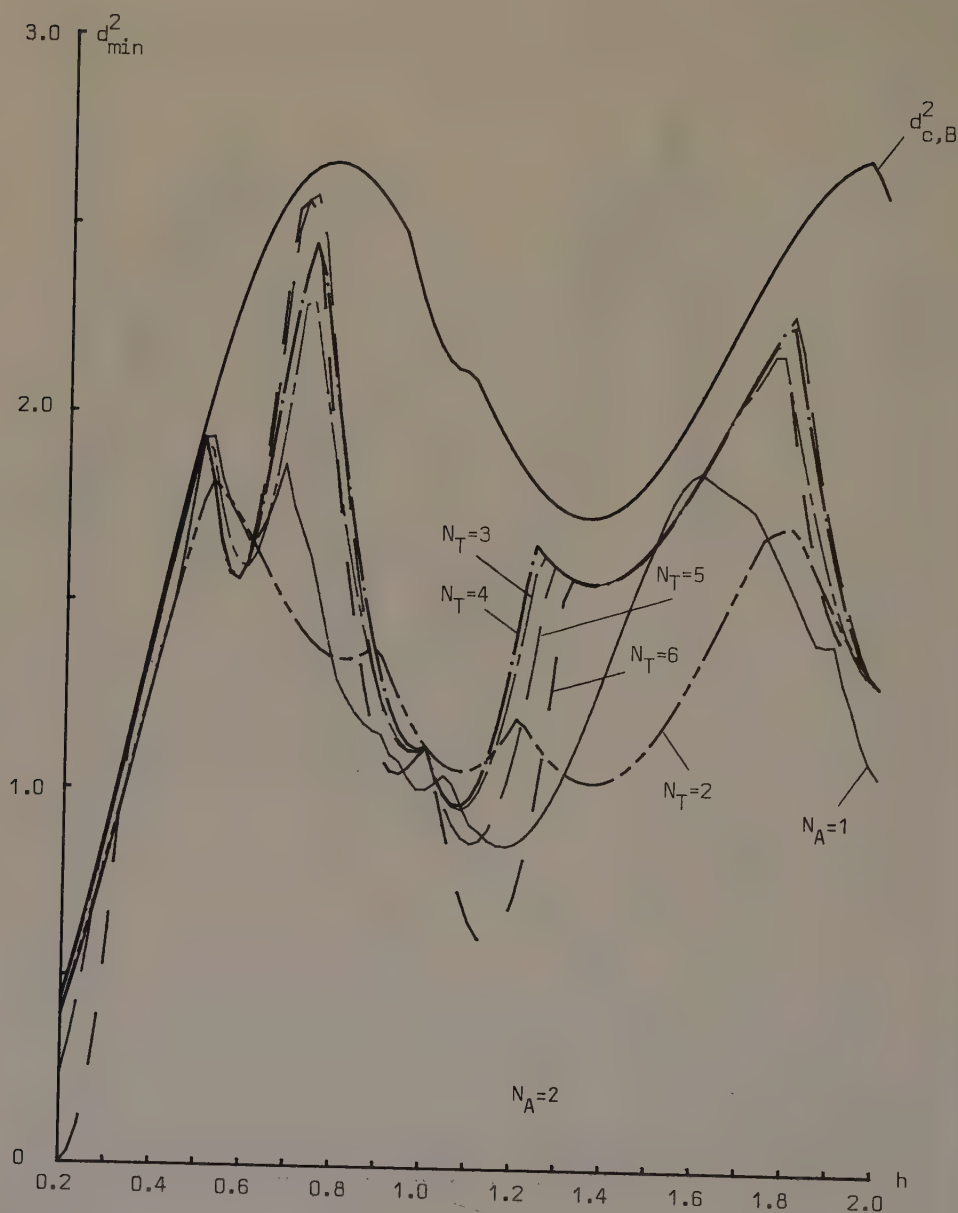


Figure 12. Minimum Euclidean distance for a binary 2RC AMF receiver with $N_A=2$ as a function of h and N_T . The upper bound $d_{c,B}^2$ for the optimal receiver, and maximum on $d_{\min}^2$ for an AMF receiver with $N_A=1$ and with $N_T \leq 5$ are shown as comparison.

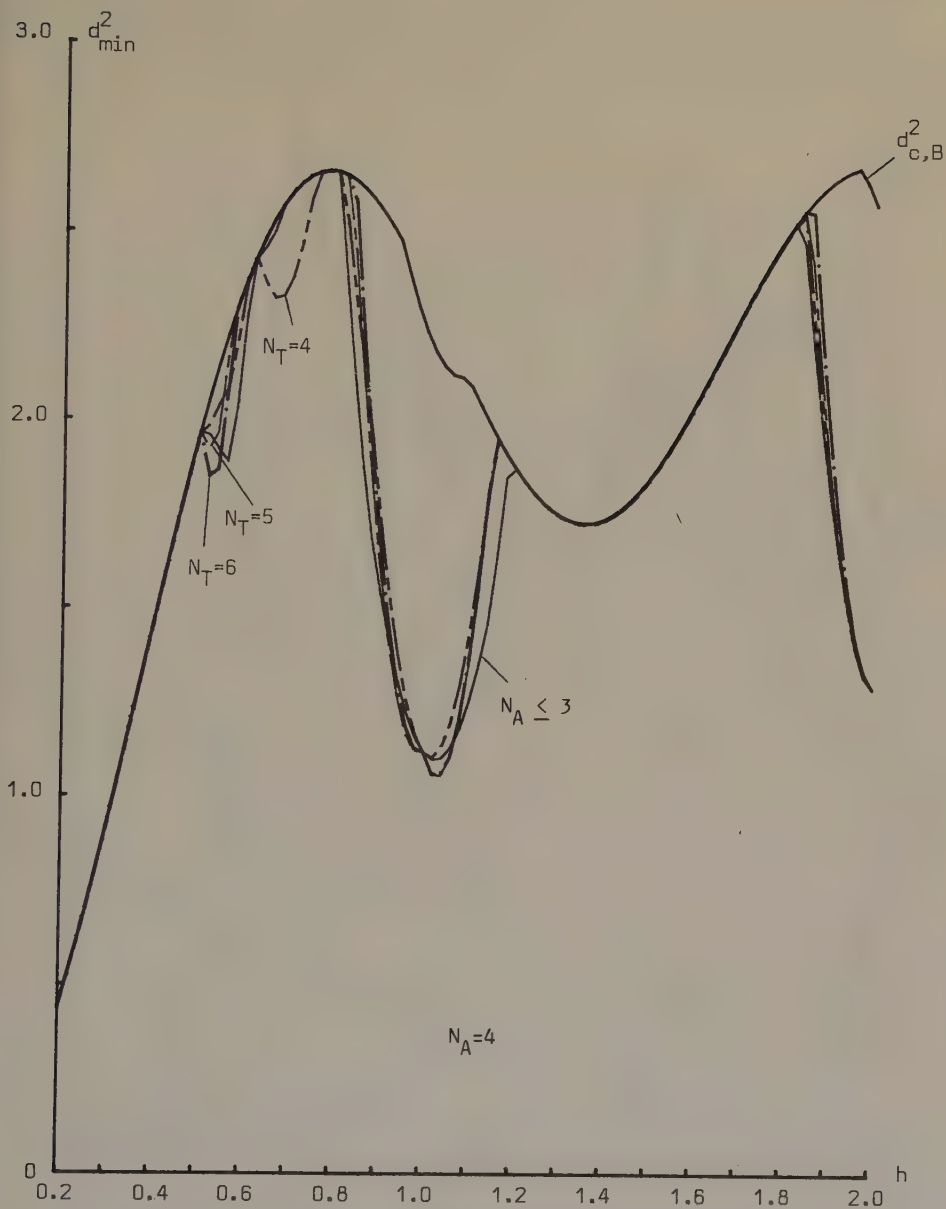


Figure 13. Minimum distance for a binary 2RC AMF receiver with $N_A=4$ as a function of h and N_T . The upper bound $d_{c,B}^2$ for the optimal receiver, and maximum on $d_{\min}^2$ for the AMF receivers with $N_A \leq 3$ are shown as comparison.

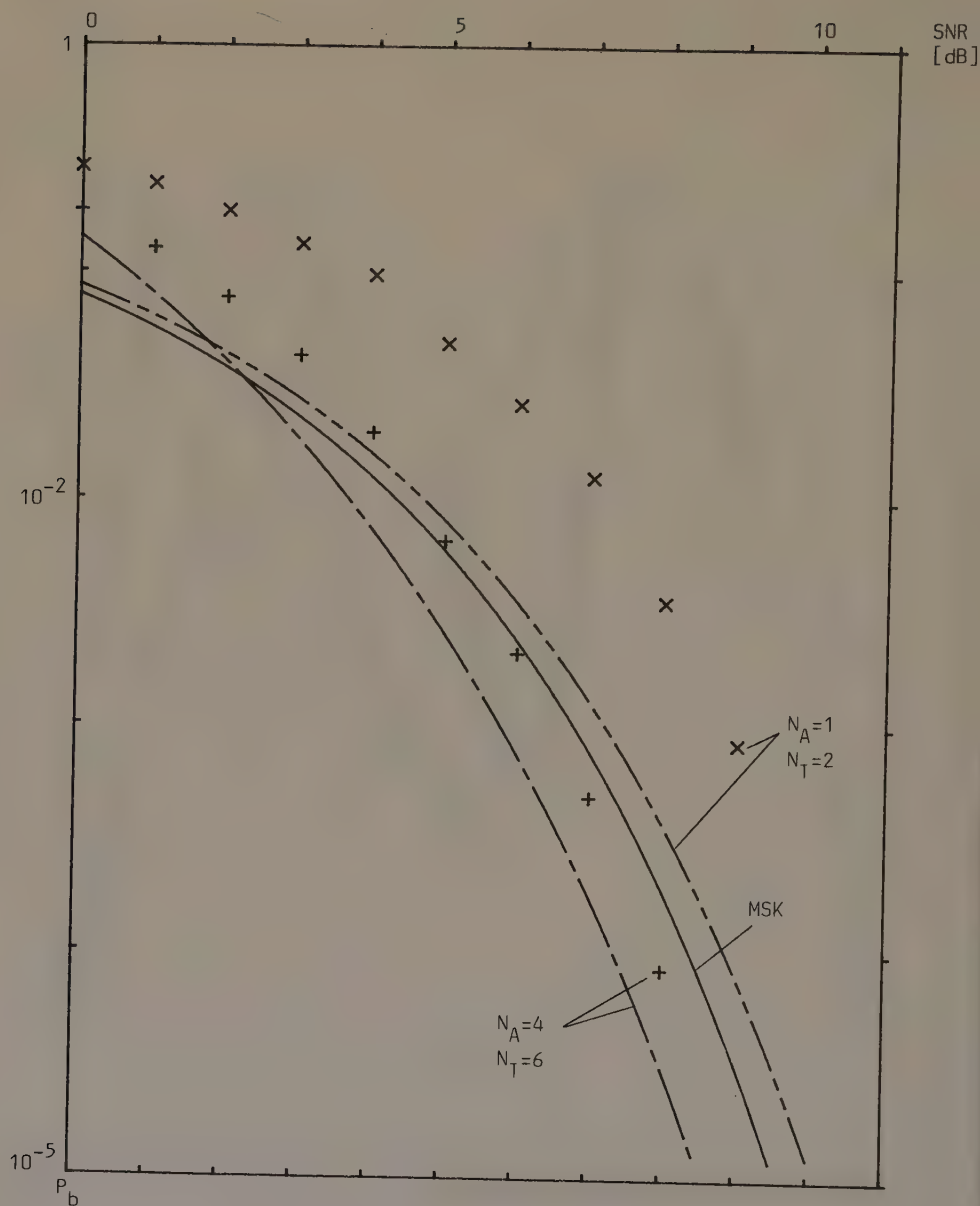


Figure 14. Simulated and calculated error probabilities for 2RC with $h=0.6$. The receivers are given above. The calculated result for $N_A=4$ is an upper bound.

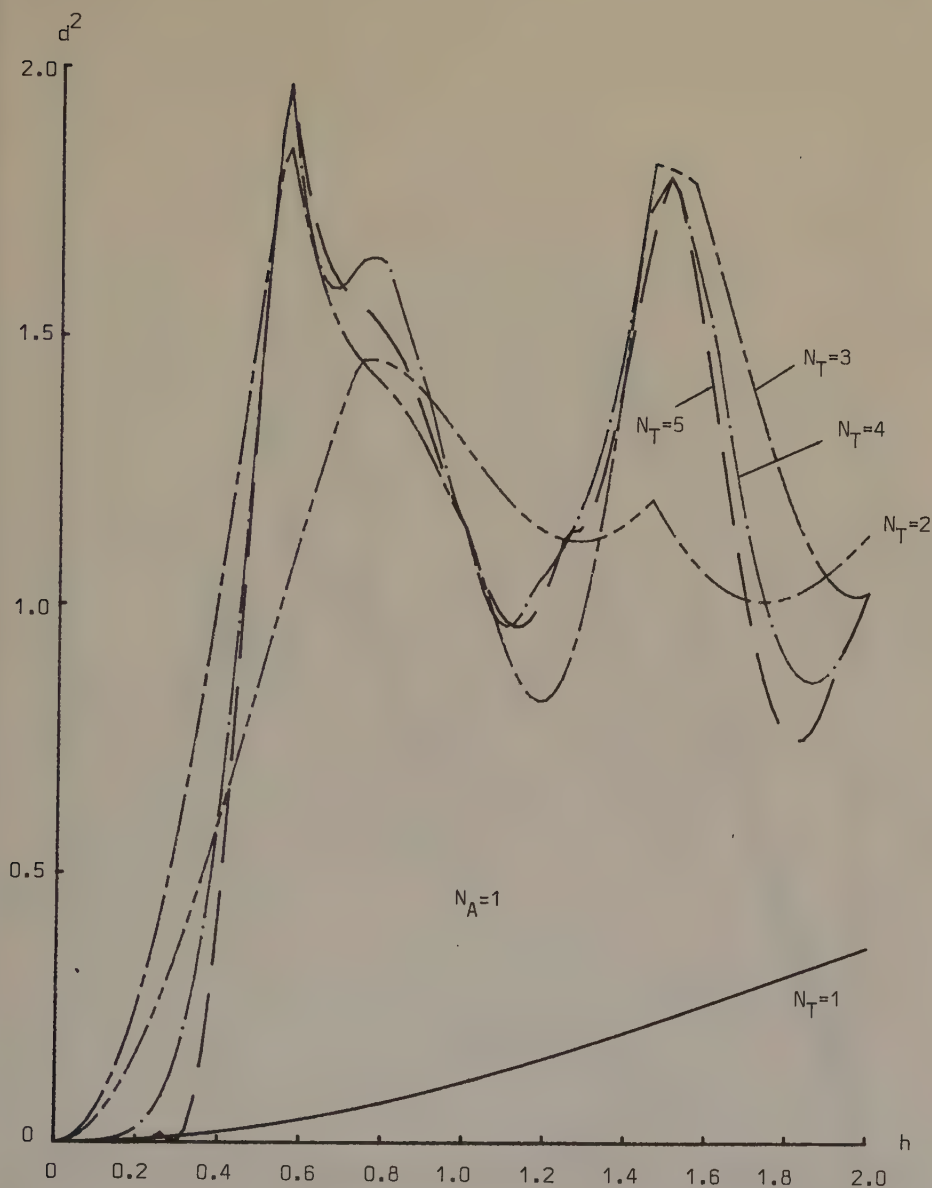


Figure 15. Minimum normalized squared Euclidean distance for a binary 3RC AMF receiver with $N_A=1$ as a function of h and N_T .

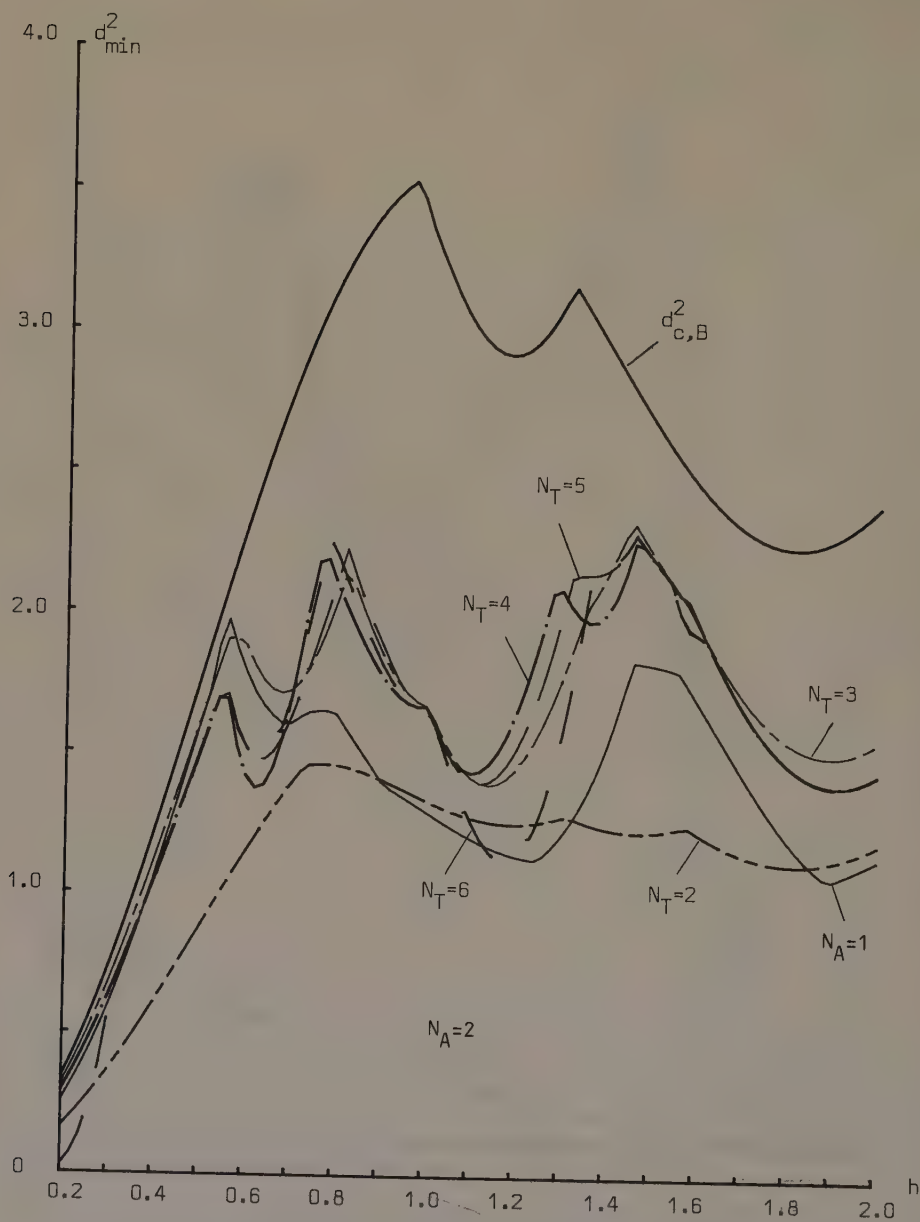


Figure 16. Minimum Euclidean distance for a binary 3RC AMF receiver with $N_A=2$ as a function of h and N_T . The upper bound $d_{C,B}^2$ for the optimal receiver, and maximum on $d_{\min}^2$ for the AMF receiver with $N_A=1$ and $N_T \leq 5$ are shown as comparison.

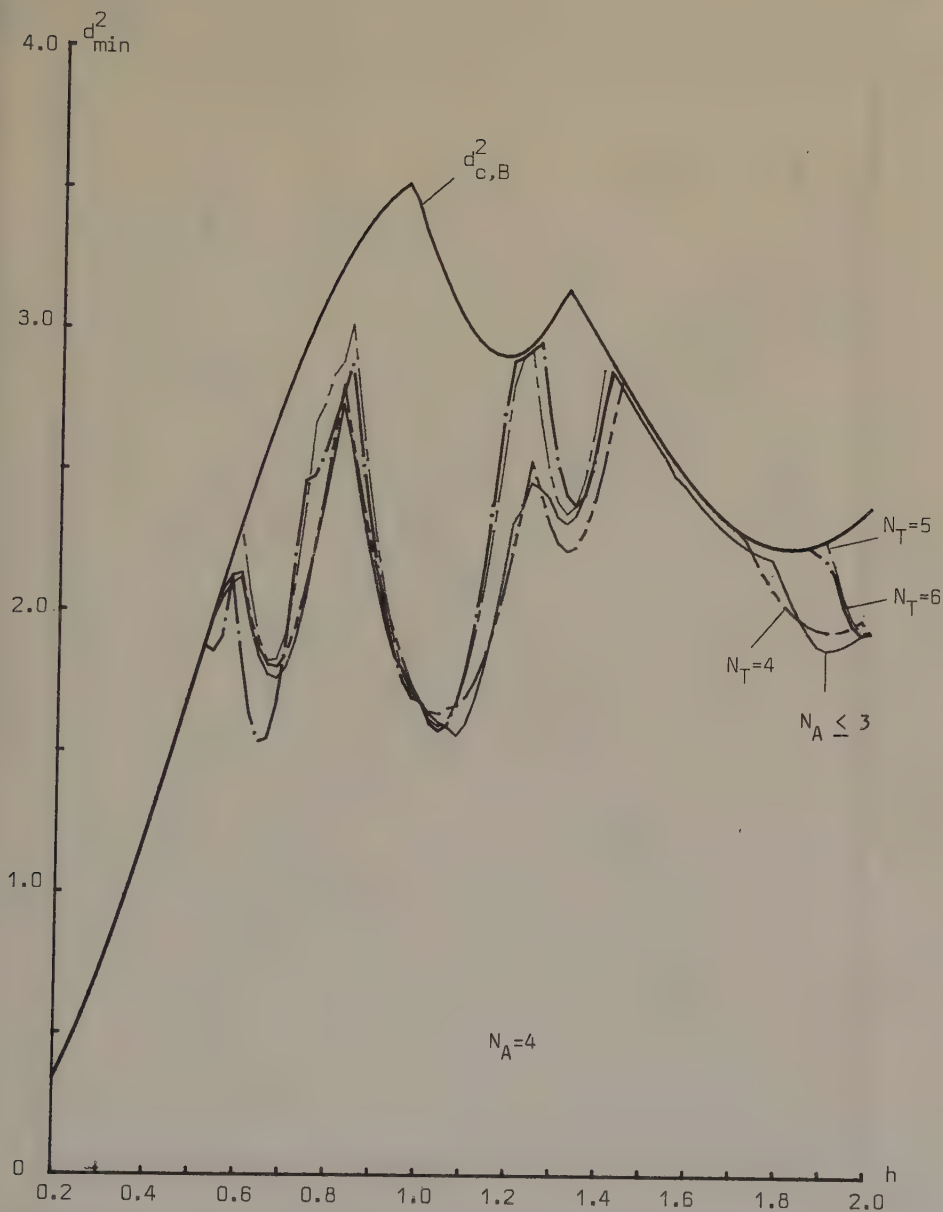


Figure 17. Minimum Euclidean distance for a binary 3RC AMF receiver with $N_A=4$ as a function of h and N_T . The upper bound for the optimal receiver and maximum on $d_{\min}^2$ for the AMF receivers with $N_A \leq 3$ are shown as comparison.

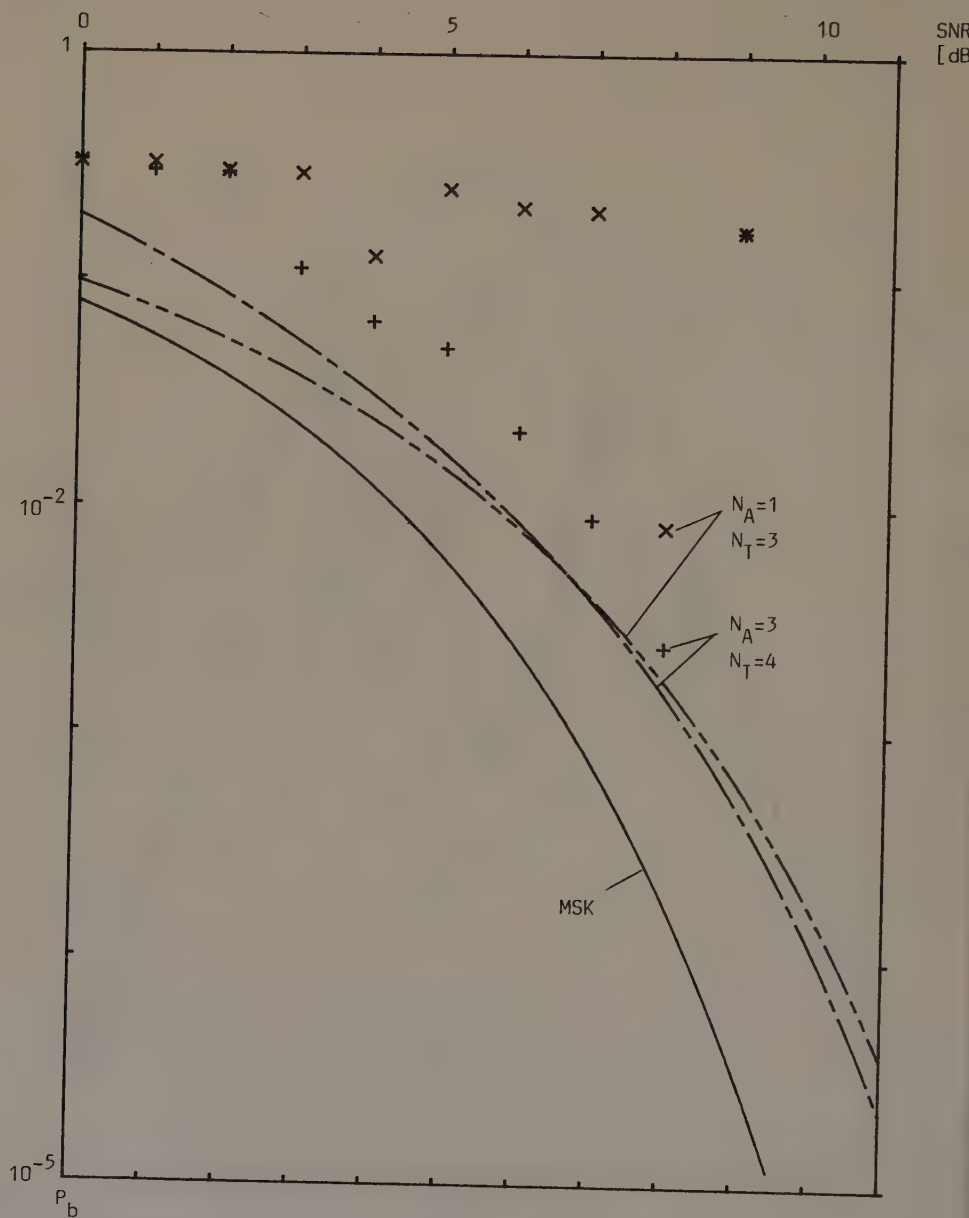


Figure 18. Simulated and calculated error probabilities for 3RC with $h=0.4$. The receivers are given above. The calculated result for $N_A=3$ is an upper bound.

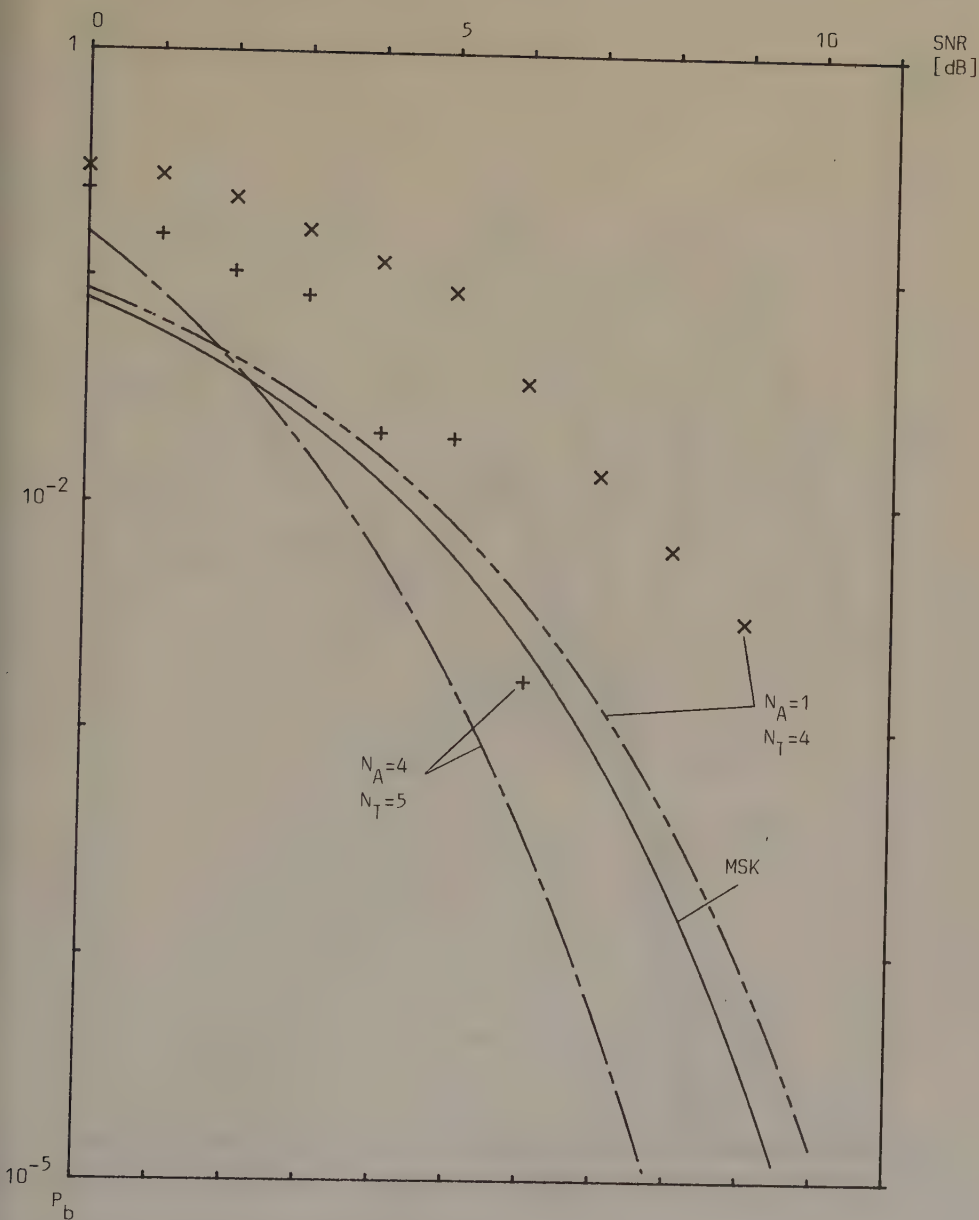


Figure 19. Simulated and calculated error probabilities for 3RC with $h=0.8$. The receivers are given above. The calculated result for $N_A=4$ is an upper bound.

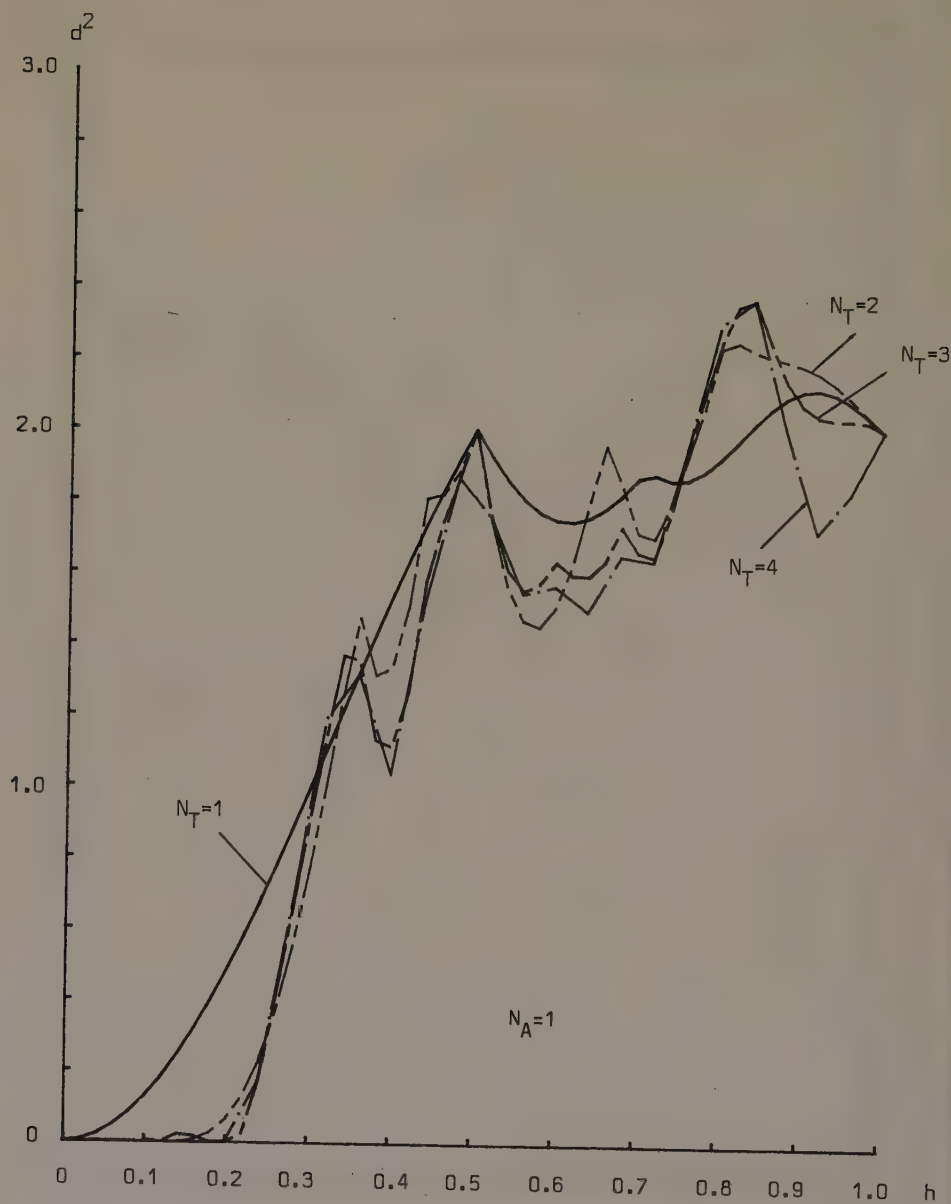


Figure 20. Minimum normalized squared Euclidean distance for a quaternary 1REC AMF receiver with $N_A=1$ as a function of h and N_T .

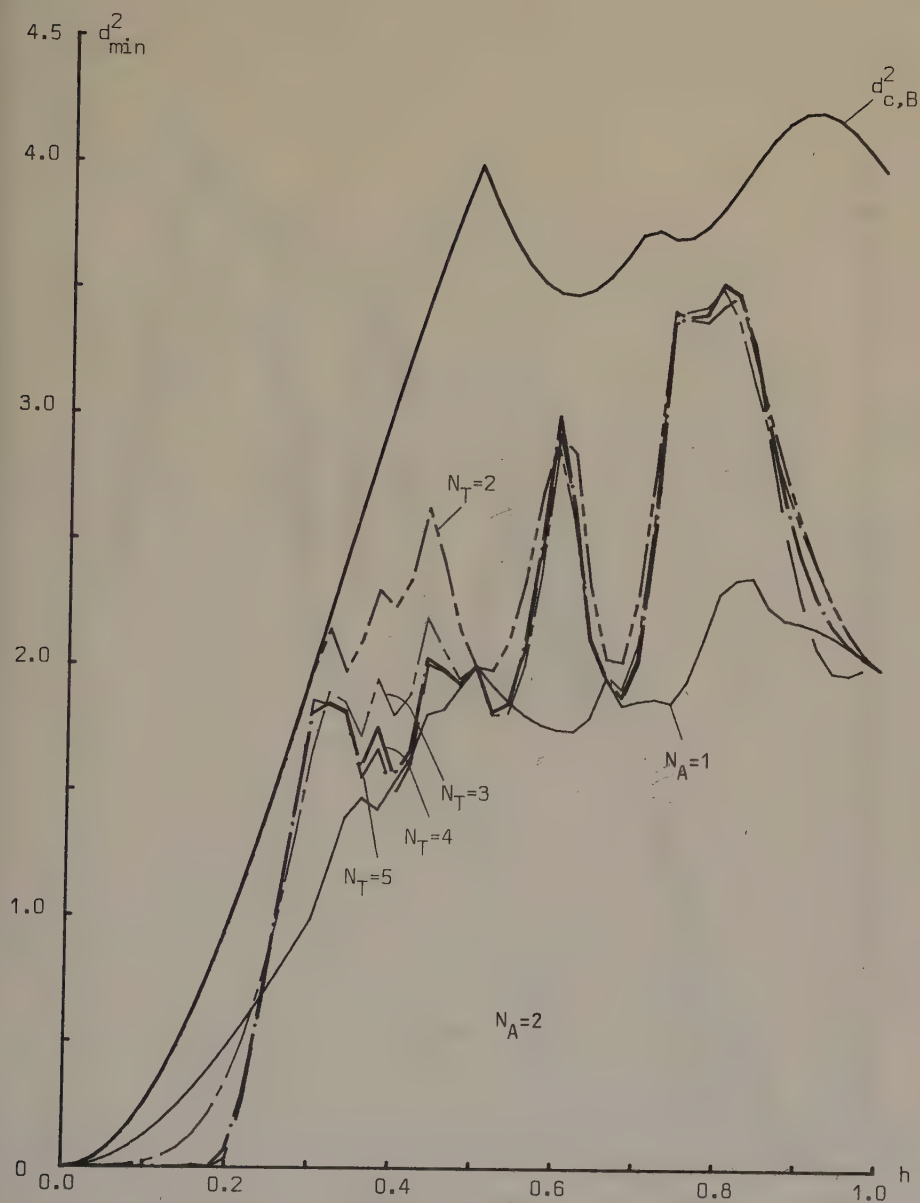


Figure 21. Minimum Euclidean distance for a quaternary 1REC AMF receiver with $N_A=2$ as a function of h and N_T . The upper bound for the optimal receiver and maximum on $d_{\min}^2$ for the AMF receiver with $N_A=1$ and $N_T \leq 4$ are shown as comparison.

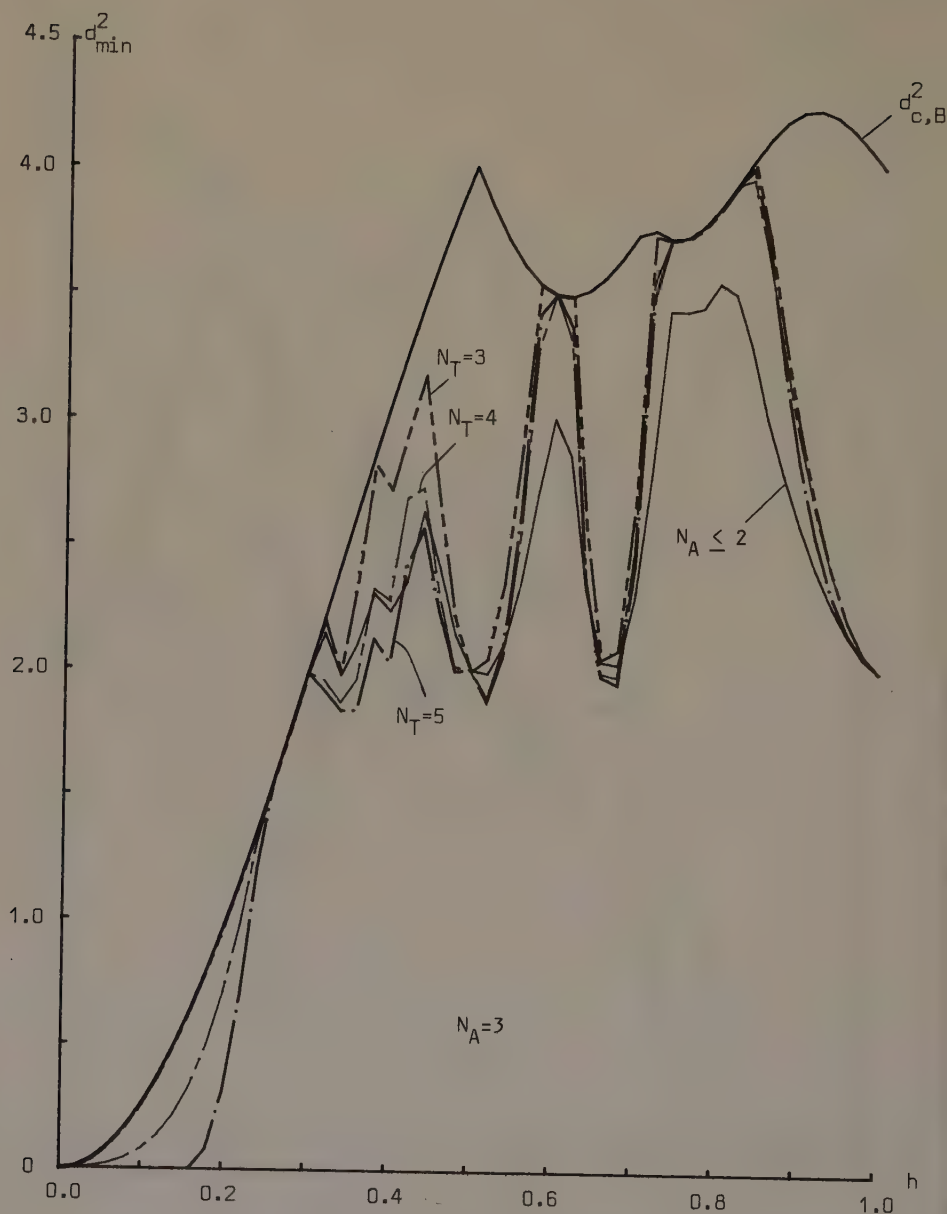


Figure 22. Minimum Euclidean distance for a quaternary 1REC AMF receiver with $N_A=3$ as a function of h and N_T . The upper bound for the optimal receiver and maximum on $d_{\min}^2$ for the AMF receivers with $N_A \leq 2$ are shown as comparison.

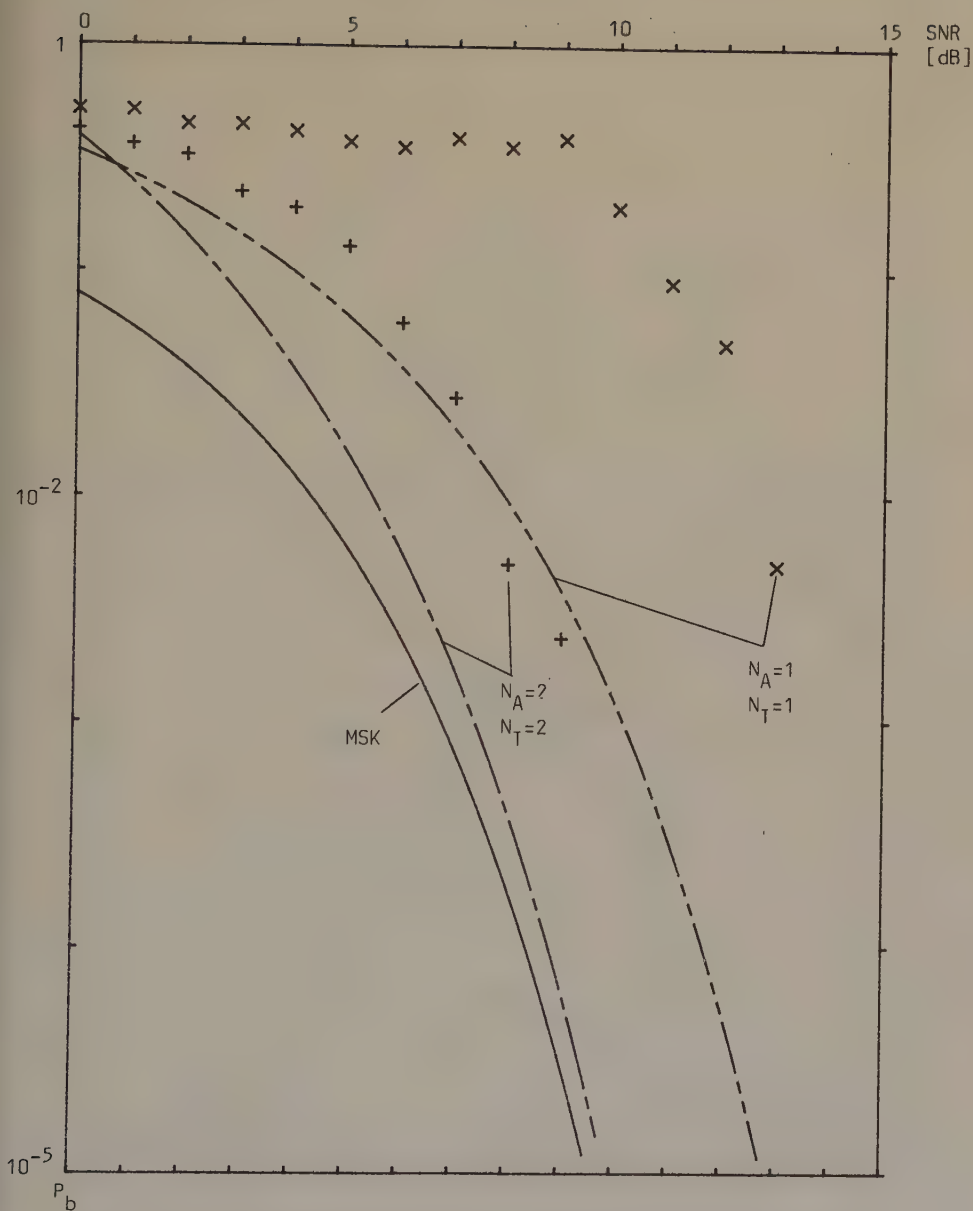


Figure 23. Simulated error probability and calculated upper bound for quaternary 1REC with $h=0.3$. The receivers are given above.

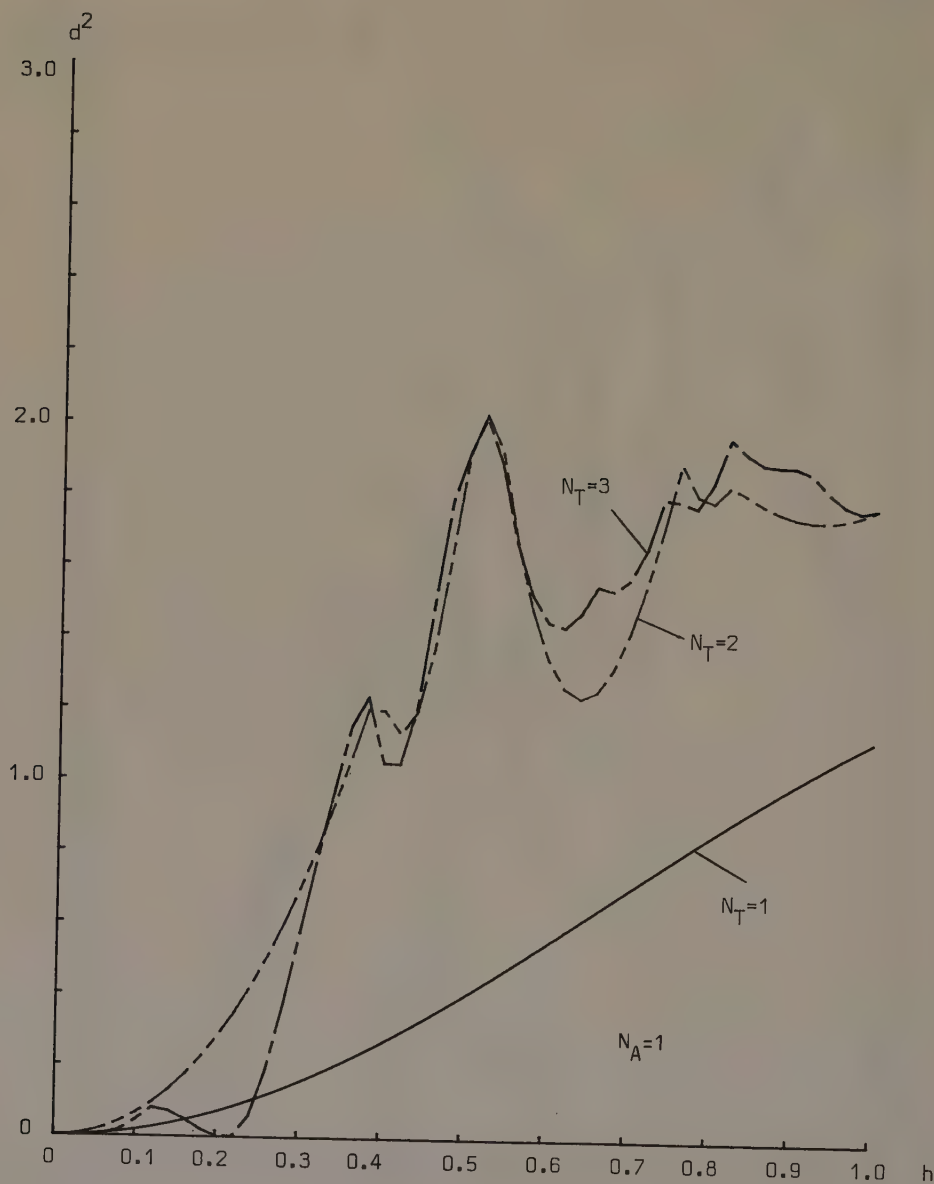


Figure 24. Minimum normalized squared Euclidean distance for a quaternary 2RC AMF receiver with $N_A=1$ as a function of h and N_T .

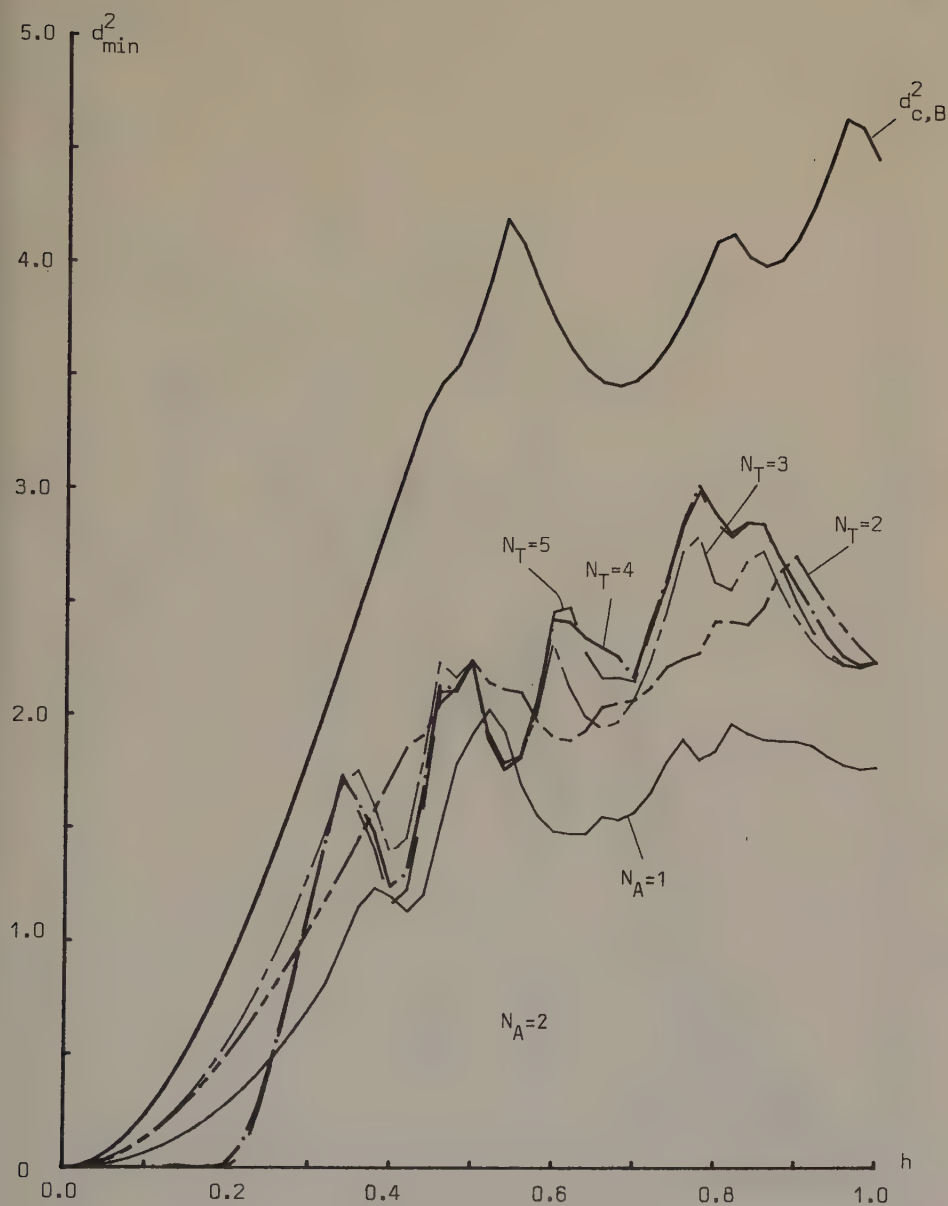


Figure 25. Minimum Euclidean distance for a quaternary 2RC AMF receiver with $N_A=2$ as a function of h and N_T . The upper bound for the optimal receiver and maximum on $d_{\min}^2$ for the AMF receiver with $N_A=1$ and $N_T \leq 4$ are also shown.

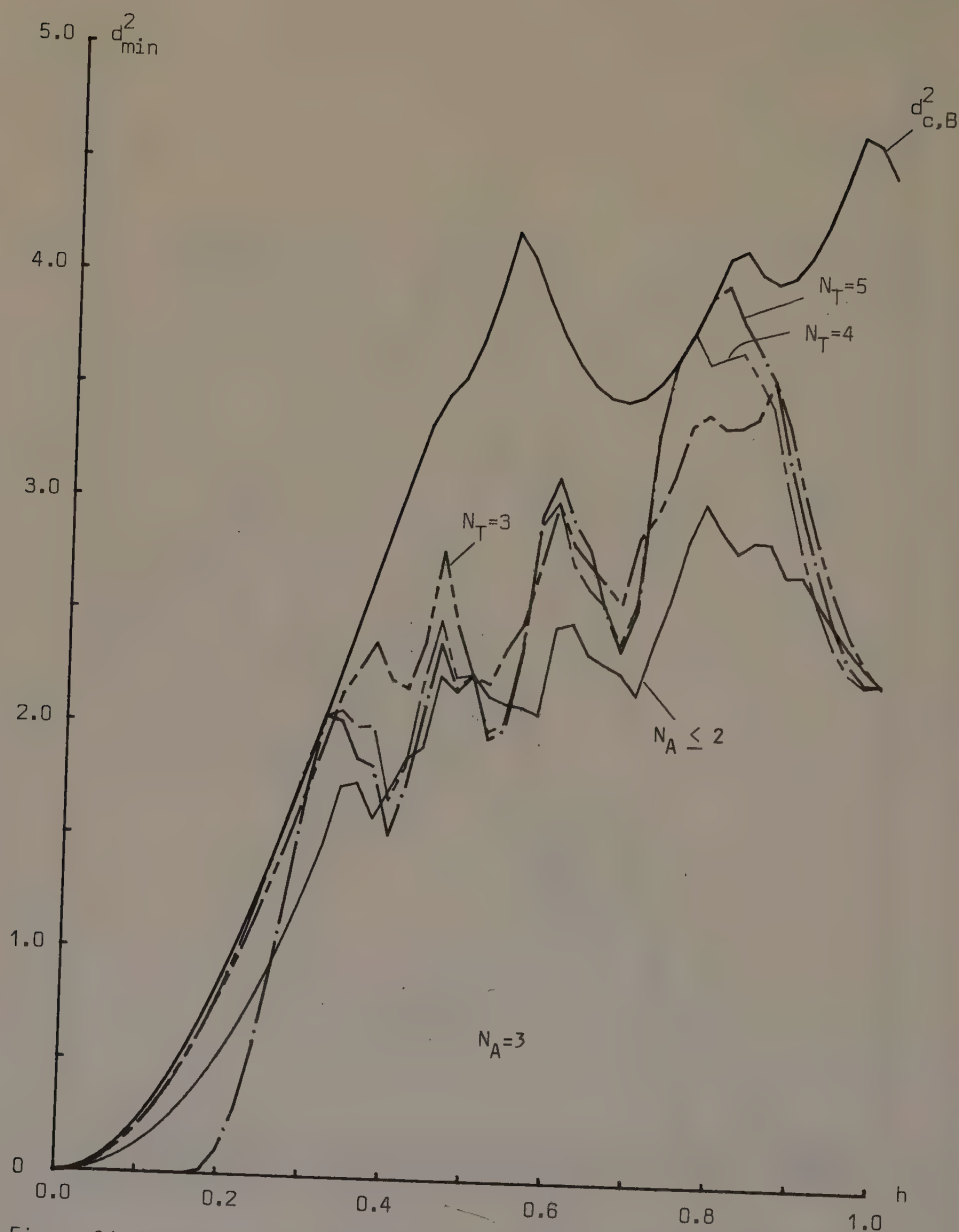


Figure 26. Minimum Euclidean distance for a quaternary 2RC AMF receiver with $N_A=3$ as a function of h and N_T . The upper bound for the optimal receiver and maximum on $d_{\min}^2$ for the AMF receivers with $N_A \leq 2$ are shown as comparison.

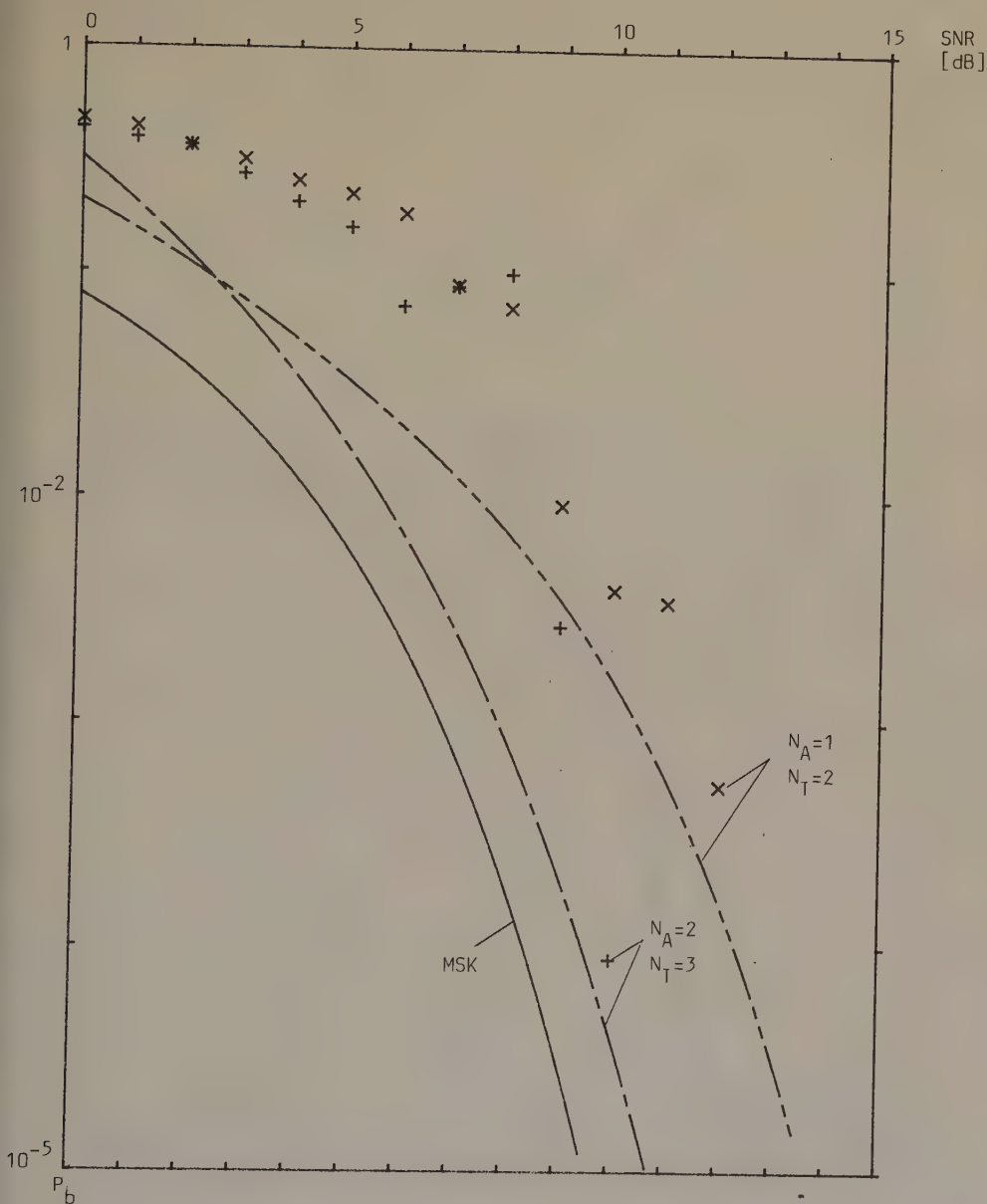


Figure 27. Simulated error probability and calculated upper bound for quaternary 2RC with $h=0.3$. The receivers are given above.

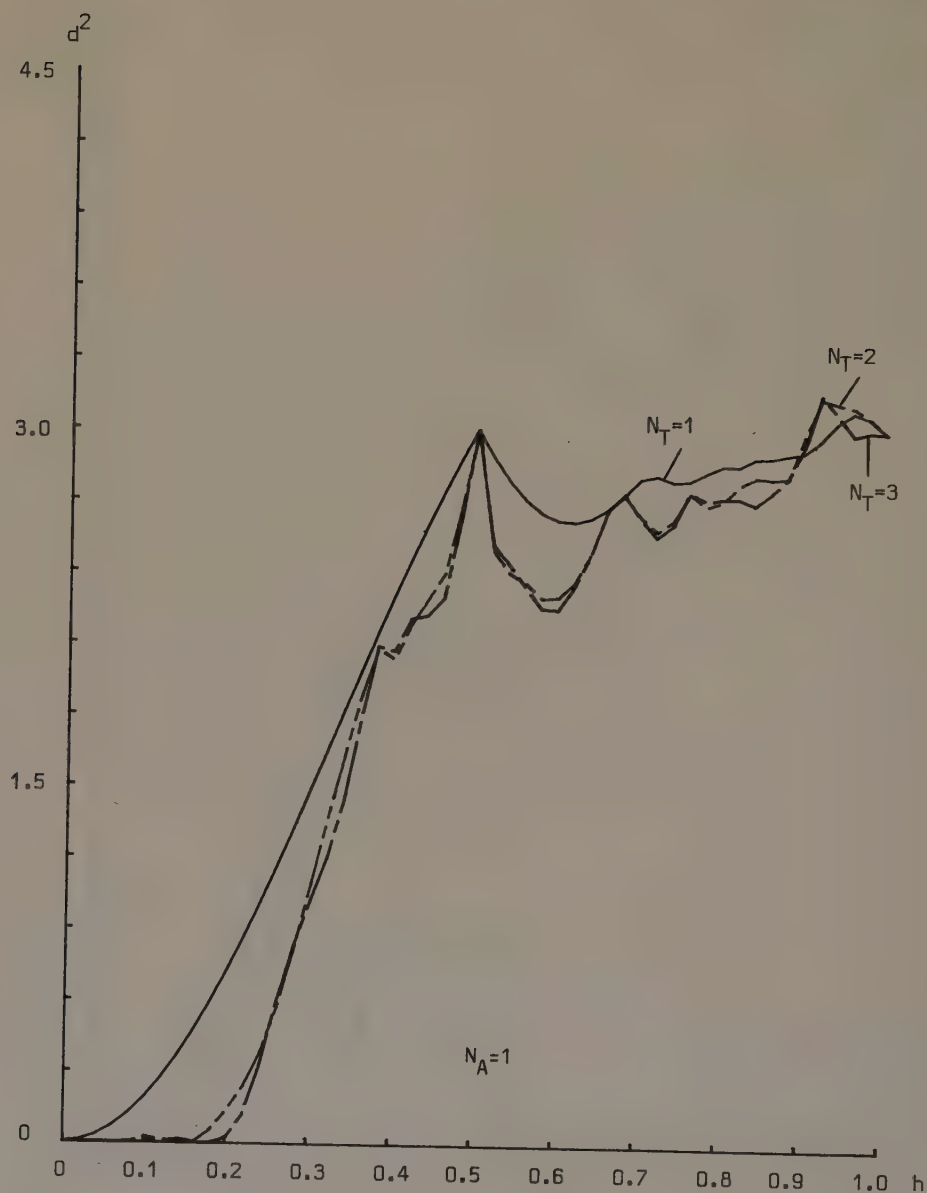


Figure 28. Minimum normalized squared Euclidean distance for a 1REC AMF receiver with $N_A=1$, when $M=8$, as a function of h and N_T .

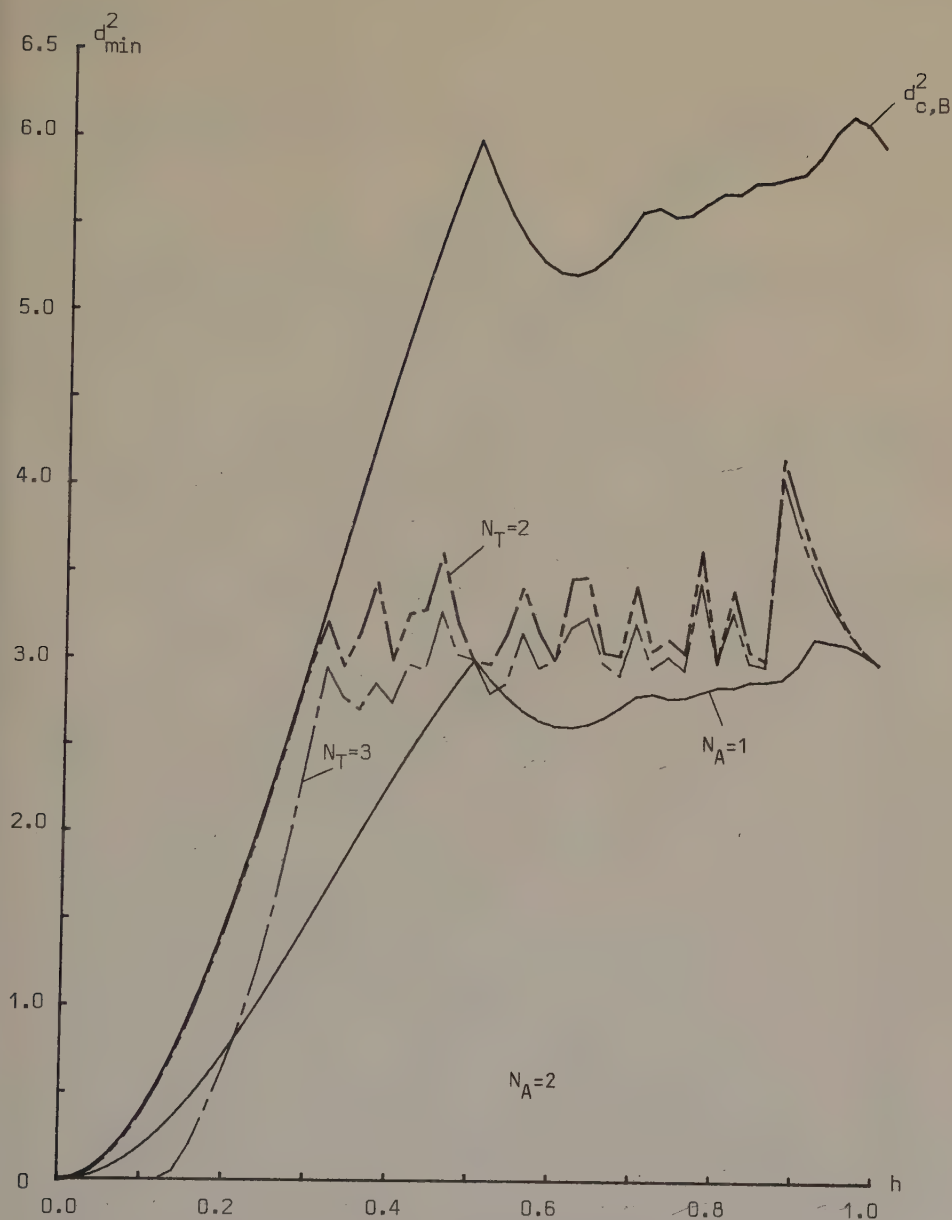


Figure 29. Minimum Euclidean distance for an octal 1REC AMF receiver with $N_A=2$ as a function of h and N_T . The upper bound for the optimal receiver and maximum on $d_{\min}^2$ for the AMF receiver with $N_A=1$ and $N_T \leq 2$ are shown as comparison.

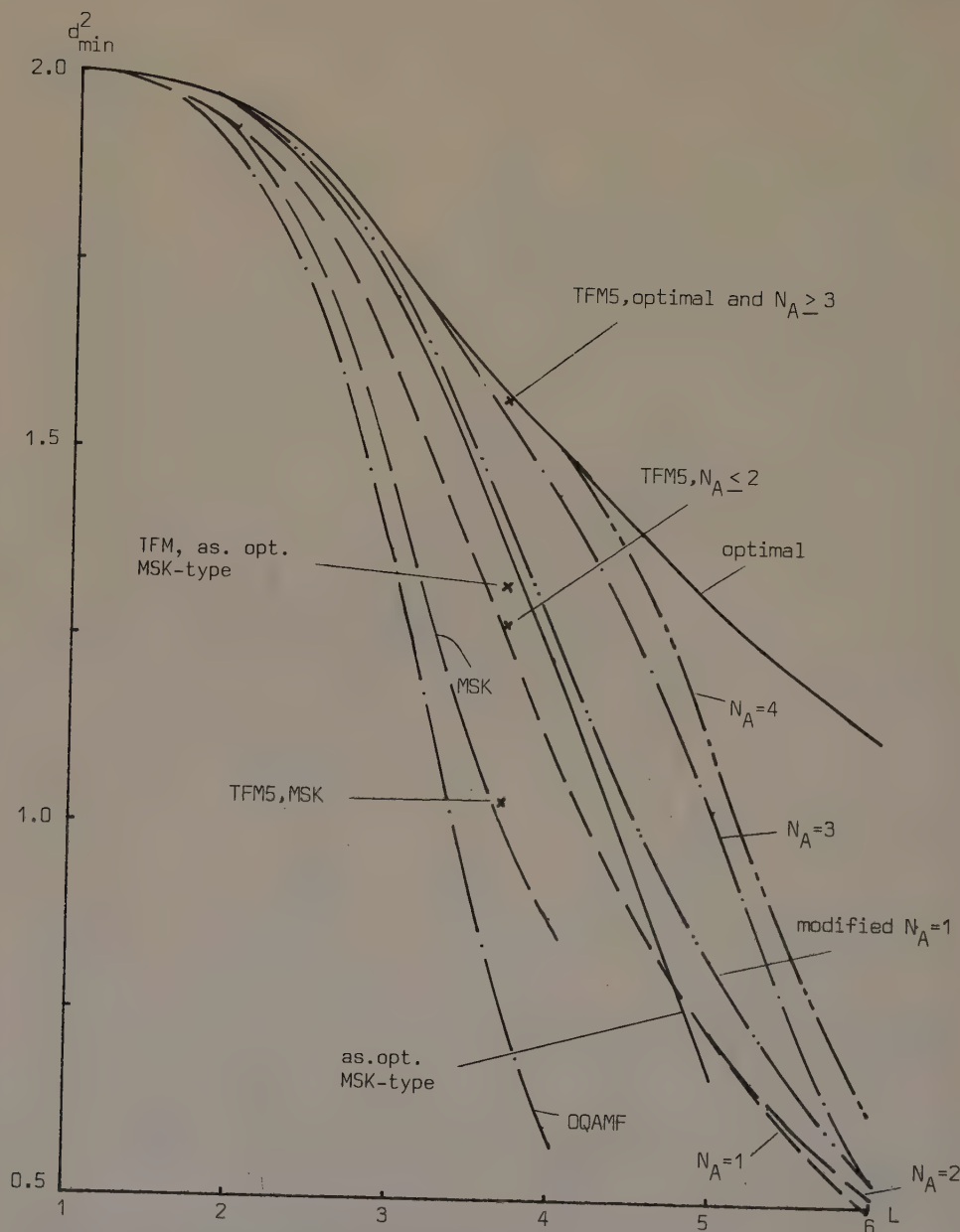


Figure 30. Minimum Euclidean distance versus the frequency pulse duration L , for the binary RC-family, with AMF receivers with N_A fixed symbols. The minimum distance for the optimal receiver and the offset quadrature type of receivers considered in [10]–[12] are also shown as comparison. The points marked with (x) are the distances for a TFM5 scheme with various receivers. The modulation index is $h=1/2$.

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